Common Core State Standards

Mathematics

What difference will it make?

Nothing is easier than sitting around agreeing on what other people should do, especially what other people's children should do. If that's all standards were, we should be ashamed. Standards must also be a promise to students of mathematics they can take with them. We haven't kept our old promise and now we make a new one. What difference will it make?

You are Leaders

- The mistake of the Head
- The mistake of the Tail
- The way of the good Leader

Lessons Learned #1

- After two decades of standards based accountability:
- “cover” at “pace” is a failure
  - Tells teachers to ignore students
  - Turn the page regardless
  - Shrug your shoulders and do what “they” say
  - Mathematics is not a list of topics to cover
- Teach to (what they say is) the test
- Singapore: “Teach less, learn more”

Lessons Learned #2

- TIMSS: math performance in the US is being compromised by a lack of focus and coherence in the “mile wide, inch deep” curriculum
- Hong Kong students outscore U.S. students on the grade 4 TIMSS, even though Hong Kong only teaches about ***half*** of the tested topics. U.S. covers over 80% of the tested topics.
- High-performing countries spend more time on mathematically central concepts: greater depth and coherence.

#3 What does “higher standards” mean?

- more topics? but the U.S. curriculum is already cluttered with too many topics.
- earlier grades? but this does not follow from the evidence. In Singapore, division of fractions: grade 6, whereas in the U.S. : grade 5 (or 4).

Standards are high for what students take away

- In top performing countries.
- U.S. standards often tell teachers what to cover, whether students learn it well or not. The CCSS: tell teachers when to *slow down.*
- What problem does "pacing" solve? What is the theory? Is it that students don't understand because it hasn't been "covered"? Low expectations?

Standards for Mathematical Practice

"Proficient students of all ages expect mathematics to make sense. They take an active stance in solving mathematical problems. When faced with a non-routine problem, they have the courage to plunge in and try something, and they have the procedural and conceptual tools to continue. They are experimenters and inventors, and can adapt known strategies to new problems. They think strategically. "

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Practices: developing expertise

"Students who engage in these practices, individually and with their classmates, discover ideas and gain insights that spur them to pursue mathematics beyond the classroom walls. They learn that effort counts in mathematical achievement. Encouraging these practices in students of all ages should be **as much a goal of the mathematics curriculum as the learning of specific content**."

Common Core State Standards

Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.
2 Reason abstractly and quantitatively.
3 Construct viable arguments and critique the reasoning of others.
4 Model with mathematics.
5 Use appropriate tools strategically.
6 Attend to precision.
7 Look for and make use of structure.
8 Look for and express regularity in repeated reasoning.

6 Reason abstractly and quantitatively.

Mathematically proficient students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Assessing Practices

Depth of practice demanded by a task:

Need assessments that vary in depth of practice from beginner to novice to expert demand.

We are assessing the development of expertise … apprenticeship model…

Like assessing reading above grade 3: the demand of the passages increases, higher complexity, but ‘comprehension’ is comprehension.

Concepts *and* Skills

- Understand and solve
- What does "understand" mean?

Take the number apart?

Tina, Emma, and Jen discuss this expression:

- $6 \times 5\ {}^{1}/_{3}$
- Tina: I know a way to multiply with a mixed number that is different from the one we learned in class. I call my way "take the number apart." I'll show you. First, I multiply the 5 by the 6 and get 30. Then I multiply the 1/3 by the 6 and get 2. Finally, I add the 30 and the 2 to get my answer, which is 32.

Which of the three girls do you think is right?

Tina: It works whenever I have to multiply a mixed number by a whole number.

Emma: Sorry Tina, but that answer is wrong!

Jen: No, Tina's answer is right for this one problem, but "take the number apart" doesn't work for other fraction problems.

Justify your answer mathematically.

Distributive property

5 1/3 = 5 + 1/3

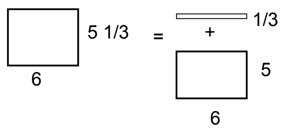
6 x 5 1/3 = 6(5 + 1/3)

6(5 + 1/3) = 6x5 + 6x 1/3

since a(b + c) = ab + ac

Could illustrate with area of a rectangle

6 by 5 1/3



"Understand" is used in these standards to mean that students can

explain the concept with mathematical reasoning, including concrete illustrations, mathematical representations, and example applications.

Students who understand a concept can

(a) use it to make sense of and explain quantitative situations (see "Model with Mathematics" in Practices)

(b) incorporate it into their own arguments and use it to evaluate the arguments of others (see " Construct viable arguments and critique the reasoning of others" in Practices)

(c) bring it to bear on the solutions to problems (see " Make sense of problems and persevere in solving them")

(d) make connections between it and related concepts

“Understand” expectation

At a given grade level, students demonstrate all of a. through d. over the collection of "understand" standards. For any given "understand" standard, students demonstrate at least two of a. through d.

Mental math

Quantity and measurement

- Number line
- Variable
- Co-variation and function

Number and expressions

- Representing numbers
- Letters
- Expressions
- Statements that two expressions refer to the same number

Modeling with functions vs.. solving equations

- From value of an expression to value of a variable
- From value of a variable to value of an expression

Summative and Formative Assessments

Next Steps

Blueprint

- On the next slide

Blueprint: Common Core College Readiness Level Examination % allocation of examination time

Depth of *Mathematical Practice* of the task:		Beginner Tasks	Novice Tasks	Expert Tasks
Number and quantity	5	2	2	1
Algebra	25	10	8	6
Seeing Structure in Expressions	6	2	2	2
Arithmetic with Polynomials and Rational expressions	6	4	2	0
Creating Equations that Describe Numbers or Relationships	6	2	2	2
Reasoning with Equations and Inequalities	7	2	2	3
Functions	25	9	9	6
Interpreting Functions	7	2	3	2
Building Functions	7	2	3	2
Linear, Quadratic, and Exponential models	8	2	3	3
Trigonometric Functions	3	3	0	0
Geometry	15	5	6	4
Statistics and probability	15	5	6	4
Modeling	15	0	6	9

Summative Test: item type by depth of mathematical practice

- **Beginner Items**: **[mc-80, cr-20, ecr-0]** Accessible tasks designed to detect growth in the low end of distribution. Focus on the most accessible important mathematics in the Standards.
- **Novice Items: [mc-50,cr-30,ecr-20]** Tasks are designed to make *mathematical practices*, adaptive reasoning, strategic competence (see *Adding It Up*) useful, while focusing on priority mathematical understandings and skills.
- **Expert Items: [mc-0, cr-40, ecr-60]** *Mathematical practices,* adaptive reasoning, and strategic competence (see *Adding It Up*) are needed to make sense of the task and formulate a mathematically valid response that achieves the goal of the task.

Quantity

- Bodies of water
  - According to the Runners' World: *On average, the human body is more than 50 percent water. Runners and other endurance athletes average around 60 percent. This equals about 120 soda cans' worth of water in a 160-pound runner!*
  - Check out the Runners' World calculation. Are there really about 120 soda cans' worth of water in the body of a 160-pound runner? Here are some facts:
  - A typical soda can holds 12 fluid ounces.
  - 16 fluid ounces (one pint) of water weighs one pound.

Sidewalk patterns

This task embodies key elements of algebra

- Generalization
- Symbolic formulation
- Linear and quadratic scaling

The first version is the “core” task

With some teaching of problem solving strategies

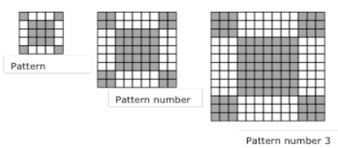
- try simple cases, organize information, find general rules, find a good representation, check,…

this version could be a test task; currently the scaffolded version is more appropriate.

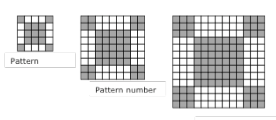
Sidewalk patterns

In Prague some sidewalks are made of small square blocks of stone.

The blocks are in different shades to make patterns that are in various sizes.

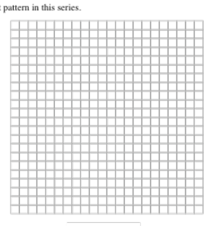


Write formulas for the number W of white tiles, and G of grey tiles, in terms of the pattern number n.



1. Draw the next pattern in this series.

You may not need to use all of the squares on this grid.



2. Complete the table below.

Pattern number, n	1	2	3	4
Number of white blocks	12	40		
Number of gray blocks	13			
Total number of blocks	25			

3. What do you notice about the number of white blocks and the number of gray blocks?

____

4. a. How many blocks in total will pattern number 5 need? ______________

b. How many blocks will pattern number *n* need? ______________

5. a. If you know the total number of blocks in a pattern you can work out the number of white blocks in it. Explain how you can do this.

____

____

b. Pattern number 6 has a total of 625 blocks. How many white blocks are needed for pattern number 6? Show how you figured this out.

Why do students have to do math problems?

Why do students have to do math problems?

1. to get answers because Homeland Security needs them, pronto
2. I had to, why shouldn't they?
3. so they will listen in class
4. to learn mathematics

Why give students problems to solve?

To learn mathematics.

Answers are part of the process, they are not the product.

The product is the student's mathematical knowledge and know-how.

The 'correctness' of answers is also part of the process. Yes, an important part.

Wrong Answers

- Are part of the process, too
- What was the student thinking?
- Was it an error of haste or a stubborn misconception?

Three Responses to a Math Problem

1. Answer getting
2. Making sense of the problem situation
3. Making sense of the mathematics you can learn from working on the problem

Answers are a black hole: hard to escape the pull

- Answer getting short circuits mathematics, making mathematical sense
- Very habituated in US teachers versus Japanese teachers
- Devised methods for slowing down, postponing answer getting

Answer getting vs.. learning mathematics

- USA:

How can I teach my kids to get the answer to this problem?

Use mathematics they already know. Easy, reliable, works with bottom half, good for classroom management.

- Japanese:

How can I use this problem to teach mathematics they don't already know?

Teaching against the test

3 + 5 = [ ]
3 + [ ] = 8
[ ] + 5 = 8

8 - 3 = 5
8 - 5 = 3

N stands for the number of hours of sleep Ken gets each night. Which of the following represents the number of hours of sleep Ken gets in 1 week?

Ⓐ $N + 7$

Ⓑ $N - 7$

Ⓒ $N \times 7$

Ⓓ $N \div 7$

SOURCE: U.S. Department of Education, National Center for Education Statistics, *National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment*, Grade 4, Block Z1M12 #12, 2005.

Anna bought 3 bags of red gumballs and 5 bags of white gumballs. Each bag of gumballs had 7 pieces in it. Which expression could Anna use to find the total number of gumballs she bought?

A (7 X 3) + 5 =
B (7 X 5) + 3 =
C 7 X (5 + 3) =
D 7 + (5 X 3) =