Student-Focused Assessment Cycle

www.svmimac.org

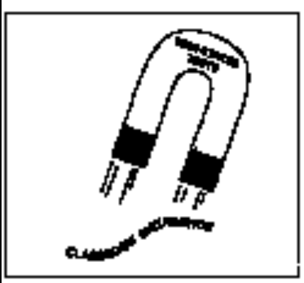
The Silicon Valley Mathematics Initiative

NCSM. Leadership That Inspires Action.

Join NCSM

The state of mathematics education in America

Where are we after a decade of High Stakes Accountability?



WYTIWYG

What You Test Is What You Get!

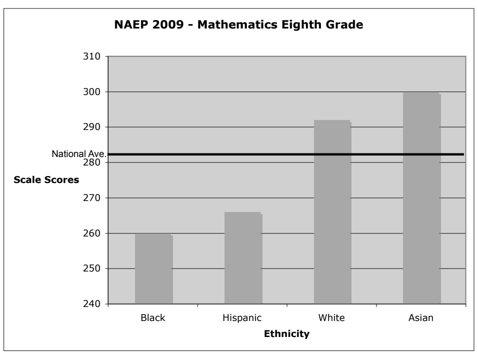
The Achievement Gap

NAEP 2009	US 4th Grade	US 8th Grade
All Students	240	283
Black	222	260
White	248	292
Hispanic	227	266
Asian	255	300
Low Income	228	266
Mid-High Income	250	293
English Learner	218	243
English Fluent	242	284

Source: U.S. Department of Education

National Achievement of Education Progress (NAEP 2009 Math) The Nation's Report Card

Approximately 10 scale points is equivalent to a grade level of learning



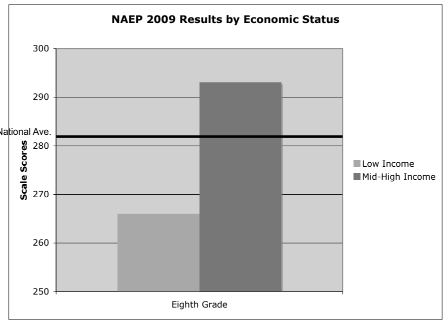
Approximately 10 scale points is equivalent to a grade level of learning

NY TIMES April 29, 2009

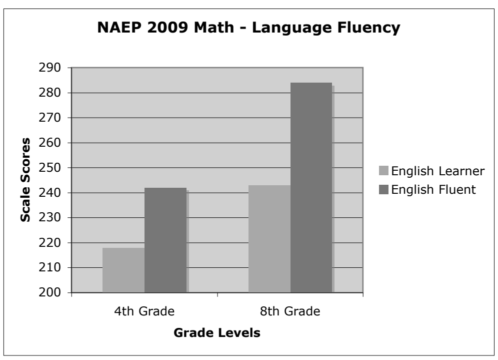
Persistent Racial Gap Seen in Students Test Scores

By SAM DILLON

The achievement gap between white and minority students has not narrowed in recent years, despite the focus of the No Child Left Behind law on improving black and Hispanic scores, according to results of a federal test considered to be the nation's best measure of long-term trends in math and reading proficiency.



Approximately 10 scale points is equivalent to a grade level of learning



Approximately 10 scale points is equivalent to a grade level of learning

NAEP 2009 8th grade

National Ave. 282

Massachusetts (1st) 299

California (49th) 270

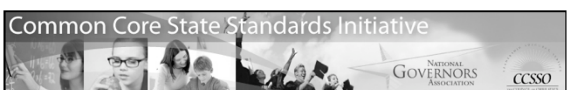
Where you live and your background correlates to how you score on tests.

Link Assessment and Learning

"Assessment should be an integral part of teaching. It is the mechanism whereby teachers can learn how students think about mathematics as well as what students are able to accomplish."

Everybody Counts

A New Direction in Assessment



The Common Core State Standards Initiative is a joint effort by the National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO) in partnership with Achieve, ACT and the College Board. Governors and state commissioners of education from across the country committed to joining a state-led process to develop a common core of state standards in English-language arts and mathematics for grades K-12.These standards will be research and evidence-based, internationally benchmarked, aligned with college and work expectations and include rigorous content and skills. The NGA Center and CCSSO are coordinating the process to develop these standards and have created an expert validation committee to provide an independent review of the common core state standards, as well as the grade-by-grade standards. The college and career ready standards are expected to be publicly released in September 2009. The grade-by-grade standards work is expected to be completed in December 2009.

National Governors Association

CCSSO
The Council of Chief State School Officers

Clearer, Fewer and Higher

Number. Procedural fluency in operations with real numbers and strategic competence in approximation are grounded in an understanding of place value. The rules of arithmetic govern operations and are the foundation of algebra.

Expressions. Expressions use symbols and efficient notational conventions about order of operations, fractions and exponents to express verbal descriptions of computations in a compact form.

Equations. An equation is a statement that two expressions are equal, which may result from expressing the same quantity in two different ways, or from asking when two different quantities have the same value. Solving an equation means finding the values of the variables in it that make it true.

Functions. Functions describe the dependence of one quantity on another. For example, the return on an investment is a function of the interest rate. Because nature and society are full of dependencies, functions are important tools in the construction of mathematical models.

Quantity. A quantity is an attribute of an object or phenomenon that can be measured using numbers. Specifying a quantity pairs a number with a unit of measure, such as 2.7 centimeters, 42 questions or 28 miles per gallon.

Modeling. Modeling uses mathematics to help us make sense of the real world—to understand quantitative relationships, make predictions, and propose solutions.

Shape. Shapes, their attributes, and the relations among them can be analyzed and generalized using the deductive method first developed by Euclid, generating a rich body of theorems from a few axioms.

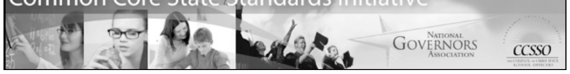
Coordinates. Applying a coordinate system to Euclidean space connects algebra and geometry, resulting in powerful methods of analysis and problem solving.

Probability. Probability assesses the likelihood of an event. It allows for the quantification of uncertainty, describing the degree of certainty that an event will happen as a number from 0 through 1.

Statistics. We often base decisions or predictions on data. The decisions or predictions would be easy to make if the data always sent a clear signal, but the signal is usually obscured by noise. Statistical analysis aims to account for both the signal and the noise, allowing decisions to be as well informed as possible.

http://www.corestandards.org/Standards/index.htm

Common Core State Standards Initiative



Race to the Top

Now 2 of 16 Finalist States for Phase 1

Colorado, Delaware, District of Columbia, Florida, Georgia, Illinois, Kentucky, Louisiana, Massachusetts, New York, North Carolina, Ohio, Pennsylvania, Rhode Island, South Carolina and Tennessee

ED.gov

Goals of Assessment

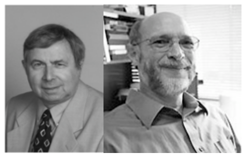
"We must ensure that tests measure what is of value, not just what is easy to test. If we want students to investigate, explore, and discover, assessment must not measure just mimicry mathematics."

Everybody Counts

WYTIWYG

Gareth Mills at QCA

What you TEST is What you GET!



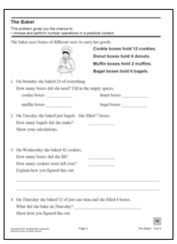
BILL & MELINDA GATES foundation

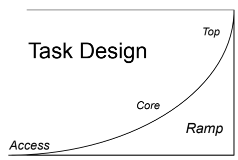
MARS Team
Mathematics Assessment Resource Service

Next Generation MARS Tasks - Formative Assessment Lesson Units and Professional Development Modules

Performance Assessments

To Inform Instruction And Measure Higher Level Thinking





Entry level (access into task)
Core Mathematics - (meeting standards)
Top of Ramp (conceptually deeper, beyond)

- The Mathematics Assessment Resource Service (MARS) is an NSF funded collaboration between U.C. Berkeley and the Shell Centre in Nottingham England.
- The Assessments target grades 2- Geometry and are aligned with the State and NCTM National Math Standards.

SHELL CENTRE BALANCED ASSESSMENT MARS

2009 Comparison Between CST & MARS

Grade 5	MARS 1	MARS 2	MARS 3	MARS 4	Total
Far Below	3.7%	0.9%	0.1%	0.0%	4.7%
Below	6.9%	4.6%	1.0%	0.0%	12.5%
Basic	4.3%	8.4%	3.9%	0.5%	17.1%
Proficient	2.2%	7.7%	12.9%	6.2%	29.0%
Advanced	0.2%	1.7%	8.0%	26.5%	36.4%
Totals	17.3%	23.3%	25.9%	33.2%	100%

Grade 5	MARS vStd	MARS^STD	Totals
CST Bas&v	28.8%	5.5%	34.3%
CST Prof&^	11.8%	53.6%	65.4%
Totals	40.6%	59.1%	100%

Trends Grade to Grade

Grade 2	MARS vStd	MARS^STD	Totals
CST Bas&v	18.4%	6.2%	24.6%
CST Prof&^	10.7%	64.8%	75.5%
Totals	29.1%	71.0%	100%

Grade 3	MARS vStd	MARS^STD	Totals
CST Bas&v	16.4%	9.2%	25.6%
CST Prof&^	8.3%	66.1%	74.4%
Totals	24.7%	75.3%	100%

Grade 4	MARS vStd	MARS^STD	Totals
CST Bas&v	16.6%	6.6%	23.2%
CST Prof&^	9.6%	67.4%	77.0%
Totals	26.2%	74.0%	100.2%

Grade 5	MARS vStd	MARS^STD	Totals
CST Bas&v	28.8%	5.5%	34.3%
CST Prof&^	11.8%	53.6%	65.4%
Totals	40.6%	59.1%	100%

Grade 6	MARS vStd	MARS^STD	Totals
CST Bas&v	36.3%	8.3%	44.6%
CST Prof&^	8.6%	46.7%	55.3%
Totals	44.9%	55.0%	100%

Grade 7	MARS vStd	MARS^STD	Totals
CST Bas&v	44.7%	2.7%	47.4%
CST Prof&^	19.0%	33.7%	52.7%
Totals	63.7%	36.4%	100%

Grade 8	MARS vStd	MARS^STD	Totals
CST Bas&v	64.1%	1.0%	65.1%
CST Prof&^	30.9%	6.0%	36.9%
Totals	95.0%	7.0%	102%

Algebra 1	MARS vStd	MARS^STD	Totals
CST Bas&v	37.3%	4.1%	41.4%
CST Prof&^	22.8%	35.6%	58.4%
Totals	60.1%	39.7%	100%

Geometry	MARS vStd	MARS^STD	Totals
CST Bas&v	4.4%	0.0%	4.4%
CST Prof&^	21.4%	77.8%	99.2%
Totals	25.8%	77.8%	104%

40 states have signed on to develop performance assessment for state accountability in a RttT education consortium.

Stanford University School of Education

Linda Darling Hammond

BILL & MELINDA GATES foundation

Formative Assessment

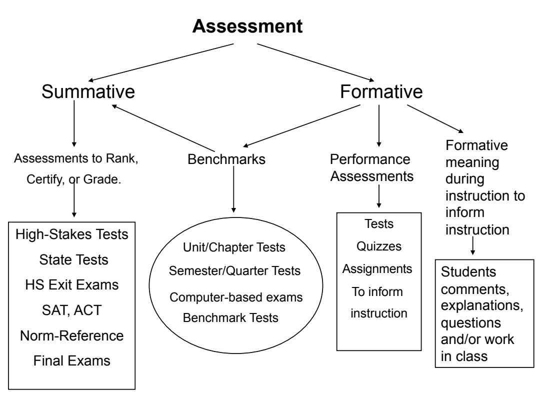
Looking at Student Work

The process of studying student work is a meaningful and challenging way to be data-driven, to reflect critically on our instructional practices, and to identify the research we might study to help us think more deeply and carefully about the challenges our students provide us. Rich, complex work samples show us how students are thinking, the fullness of their factual knowledge, the connections they are making. Talking about them together in an accountable way helps us to learn how to adjust instruction to meet the needs of our students.

Annenberg Institute of School Reform

Educational Research: Formative Assessment and Student Work to Inform Instruction

- *Assessing Student Outcomes*; Marzano, Pickering, McTighe
- *Inside the Black Box*; Black,Wiliams
- *Understanding by Design*; Wiggins, McTighe
- *Results Now*; Schmoker
- *Professional Learning Communities at Work*; Dufour, Eaker
- *Accountability for Learning*; Reeves
- *Math Talk Learning Community*; Fuson, et al
- *Normalizing Problems of Practice*; Little, Horn
- *Change the Terms for Teacher Learning*; Fullan
- *Working toward a continuum of professional development*; Loucks-Horsley, et al.



Inside the Black Box

by Paul Black and Dylan Wiliam, *Phi Delta Kappan*, copyright 1998 http://blog.discoveryeducation.com/assessment/files/2009/02/blackbox_article.pdf

Follow up research:

Working Inside the Black Box

Inside the Black Box

Raising Standards Through Classroom Assessment

By Paul Black and Dylan Wiliam

Firm evidence shows that formative assessment is an essential component of classroom work and that its development can raise standards of achievement, Mr. Black and Mr. Wiliam point out. Indeed, they know of no other way of raising standards for which such a strong prima facie case can be made.

RAISING the standards of learning that are achieved through schooling is an important national priority. In recent years, governments throughout the world have been more and more vigorous in making changes in pursuit of this aim. National, state, and district standards; target setting; enhanced programs for the external testing of students' performance; surveys such as NAEP (National Assessment of Educational Progress) and TIMSS (Third International Mathematics and Science Study); initiatives to improve school plan-

PAUL BLACK is professor emeritus in the School of Education, King's College, London, where DYLAN WILIAM is head of school and professor of educational assessment.

Illustration by A. J. Garces

OCTOBER 1998 139

Formative Assessment is:

Students and teachers
Using evidence of learning
To adapt teaching and learning
To meet immediate learning needs
Minute-to-minute and day to day

Dylan Wiliam, University of London

Educational Interventions

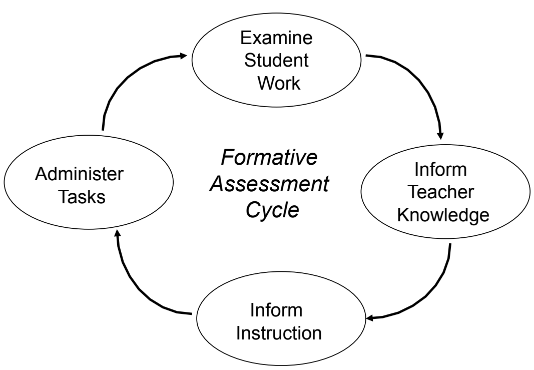
Intervention	Extra months of learning per year	Classroom cost per year
Class-size reduction (from 20 to 30)	3	**$30,000**
Increase teacher content knowledge (2 sd)	1.5	**Unknown**
Formative Assessment	6 to 9	**$3,000**

Black & Wiliam

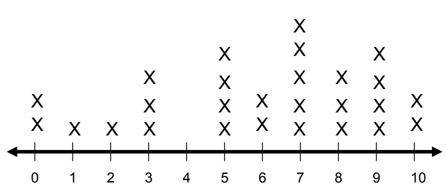
Effective Formative Assessment Strategies

- Clarifying learning intentions and sharing criteria for success
- Engineering effective classroom discussions.
- Providing feedback that moves learners forward.
- Activating students as the owners of their own learning.
- Activating students as instructional resources for one another.

Dylan Wiliam, University of London



The Results from an Assessment



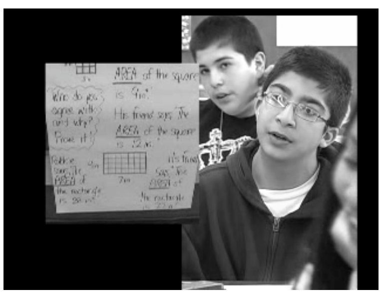
Students' performances are across the continuum

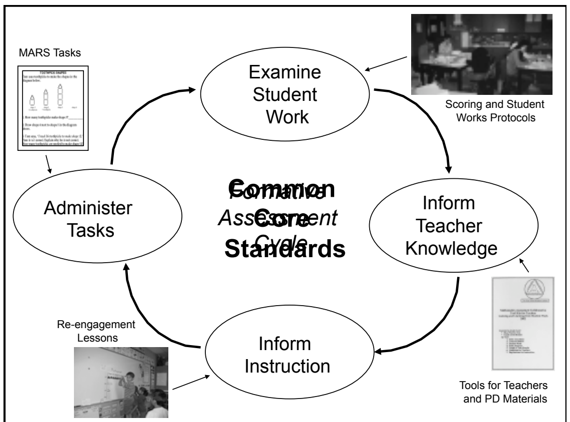
Traditionally Teachers Choose One of Three Options

- Go back and re-teach the topic with the entire class.
- Identify the students needing remediation and find some time/ opportunity to re-teach the topic while the rest of the class continues on.
- Feeling the pressure of the over packed curriculum the teacher ventures on to the next topic.

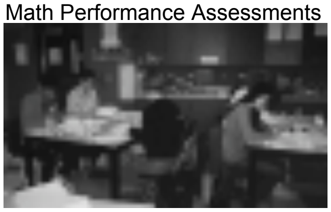
Re-engagement:

Completing the Formative Assessment Cycle





The MAC/MARS
Math Performance Assessments



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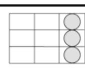
- The Assessments target grades 2- Geometry and are aligned with the State and NCTM National Math Standards.

Candies

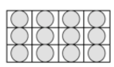
This problem gives you the chance to:
- work with fractions and ratios

1. This is Amy's box of candies.
   She has already eaten 6 of them.



   What fraction of the candies has Amy eaten? ______________

2. Valerie shares some of the 12 candies from this box.
   She gives Cindy 1 candy for every 3 candies she eats herself.



   How many candies does she give to Cindy? ______________
   Show how you figured this out.

3. In a packet of mixed candies there are 2 fruit centers for every 3 caramel centers.
   There are 30 candies in the packet.

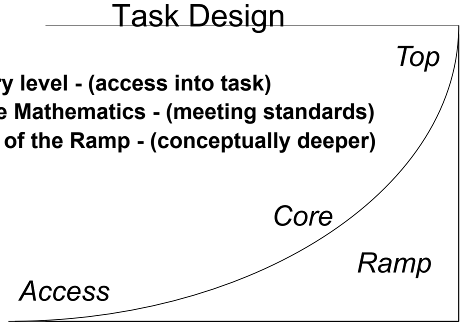
   How many caramel centers are there? ______________
   Show how you figured this out.

4. Anthony makes candies.
   First, he mixes 1 cup of cream with 2 cups of chocolate.
   In all, he uses 9 cups of these two ingredients.
   How many cups of chocolate does he use in this candy recipe? ______________

   Explain how you figured this out.

MARS Performance Assessments
Task Design

Entry level - (access into task)
Core Mathematics - (meeting standards)
Top of the Ramp - (conceptually deeper)



Designing Re-engagement

The Mathematics Assessment Collaborative uses a process of formative assessment that use student work and assessment results to inform instruction and design lesson to re-engage students in learning the mathematics.

Opening- Setting the Stage

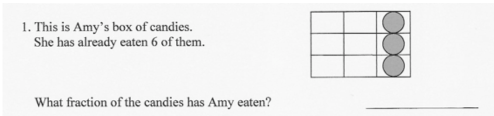
Give student “think” sheets.

“Today we are going to look at the Candies task and different ways student solved the problems.”

Goals:

- Examine how others solved the problems
- Discuss like mathematicians why the approach makes sense
- Listen to see if you hear something that makes you change your mind

Opening question: 6/9
Where's the 6? Where's the 9?



1. This is Amy's box of candies.
She has already eaten 6 of them.

What fraction of the candies has Amy eaten? __________

Another student put 2/3? Could that be correct?

Where do you see the 2? The 3?

Take a minute to think through how a student might answer these questions.

Small Group Discussions

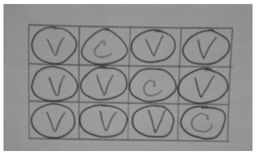
How was the re-engagement activity designed to provide access?

What were the basic concepts the students needed to learn and understand?

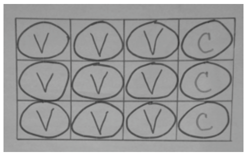
What did the students communicate?

What did the teachers emphasize?

Valerie shares some of the 12 candies. She gives Cindy 1 candy for every 3 candies she eats herself. How many does she give Cindy?

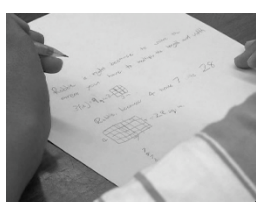


What is the top student thinking? Think, pair, share.



I want to share this problem With a class. Which one is more clear? Why?

Small Group Discussions



How were students' representations used to promote understanding of the mathematics of the task?

What do think was the teacher's purpose in sharing the student work?

Describe evidence of students being instruction resources for one another.

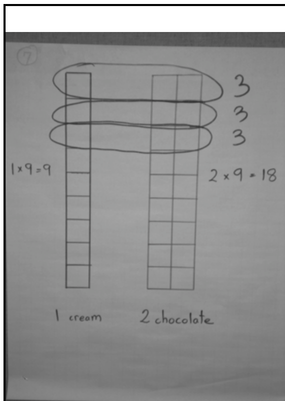
New Recipe

Anthony makes candies.

First, he mixes 1 cup of cream with 2 cups of chocolate.

In all, he uses 9 cups of these 2 ingredients.

How many cups of chocolate does he use in this recipe?



Confronting misconceptions:

Students confronted With misconception. Students asked, "What do you think the student Was thinking? Then: "How could the student use this drawing to get the correct solution?"

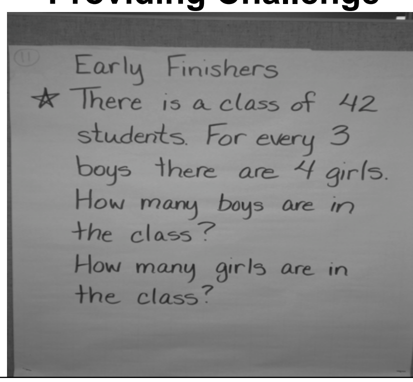
Small Group Discussions

How was the student's work used to deepen the understanding of the core mathematical concepts?

Describe the components of the activity that supported students to work at high cognitive levels?

What did the teachers do to facilitate deeper student thinking?

Providing Challenge



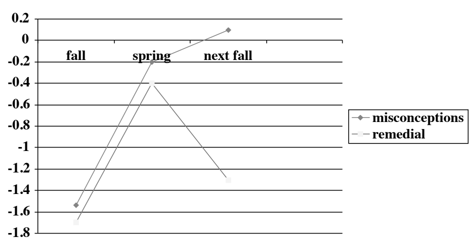
Some Reasons for Using Re-engagement

- Clarify an idea
- Compare strategies and explain why they work
- Make generalizations about types of problems (move away from specific answers to strategies for types of problems)

Some Reasons for Using Re-engagement

- Confronting misconceptions to understand the error in the logic
- Provides immediate specific feedback on student work
- Model qualities or characteristics of desired performance

Bell and Swan study



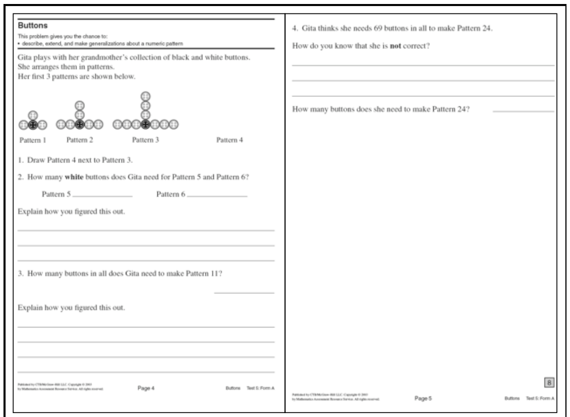
Re-teaching vs. Re-engagement

- Teach the unit again
- Address basic skills that are that are missing.
- Do the same or similar problems over.
- Practice more to make sure student learn the procedures.
- Focus mostly on underachievers.
- Cognitive level are usually lower.

- Revisit student thinking.
- Address conceptual understanding.
- Examine task from different perspective.
- Critique student approaches/solutions to make connections.
- The entire class is engaged in the math.
- Cognitive level are usually higher.

Prepare to Design a Re-engagement

- Do the task and consider the math
- Consider how the students might approach the math, where and how would the be successful, what challenges or misconception may arise?
- Look through the student work. Categorize solution strategies, approaches and where students struggled. What is the story of the task?
- Use the *Tools for Teachers* to compare your findings with the history of the tasks.



Buttons

This problem gives you the chance to:
- describe, extend, and make generalizations about a numeric pattern

Gita plays with her grandmother's collection of black and white buttons.
She arranges them in patterns.
Her first 3 patterns are shown below.

Pattern 1 Pattern 2 Pattern 3 Pattern 4

1. Draw Pattern 4 next to Pattern 3.
2. How many **white** buttons does Gita need for Pattern 5 and Pattern 6?

Pattern 5 ______ Pattern 6 ______

Explain how you figured this out.

3. How many buttons in all does Gita need to make Pattern 11?

Explain how you figured this out.

Page 4 Buttons Test 5: Form A

4. Gita thinks she needs 69 buttons in all to make Pattern 24.

How do you know that she is **not** correct?

How many buttons does she need to make Pattern 24?

8

Page 5 Buttons Test 5: Form A

What is the mathematical story of this task?

- What are the big mathematical ideas in the task?
- What are the themes that emerge from the student work?
- What might be underlying causes for problems?

MARS Task Anticipation Sheet

Task Name:________________ Grade:_____ Year:______ Tot Pts. _____ Core Pts._____

In anticipating the student work where will students show success?

What parts of the task will students be successful?	In terms of knowing and doing mathematics what does this indicate?

In anticipating the student work where will students struggle?

What parts of the task will students be unsuccessful?	In terms of knowing and doing mathematics what does this indicate? What understandings or skills do the students need to learn?

Considering strengths and weaknesses from students, what are plans for future teaching?

What are the implications for future instruction?	What specific instruction or lesson experiences will you design students?

Buttons	Test 5 Form A Rubric	
The core elements of performance required by this task are: • describe, extend, and make generalizations about a numeric pattern Based on these, credit for specific aspects of performance should be assigned as follows:	Points	Section Points
1. Draws correct Pattern 4.	1	1
2. Gives correct answers as: Pattern 5 = **15** white buttons Pattern 6 = **18** white buttons	1 1	
Gives explanation such as: Pattern 5 has 3 rows of 5 white buttons = 15 white buttons and Pattern 6 has 3 rows of 6 white buttons = 18 white buttons.	1	
Special case: Gives answers 16 and 19 with correct explanations including black buttons.	2 s c	3
3. Gives correct answer as: **34**	1	
Gives explanation such as: Pattern 11 has 3 rows of 11 white buttons = 33 white buttons and 1 black button.	1	
Special case: Gives answer as 33 with a correct explanation for the white buttons.	1 s c	2
4. Gives explanation such as: Pattern 24 needs 3 rows of 24 white buttons = 72 white buttons and 1 black button, 73 buttons in all. *Accept alternative correct explanations such as: 69 is divisible by 3, so it cannot be correct.*	1	
Gives correct answer as: **73**	1	2
Total Points		**8**

MARS ANALYSIS

Grade ____ Task _______________ Year______

Range: _______ 1. Fill in the line plot.
Mode: _______ 2. Sort by score.
Median: ______ 3. Look for patterns of understandings and misconceptions for each score.
Mean: _______

Teaching Implications: S tudents know : S tudents are struggling with:

0

<u>What might I keep and/or change in my instruction to address these issues?</u>

Tools for Teachers
Linking Assessment and Learning

"Assessment should be an integral part of teaching. It is the mechanism whereby teachers can learn how students think about mathematics as well as what students are able to accomplish."

Everybody Counts

5^{th} grade **Task 3** **Buttons**

Student Task	Use a button arrangement pattern to describe, extend, and make generalization about its numeric pattern.
Core Idea 3 Patterns and Functions	**Understand patterns and use mathematical models such as algebraic symbols and graphs to represent and understand quantitative relationships.** • Describe and extend numeric patterns (3^{rd} grade) • Represent and analyze patterns and functions using words (4^{th} grade) • Investigate how a change in one variable relates to a change in a second variable

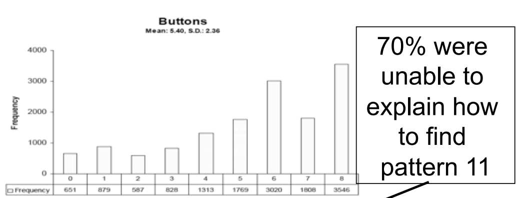
Based on teacher observations, this is what fifth grade students know and are able to do:

- Continue a pattern using pictures and numbers
- Explain how a pattern grows and use that algorithm to solve for larger numbers in the pattern

Areas of difficulty for fifth graders, fifth grade students struggled with:

- Distinguishing between part of a pattern and the whole pattern
- Explaining a pattern in words

Grade 5 – Buttons



Score	0	1	2	3	4	5	6	7	8
Frequency	651	879	587	828	1313	1769	3020	1808	3546

Score:	0	1	2	3	4	5	6	7	8
% < =	4.5%	10.6%	14.7%	20.4%	29.6%	41.9%	62.8%	75.4%	100.0%
% > =	100.0%	95.5%	89.4%	85.3%	79.6%	70.4%	58.1%	37.2%	37.2%

The maximum score available for this task is 8 points.
The cut score for a level 3 response is 4 points.

Most students (about 89%) could draw pattern 4 and give the correct number of buttons for pattern 5 and pattern 6. Many students (about 80%) could draw and extend the pattern and explain in words how the patterned worked. More than half the students (about 60%) could draw the pattern, extend the pattern for 5 and 6 and explain how it grew, find and explain the number of white buttons for pattern 11, and could find the total buttons for pattern number 24. Almost 26% of the students met all the demands of the task. About 5% of the students scored no points on this task.

Questions for Reflection on Buttons:

- What are some rich problems that your students have done this year? What are some good resources for pattern problems?
- Do you ask questions like: "What stays the same?" and "What changes?" to help students develop the ability to form generalizations?
- Do students have opportunities to connect their number sentences to geometric patterns and share how they visualize the growth pattern?

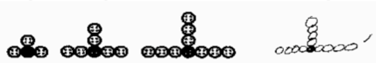
Look carefully at your student work. What strategies did they use in Part 4?

Draw a picture	24 x 3 or 24+24+24	24 x 3 + 1 or 24+24+24+1	Extend a table	Repeated addition of 3	Used a doubling strategy from a previous part of the problem

Teacher Notes:

Looking at Student Work – Buttons

Many students at this grade level are able to see the pattern and form a generalization in words or in number algorithms. These generalizations could easily be converted to algebraic symbols at later grade levels. Student A has a nice description of how the pattern grows and an algorithm for find the total number of buttons in part 4.



Pattern 1 Pattern 2 Pattern 3 Pattern 4

1. Draw Pattern 4 next to Pattern 3.
2. How many **white** buttons does Gita need for Pattern 5 and Pattern 6?

Pattern 5 15 buttons Pattern 6 18 buttons

Explain how you figured this out.

For each pattern there is the same number of white buttons on 3 sides of the black button, so you count the number of buttons on one side of the black buttons and multiply that by three.

3. How many buttons in all does Gita need to make Pattern 11?

34

Explain how you figured this out.

I did the same process in the problem above but I added 1 to my answer for the black button.

Student A, part 2

4. Gita thinks she needs 69 buttons in all to make Pattern 24.

How do you know that she is **not** correct?

Because if you follow the pattern of multipliying 3 to the number of buttons on one side and plus one for black button it doesn't equal 69.

How many buttons does she need to make Pattern 24? 73 buttons

$$\begin{array}{r} 24 \\ \times\ 3 \\ \hline 72 \\ +\ 1 \\ \hline 73 \end{array}$$

For further examples of making generalizations and algorithms, look at the work of Student B and C.

Student B

How do you know that she is **not** correct?

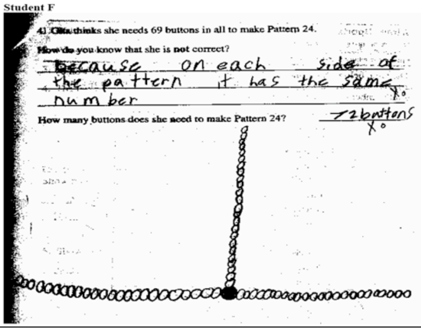
I know because (24X3)+1 does NOT equal 69.

How many buttons does she need to make Pattern 24? 73

$$\begin{array}{r} 24 \\ \times 3 \\ \hline 72 \end{array} +1=73$$

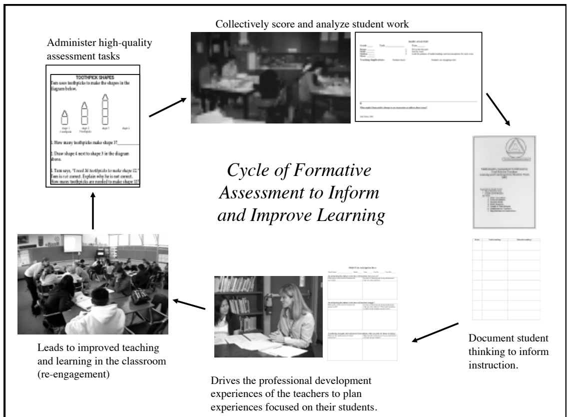
Some students can find the answer or come close to a solution, but use strategies that are inefficient, tedious, and may lead to errors. Student F uses a drawing and counting strategy. Student G does not see the complete pattern and solves for the white buttons every time. Student G uses repeated addition, which is a correct process, but makes a calculation error. Student H uses graphing, but because of the inaccuracies of the graph, the student's answer is incorrect.

Student F



Buttons

Points	Understandings	Misunderstandings
0	Most students attempted this task.	
3	Students could make a drawing of the next pattern in a sequence. They could also find the number of white buttons for pattern 5 and 6.	Students had difficulty explaining how they figured out the white buttons in the pattern. They said things like, "I added", without specifying what or why.
6	Students could find and extend a pattern. To find a solution in part 4,	They did not convert from white buttons to finding the pattern for all buttons in part 3 and 4. 37% of all the students multiplied 24 by 3 in part 4, but did not add on the black button.
7	31% of all students used the expression of 24 times 3 plus 1 to solve for part 4. Another 7% used 24+24+24+1.	Some students used inefficient strategies, which may have led to errors. 13% drew a picture. 8% made a table and extended it, 3% just kept adding 3, and some students drew a graph.
8	Students could continue a pattern using a drawing. They could also find rules for extending it without drawing.	



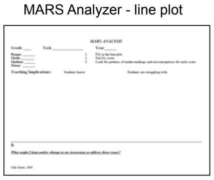
Opportunity to Analyze Student Work

Analyzing Student Work

In each group

1. Do the task
2. From your student work complete MARS Analyzer (line plot and measures of center)
3. Flag interesting Work - unusual strategy, common misconceptions, good answer or "what were they thinking????"
4. Discuss in group and complete *Score Analysis Chart*
5. Complete the MARS Task Analysis Sheet

MARS Analyzer - line plot



Score Analysis Chart

MARS Analysis Sheet

Re-Engagement Protocol

Initial Activity

Describe the 'Story of the Task'

Examine the student work. What important mathematical ideas surfaced in the initial lesson and in students' written work?

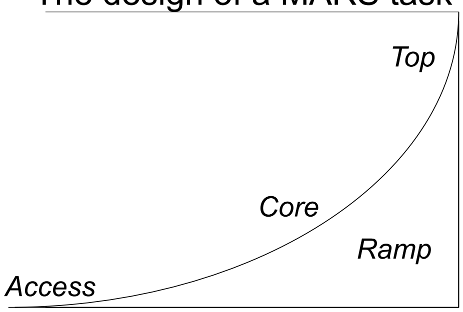
Pick a few key examples from student work

They might represent common strategies, novel approaches, misconceptions, or lead to contradictory answers.

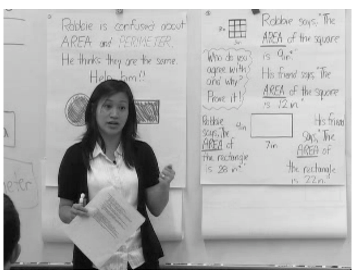
Design a Re-Engagement prompt

Use excerpts from the examples of student work to formulate a question to re-engage all students in the mathematics of the task.

The design of a MARS task



Viewing a Re-engagement Lessons



The Mathematics Assessment Collaborative uses a process of formative assessment that use student work and assessment results to inform instruction and design lesson to re-engage students in learning the mathematics.

Examine Learner A's Work

Learner A

3. How many buttons in all does Gita need to make Pattern 11?

34 Buttons

Explain how you figured this out.

I x (11 x 3) + 1 = 34 buttons
I added one for the black button in the middle

How is Learner A making sense of the mathematics?

Examine Learner B's Work

Learner B

3. How many buttons in all does Gita need to make Pattern 11?

34

Explain how you figured this out.

I added 4 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 34 which is the # of buttons.

How is Learner B making sense of the mathematics?

Small Group Discussions

How was the re-engagement lesson designed to provide access, address the core, and create high cognitive demands?

How were the students making sense of the mathematics?

How did re-engagement promote student discourse?

Describe important teacher moves that facilitated the discourse.

Re-engagement

- Makes use of actual student work - including unique thinking, misconceptions and strategies.
- Has all students re-work a task from different perspectives.
- Confronts misconceptions, so that they can be dealt with and let go.
- Gives some students strategies for solving problem
- Helps other students solidify, connect, and clarify their ideas.

Effective Formative Assessment Strategies

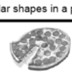
- Clarifying learning intentions and sharing criteria for success
- Engineering effective classroom discussions.
- Providing feedback that moves learners forward.
- Activating students as the owners of their own learning.
- Activating students as instructional resources for one another.

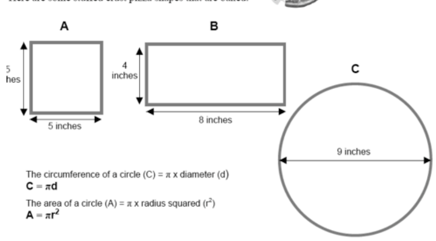
Dylan Wiliam, University of London

Pizza Crusts

This problem gives you the chance to:
- find areas and perimeters of rectangular and circular shapes in a practical context

Robbie loves the stuffed crusts on pizzas.
Here are some stuffed crust pizza shapes that are baked.





The circumference of a circle (C) = π x diameter (d)
C = πd
The area of a circle (A) = π x radius squared (r^2)
A = πr^2

1. How many inches of stuffed crust are put around the edge of each of these pizzas?

A __________ inches **B** _________ inches **C** _________ inches

2. Here is a square pizza with an area of 36 square inches.

36 square inches

(a) What length of stuffed crust will be around the edge?

__________ inches

(b) Design two rectangular pizzas, each with an area of 36 square inches, with different perimeters, so that Robbie will have more crust than on the square pizza.
In each case calculate what the perimeter will be.

Pizza 1 Pizza 2

Perimeter of Pizza 1 _________ inches Perimeter of Pizza 2 _________ inches

3. What is the circumference of a round pizza with an area of 36 square inches?

_____________ inches

Explain how you figured this out.

Pizza Crusts Rubric

The core elements of performance required by this task are: • find areas and perimeters of rectangular and circular shapes in a practical context Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answers: A: **20** inches and shows work such as: 5 x 4 B: **24** inches and shows work such as: 8 x 2 plus 4 x 2 C: **28.3** inches accept 28 - 29 and shows work such as: 9 x π = *Partial credit* Three correct answers –no work shown	 1 1 1 (1)	3
2. (a) Gives correct answer: **24** inches (b) Labels a rectangular pizza with dimensions such as: 12 x 3 = 36 This has a perimeter of 30 inches. 9 x 4 = 36 This has a perimeter of 26 inches.	1 1 1	3
3. Gives correct answer: **21.4** inches (accept **21** inches) Gives correct explanation such as: If $\pi r^2 = 36$ $r = 3.4$ $C = \pi \times 2 \times 3.4$ $= 21.4$ *Partial credit* Finds radius r = 3.4	1 1 (1)	2
Total Points		**8**

Tools for Teachers

7th Grade **Task 3** **Pizza Crusts**

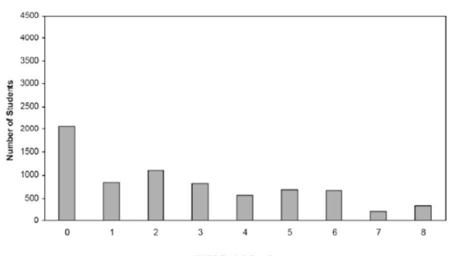
Student Task	Find areas and perimeters of rectangular and circular shapes in a practical context.
Core Idea 4 Geometry and Measurement	**Apply appropriate techniques, tools, and formulas to determine measurements.**

Based on teacher observation, this is what seventh graders knew and were able to do:
- Find the perimeter of a square and rectangle
- Work backwards from the area of a square to the perimeter

Areas of difficulty for seventh graders:
- Distinguishing between area and perimeter in a practical context
- Understanding radius and diameter when solving a problem
- Distinguishing between area and circumference in a practical context
- Working backwards
- Finding square roots
- Designing a rectangle with a given area

Strategies used by successful students:
- Labeling answers or making diagrams to show what was needed
- Skills at transforming equations to work backwards
- Writing equations and showing substitution

7th Grade Pizza Crust 2006



The maximum score available for this task is 8 points.
The minimum score for a level 3 response, meeting standards, is 4 points.

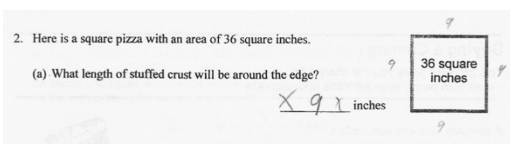
7th Grade – Task 3: Pizza Crusts
Work the task and examine the rubric.
What do you think are the key mathematics the task is trying to assess?

Look at student work in part 1a and 1b. How many of your students are confusing
area and perimeter?

Now look at student work for 2a.
• How many of your students confused 36 sq. in. for the perimeter instead of the area? What would their answer have been?
• How many of your students put the length of one edge instead of the total length of the crust?

Look at the work for designing a rectangle.
• How many of your students gave dimensions that would yield an area different than 36 sq. inches?
• How many of your students gave dimensions that would yield a perimeter of 36 inches?
• How many of your students gave dimensions that would not create a rectangle?
• What other types of errors did you find?

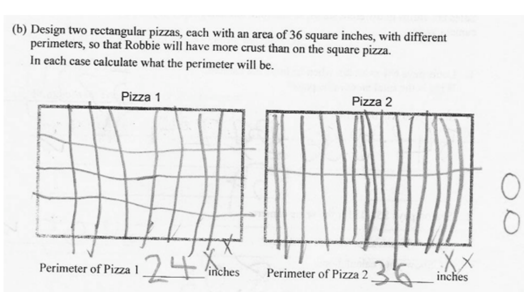
Student G makes the common error of confusing area and perimeter in part 2a.


2. Here is a square pizza with an area of 36 square inches.

(a) What length of stuffed crust will be around the edge?

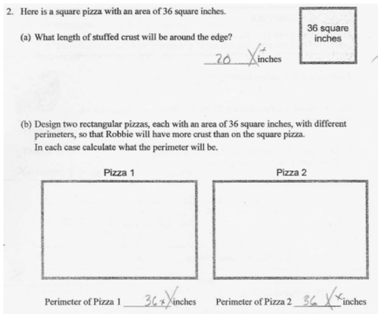
X 9 X inches

Student H draws in the units to find the perimeter in part 2b. However, for both rectangles the area is 32 sq. in. instead of 36 square in.


(b) Design two rectangular pizzas, each with an area of 36 square inches, with different perimeters, so that Robbie will have more crust than on the square pizza.
In each case calculate what the perimeter will be.

Perimeter of Pizza 1 24 X inches Perimeter of Pizza 2 36 XX inches

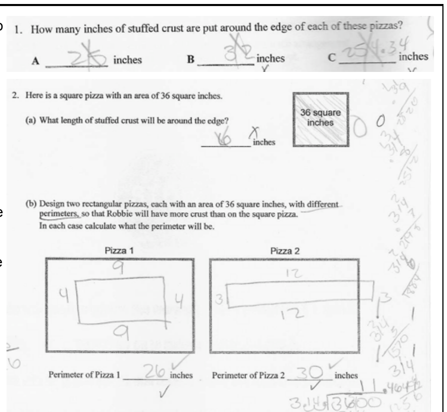
Student J treats all geometric shapes as squares for finding the perimeter. The thinking stays consistent even in part 3. What does it take to help shake a student from a misconception? Notice that the student confuses area and perimeter in part 2b and just gives a perimeter of 36in. with no dimensions.



Student K tries to find the area of all the pizzas in part 1 instead of the perimeter.

In part 2a the student finds the length of one edge instead of the total length of the edge.

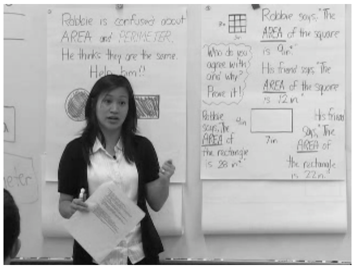
But in part 2b the students is successful at finding the dimensions of two rectangles with same areas and different perimeters.



Pizza Crusts

Points	Understandings	Misunderstandings
0	90% of the students with this score attempted the task.	Many students found area instead of perimeter in part one of the task. Many students lost points for not showing their work. Some students tried to multiply the length by 4 to find the perimeter of a rectangle.
2	Students could find the perimeter of a square and a rectangle and show their work.	Students had difficulty finding the circumference of the circle. 5% thought the circumference was 9 in. (the same as the diameter). 6% multiplied the diameter by 2 (confusing diameter and radius). 3% continued the pattern of 1A, multiplying by 4.
4	Students could find the perimeter for squares and rectangles, given the dimensions or working backwards from the area.	17% of the students put the length of the side, 6 in., instead of the total length of the crust. 15% of the students divided the area by 4, probably confusing area with perimeter. 5% thought the perimeter was 36, the same as the area. Students also had trouble designing rectangles with an area of 36.
6	Students could find the perimeter for squares and rectangles, given the dimensions or working backwards from the area. They could also design a rectangle with a given area, give the appropriate dimensions and calculate the area.	Many students designed rectangles with an area of 32 and gave the dimensions for those rectangles. A few students gave 4 different side lengths for a rectangle. Some students gave perimeters with a length of 36 inches.
8	Students could find the perimeter given the dimensions or working backwards from the area. They could also design a rectangle with a given area, give the appropriate dimensions and calculate the area. They could also work backwards from the area to find the circumference of a circle.	Students had difficulty working backwards from the area of a circle to the circumference of the circle. Some students, 5%, stopped with just the r^2, 11. 46. 8% used the 36sq. in. as a diameter to calculate the circumference. Almost 20% did not attempt this part of the task.

Viewing a Re-engagement Lessons



The Mathematics Assessment Collaborative uses a process of formative assessment that use student work and assessment results to inform instruction and design lesson to re-engage students in learning the mathematics.

Re-engagement Happens "Live"

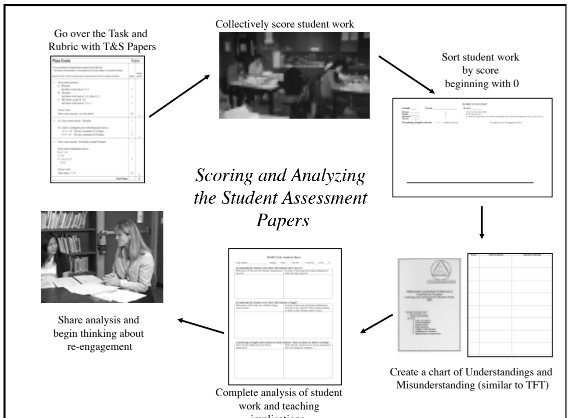
- The heart of the process is in the discussion, controversy, and convincing of the big mathematical ideas.
- This is where students have the opportunity to clarify their own thinking, confront their misconceptions to see the errors in logic, use mathematical vocabulary for a purpose, and make generalizations and connections.

Re-engagement

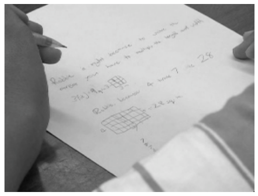
- Has all students re-work task from different perspectives.
- Confronts misconceptions, so that they can be dealt with and let go.
- Gives some students strategies for solving problem
- Helps other students solidify and clarify their ideas.

Re-engagement

- Use student work and their thinking to surface understanding and misconceptions.
- Use student work to address access into basic concepts and foundational understanding.



Small Group Discussions



What is the core mathematics of the task?

What experiences in the lesson focused learning around the core mathematics?

How was student work used to promote understanding of the core mathematics?

Describe evidence of students demonstrating that understanding. Where are students still struggling?

Re-engagement

- Use student work to confront common misconceptions.
- Use student work to apply or adapt new or unfamiliar strategies.
- Use student work to “debug” unsuccessful approaches or flawed reasoning.

In order to learn core concepts

Small Group Discussions

- Describe the components of the lesson that supported students to work at high cognitive levels?
- What aspects of the lessons encouraged students to make connections?
- How did students demonstrate deeper understanding of the mathematical concepts?
- What did the teachers do to facilitate deeper student thinking?

Re-engagement

- Use student work to critique other students' solutions and processes in order to deepen understanding and ramp up the cognitive load of the task.

MARS Task Anticipation Sheet

Task Name:________________ Grade:_____ Year:______ Tot Pts. _____ Core Pts.______

In anticipating the student work where will students show success?

What parts of the task will students be successful?	In terms of knowing and doing mathematics what does this indicate?

In anticipating the student work where will students struggle?

What parts of the task will students be unsuccessful?	In terms of knowing and doing mathematics what does this indicate? What understandings or skills do the students need to learn?

Considering strengths and weaknesses from students, what are plans for future teaching?

What are the implications for future instruction?	What specific instruction or lesson experiences will you design students?

MARS Task Analysis Sheet

Task Name:________________ Grade:_____ Year:______ Tot Pts. _____ Core Pts._____

In analyzing the student work where did students show success?

What parts of the task did students demonstrate success?	In terms of knowing and doing mathematics what does this indicate?

In analyzing the student work where did students struggle?

What parts of the task were students being unsuccessful?	In terms of knowing and doing mathematics what does this indicate? What understandings or skills do the students need to learn?

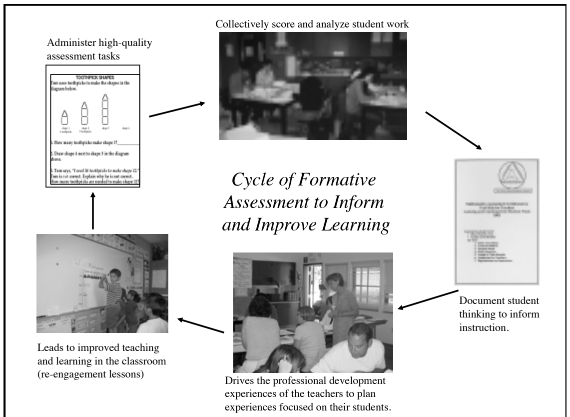
Considering strengths and weaknesses from students, what are plans for future teaching?

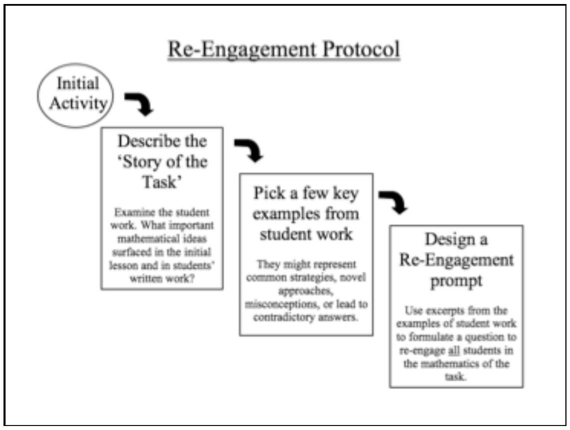
What are the implications for future instruction?	What specific instruction or lesson experiences will you design students?

Some of the Purpose of Re-engagement

Re-engagement lessons may:

- Make use of actual student work - including unique thinking, misconceptions and strategies.
- Have all students re-work a task from different perspectives.
- Confront misconceptions, so that they can be dealt with and let go.
- Give some students strategies for solving problem
- Help other students solidify, connect, and clarify their ideas.
- Allow students to critique solutions and processes in order to deepen understanding and raise the cognitive load of the task.





Design a Re-engagement Lesson

- What are the foundational concepts that need to be solidified?
- What examples of student work or errors could be presented?
- What approaches or strategies are unique or present a reasoning dilemma? What student work might be shared?
- What conceptual ideas do you want students to learn or connect? What student work would engage the students and invite high cognition.

Re-engagement Happens "Live"

- The heart of the process is in the discussion, controversy, and convincing of the big mathematical ideas.
- This is where students have the opportunity to clarify their own thinking, confront their misconceptions to see the errors in logic, use mathematical vocabulary for a purpose, and make generalizations and connections.

Re-engagement

- Makes use of actual student work - including unique thinking, misconceptions and strategies.
- Has all students re-work a task from different perspectives.
- Confronts misconceptions, so that they can be dealt with and let go.
- Gives some students strategies for solving problem
- Helps other students solidify, connect, and clarify their ideas.

Small Group Discussions

How was the re-engagement lesson designed to provide access?

What were the basic concepts the students needed to learn and understand?

How were the students making sense of the mathematics?

What did the students communicate? What did the teachers emphasize?

Designing a Lesson

- What makes a good opening?
- What is important about this piece?
- How does it lay a foundation to bring all students along?

Re-engagement

- Give ourselves permission to spend more time on a problem and its discussion.
- Give students the opportunity to really examine the mathematics and change their ideas through **rich dialogue**.
- Promotes sense-making, justification, making conjectures and testing them.
- Ups the cognitive demand of the task.

Process of Re-engagement

- Give students a purpose for re-examining the work or mathematics of a task by creating a dilemma or cognitive conflict.
- Move students from the process of solving a problem to justification and sense-making. Why did this work? Why doesn't this make sense? Involve them in the discipline of doing mathematics.

Welcome to the Inside Mathematics Website

Welcome to Inside Mathematics, a professional resource for educators passionate about improving students' mathematics learning and performance.
This site features classroom examples of innovative teaching methods and insights into student learning, tools for mathematics instruction that teachers can use immediately, and video tours of the ideas and materials on the site.
Several allied initiatives dedicated to improving math teaching have contributed to this resource. We are glad you're here and look forward to learning with youe

http://www.insidemathematics.org