For more information on OGAP:

Visit for more information
http://margepetit.com

Contact:
Marge Petit at mpetit@gmavt.net
Bob Laird at rlaird@uvm.edu

For the Power Point from this session go to:
http://grou.ps/mathedleaders

Read Recent Publications:

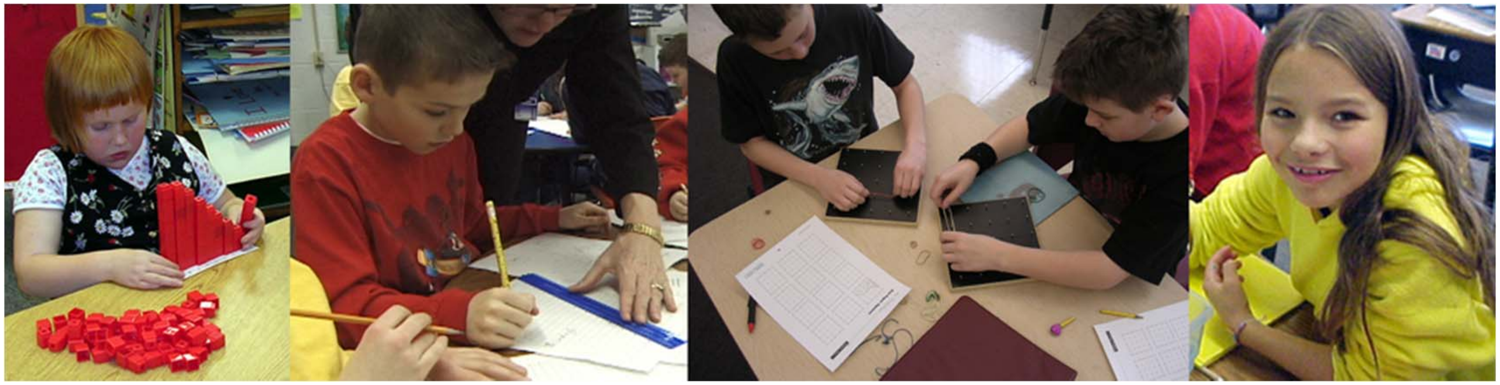
Petit, Laird, and Marsden (2010), A Focus on Fractions: Brining Research to the Classroom. Routledge, New York and London.

Petit, Laird, & Marsden (September, 2010). They get fractions as pies – but now what?. Mathematics in the Middle School, NCTM, Reston, Virginia.

Petit, Zawojewski (2010). Formative Assessment in Elementary Classrooms. Teaching and Learning Mathematics: Translating Research for Elementary School Teachers. NCTM, Reston, VA.

Petit, Zawojewski, Labaddo (2010). Formative Assessment in the Secondary School Classroom. Teaching and Learning Mathematics: Translating Research for Secondary School Teachers. NCTM, Reston, VA.

Petit (2011). Going from Research to Practice: Learning Trajectories in Action. Mathematics Learning Trajectory Report. Consortium for Policy and Research in Education, Teacher's College, Columbia University. Chapter 4.



Facilitating Use of Formative Assessments: Fractions at the Middle Grades – Ongoing Assessment Project

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Marge Petit, Marge Petit Consulting, MPC (mpetit@gmavt.net)

OGAP Sites:

- **Vermont**
- **Alabama**
- **Michigan**
- **Ohio**
- **Amman, Jordan**
- **Soon - Nebraska**

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In the end – it is the evidence of student thinking not just from assessment questions, but also from classroom discussions and activities that informs instructional decision making.

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Take Aways!

- **Teacher knowledge** about the research/learning trajectories is fundamental – this involves a real commitment to PD, NOT just creating tools and materials, but substantive professional development.

- **Evidence of Student Thinking** - it is the evidence of student thinking not just from assessment questions, but from classroom discussions and activities that informs instructional decision making.

- **Formative assessment** is a powerful tool when it is implemented systematically and intentionally coupled with the above.

- **Transitions** - One should not assume that middle school (or high school) students will naturally make the transition from knowledge ABOUT fractions to application in the new mathematical topics and concepts.

- **Students self-assessment is key!**

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In 2 hours…

What can be done ….

- …provide participants with the big idea of OGAP and ***some*** applications

What cannot be done…

- … provide participants with a deep understanding of the details and potential implications of OGAP and the research related to students developing their understanding of fractions
- …be sure that participants understand the difference between formative and summative assessment.

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The VMP Ongoing Assessment Project responds to 2 needs:

- Providing teachers instructional information as students learn, not later.
- To improve student learning in regards to state standards (and now the CCSS)

OGAP Sites:

Vermont
Alabama
Michigan
Ohio
Amman, Jordan
Soon - Nebraska

These needs are shared across the country, not just in Vermont and more.

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OGAP is a systematic and intentional formative assessment system in mathematics.

- Gathering information about pre-existing knowledge through the use of a **pre-assessment**;
- **Analysis of pre-assessment** to guide unit planning; and
- **A continuous and intentional system** of instructing, probing with instructionally embedded questions, analysis, and instructional modification.

Grades 2 - 8

- **Fractions**
- **Multiplicative reasoning**
- **Proportionality**

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In place and in use for all 3 mathematical topics

- Pre-assessments and ongoing questions
- Tools and strategies to analyze student work
- Professional development workshop materials and resources to communicate research and support the use of OGAP formative assessment system

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OGAP was Developed Based on Four Principles

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Principle # 1: *Build on pre-existing knowledge* (*How People Learn (2000)* National Research Council)

Principle # 2*: Learn (and assess) for Understanding*

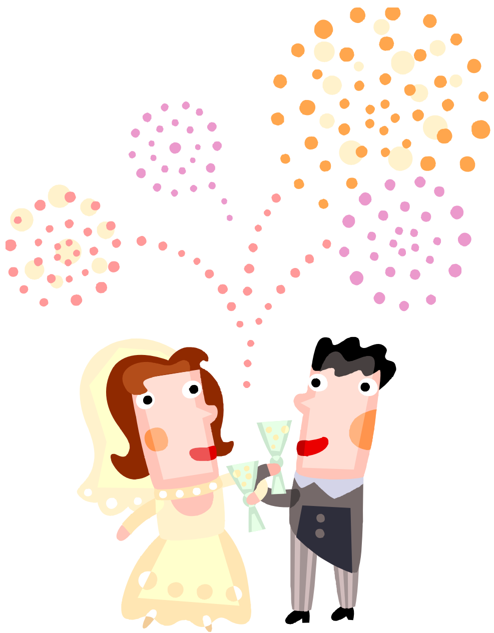
(Adding it Up! (*2001) National Research Council*)

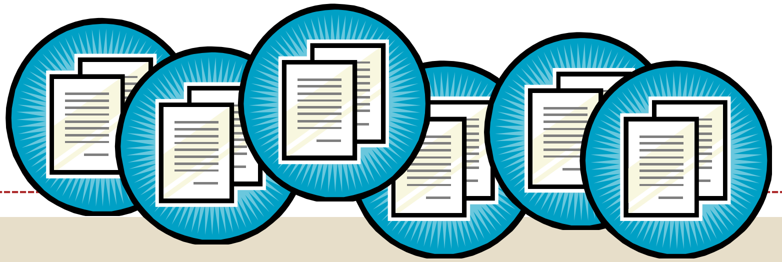
Principle # 3: *Use Frequent Formative Assessment* ***(****Inside the Black Box, (2001) Black, P, and Wiliam, D.)*

Principle # 4: *Build Assessment on Mathematics Education Research (Knowing What Students Know (2001) National Research Council)*

It is not formative assessment alone OR knowledge of cognitive research alone…

…but the marriage of the two that empowers teachers





Hundreds of research articles distilled into a frameworks and used

In design of materials

- formative assessment items
- professional development materials (case studies, activities, essays)
- Book and articles

In work with educators

- analyze student work
- inform instructional decisions
- help understand the purposes of activities in mathematics programs

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Research to Practice

Teachers say understanding the math education research help them…

- Understand the purposes of activities in math programs;
- Understand evidence in student work used to inform instruction;
- Strengthen and focus first wave instruction;
- Respond to evidence in student work as instruction proceeds.

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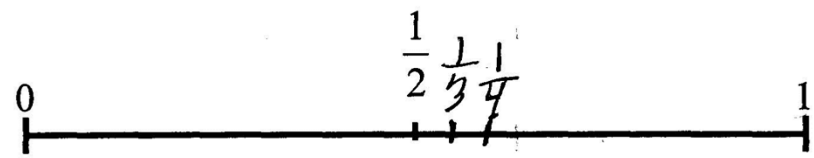
According to research, some students may see a fraction as two whole numbers (e.g., ¾ as a 3 and 4) inappropriately using whole number reasoning, not reasoning with a fraction as a single quantity. (Behr, M., Post, T., Lesh, R., and Silver, E. (1983)*;* Behr, Wachsmuth and Post, (1984); *VMP OGAP Study (2005))*

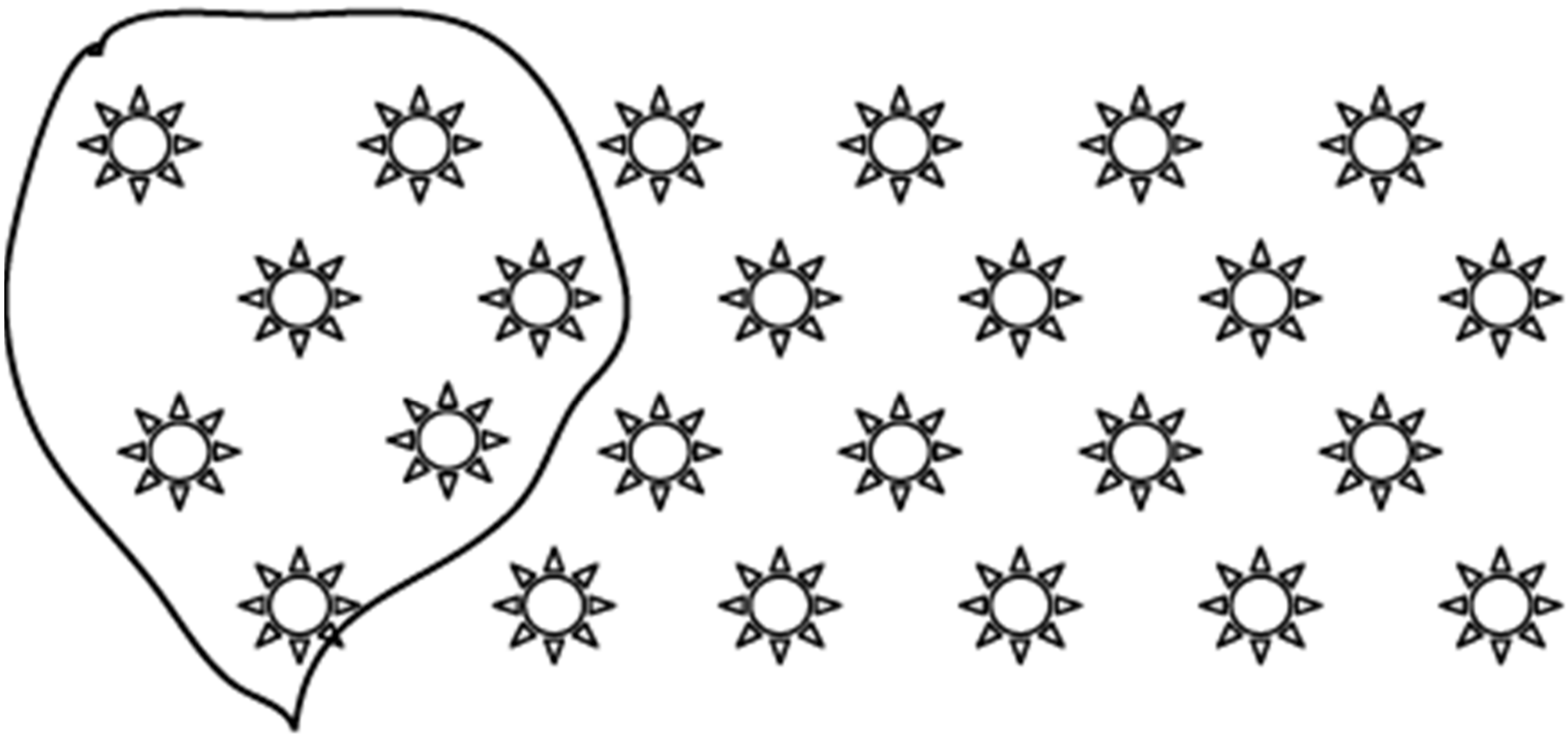
Place $\frac{1}{3}$ and $\frac{1}{4}$ in the correct location on the number line below.

Explain your answer using words or diagrams.



I chose these spots because, it says $\frac{1}{2}$, and then $\frac{1}{3}$ comes after $\frac{1}{2}$, and then $\frac{1}{4}$ after $\frac{1}{3}$ because it goes 1, 2, 3,4, and so that is how I think.

Circle 7/12 of the set of suns.



A) The sum of $\frac{1}{12} + \frac{7}{8}$ is closest to:

a) 20

b) 8

c) $\frac{1}{2}$

d) 1

Use words, pictures, or diagrams to explain your answer.

$\frac{1}{12} + \frac{7}{8} = \frac{2}{24} + \frac{21}{24} = \frac{23}{24}$ is closest to

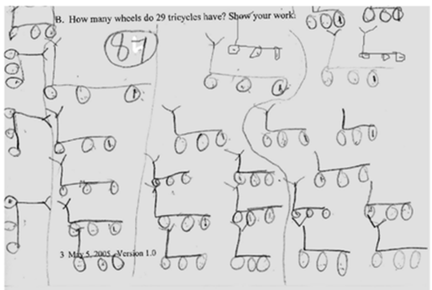
Non-fractional Reasoning

Fractional Strategy

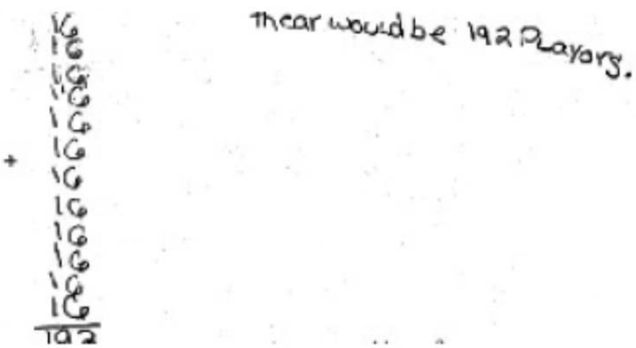
Going beyond celebrating different strategies TO…

How many wheels do 29 tricycles have?

One tricycle has three wheels.

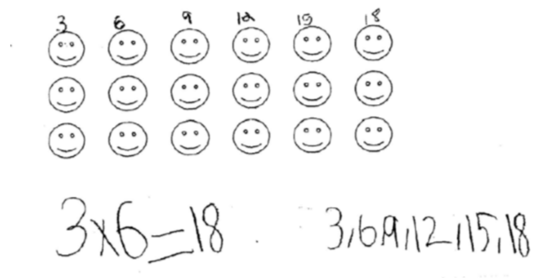


There are 16 players on a team in the Smithville Soccer League. How many players are in the league if there are 12 teams?

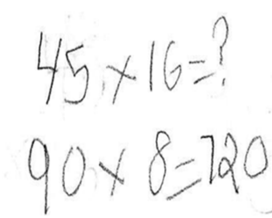


…understanding the instructional implications of the strategies and taking action

Write an equation to match this picture**.**



A class has set a goal that each student will read 45 pages this week. There are 16 students in the class. How many pages will they have read altogether by the end of the week?



The first step to helping students is understanding what they understand and can do.

OGAP Fraction Framework

Structures of Problems

Mathematical Topics
- **Equivalence and Magnitude**
- **Part to Whole Relationships**
- **Operations**

Other Structures

Evidence to Inform Instruction

- Fractional Strategy
  - Generalizes and Applies to other mathematical topics
- Transitional Fractional Strategy
- Early Fractional Strategy
- NON- Fractional Reasoning

Structures of Problems

Mathematical Topics
- Equivalence and Magnitude
- Part to Whole Relationships
- Operations

Other Structures

Structures of Fraction Problems

FRACTIONS: unit fractions, non-unit fraction, proper fractions, improper fractions, mixed numbers, negative fractions, algebraic fractions

Models
Area
Set
Linear

To solve problems
To understand concepts
To generalize concepts

Wholes
- Same sized wholes
- Different sized wholes
- Given part, find whole

Number of Parts in Whole
relative to the magnitude of the denominator
Equal
Multiples
Factors

In a model or problem situation

Partitioning Strategies
Algorithmic halving (e.g., 1/2, 1/4, 1/8)
Oddness (e.g., 1/3, 1/5, 1/7)
Eveness (e.g., 1/6, 1/10, 1/12)
Composition (e.g., for 12ths partitions into a 3x4 instead of a 1 x 12)

Number Lines
0 -1
Negative to positive
More than 2 units
Unpartitioned
Partitioned

Classes of Fractions
Same numerators, different denominators
Different numerators, same denominators
Different numerators and denominators

Reasoning Strategies
Number sense
Unit fraction
Extended unit fraction
Modeling
Benchmarks/reference points
Equivalence
Common denominators
Density of Fractions

Operations

All Operations
Estimation
Number sense
Modeling
Equivalence

Multiplication and Division
Impact of multiplying or dividing by a fraction
Partitive division
Quotative division

Derivative product of The Vermont Mathematics Partnership funded by The NSF (Award Number EHR-0227057) and the US DOE

OGAP Fraction Framework (draft April 2011)

The examples provided below do NOT represent the full set of possible solutions that represent each level.

As students learn new concepts or interact with new structures or problem situations they may move back and forth across these levels

Generalizes and Applies to other Mathematical Topics

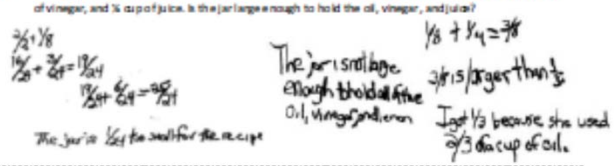
Middle School topics and concepts in which rationale number understandings are applied:

Fractional Strategy

Can accurately locate rational numbers on a number line of any length, compare and order rational numbers with a range of strategies, find equivalent rational numbers, and operate with rational numbers when solving contextual and non-contextual problems.

- Uses reasoning about relative magnitudes
- Uses benchmark reasoning
- Uses unit fraction reasoning
- Uses extended fraction reasoning
- Uses equivalence reasoning
- Uses common denominators
- Efficient algorithm
- Uses "out of equal parts" reasoning

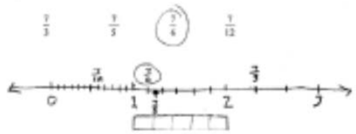
Aunt Sally has a jar that holds one cup of liquid. Her salad dressing recipe calls for 2/3 cup of oil, 1/8 cup of vinegar, and ¼ cup of juice. Is the jar large enough to hold the oil, vinegar, and juice?



Transitional Fractional Strategy

Effectively generates a model to solve contextual and non- contextual problems

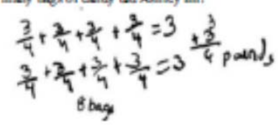
(3) Which fraction is closest to 1? Show your work and reasoning.



Strategy not efficient or generalizable (e.g., "out of parts")

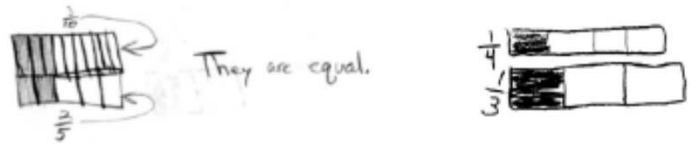
5/8 of the figure

Ashley bought 6 pounds of candy. She put the candy into bags that each hold 3/4 of a pound of candy. How many bags of candy did Ashley fill?



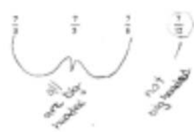
Early Fractional Strategy

Uses a fractional strategy (like modeling) or an operation appropriate for the situation but the solution includes an error(e.g., calculation, partitioning, size of the whole.)

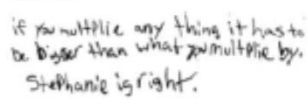


Non-Fractional Reasoning

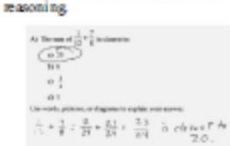
Which fraction is closest to 1?



Stephanie and Paige are discussing the answer to $3\frac{1}{2} \times \frac{1}{2}$.
Stephanie said that the answer is more than $3\frac{1}{2}$.
Paige said the answer is less than $3\frac{1}{2}$.
Who is correct?



Whole number reasoning, not fractional reasoning



This is a derivative product of the Vermont Mathematics Partnership Ongoing Assessment Project originally funded by NSF (EHR-0227057) and the US DOE (S366A020002)

Some fraction research considerations at middle school…

- Whole number reasoning may interfere with development of fraction concepts and procedural fluency (e.g., Post, Behr, Lesh & Wachsmuth, 1986; VMP OGAP, 2005)
- Fraction order and equivalence form the framework for understanding fractions as quantities that can be operated on (e.g., Post, Cramer, Behr, Lesh & Harel, 1993)
- Students may struggle with the use and understanding of formal algorithms when their knowledge is dependent primarily on memory, rather than anchored with a deeper understanding of the foundational concepts. Understanding and procedural fluency should be built in a way that brings meaning to both. (e.g., Behr et al., 1984; Behr & Post, 1992; Wong & Evans, 2007; Payne, 1976; Lesh, Landau, & Hamilton, 1983 Kieren, as cited in Huinker, 2002).
- Transitions to other mathematical topics

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Examples of teacher interventions (response to inappropriate whole number reasoning)

OGAP Exploratory Studies (2004, 2005) and 2006-2008 Roll-outs

- Use modeling to build concepts
- Emphasis on number line
- Emphasis on relative magnitude of fractions using modeling and other reasoning strategies

OGAP Whole Number Reasoning Sub-study(2005)

	Percentage of Students	Average number of incorrect responses
Pre-assessment	85% (33/39)	4.1 (33 students)
Post assessment	18% (7/39)	1.8 (7 students)

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- Transitions to other mathematical topics

Research - Comparing and Ordering Fractions

A) The sum of $\frac{1}{12} + \frac{7}{8}$ is closest to:

a) 20

b) 8

c) $\frac{1}{2}$

d) 1

D

Use words, pictures, or diagrams to explain your answer.

I think 1 because $\frac{7}{8}$ is almost one + $\frac{1}{12}$ is just going to be a little less than 1.

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Comparing Fractions

***Directions*:** Work with a partner to compare the fraction pairs below. Discuss your thinking with your partner and record the strategies you used to make your comparisons.

1. $\frac{3}{6}$ $\frac{5}{6}$

2. $\frac{11}{13}$ $\frac{9}{11}$

3. $\frac{7}{9}$ $\frac{7}{11}$

4. $\frac{3}{6}$ $\frac{7}{15}$

5. $\frac{1}{7}$ $\frac{1}{5}$

6. $\frac{15}{38}$ $\frac{5}{13}$

7. $\frac{31}{64}$ $\frac{37}{50}$

8. $\frac{8}{25}$ $\frac{15}{50}$

9. $\frac{8}{9}$ $\frac{10}{11}$

1. $\frac{3}{6}$ $\frac{5}{6}$

2. $\frac{11}{13}$ $\frac{9}{11}$

3. $\frac{7}{9}$ $\frac{7}{11}$

4. $\frac{3}{6}$ $\frac{7}{15}$

5. $\frac{1}{7}$ $\frac{1}{5}$

6. $\frac{15}{38}$ $\frac{5}{13}$

7. $\frac{31}{64}$ $\frac{37}{50}$

8. $\frac{8}{25}$ $\frac{15}{50}$

9. $\frac{8}{9}$ $\frac{10}{11}$

- **Students should understand and use flexibly the different classes of fractions:**
  - Different Numerators, Same Denominators;
  - Same Numerators, Different Denominators;
  - Different Numerators, Different Denominators.

  (Behr, M.J., Lesh, R, and Post (1981)

Identify examples of different classes of fractions.

- **Researchers found that students effectively used five types of reasoning when solving problems involving fractions**: *(Behr, M., & Lesh, R. (1992)))*
  - o Using relationships between the number of parts in the whole and the size of the part in **unit fractions** (fractions with numerators of one)
  - o **Extending unit fraction** reasoning when comparing and ordering other fractions
  - o Using a reference point.
  - o Using models (manipulatives or drawn)
  - o Using common denominators

1. $\frac{3}{6}$ $\frac{5}{6}$	4. $\frac{3}{6}$ $\frac{7}{15}$	7. $\frac{31}{64}$ $\frac{37}{50}$
2. $\frac{11}{13}$ $\frac{9}{11}$	5. $\frac{1}{7}$ $\frac{1}{5}$	8. $\frac{8}{25}$ $\frac{15}{50}$
3. $\frac{7}{9}$ $\frac{7}{11}$	6. $\frac{15}{38}$ $\frac{5}{13}$	9. $\frac{8}{9}$ $\frac{10}{11}$

Identify fraction pairs or sets that provide the opportunity for different types of reasoning.

Common Errors/Misconceptions

- Inappropriate whole number reasoning

- Ordering and comparing based on the difference between the magnitude of the numerator and the magnitude of the denominator

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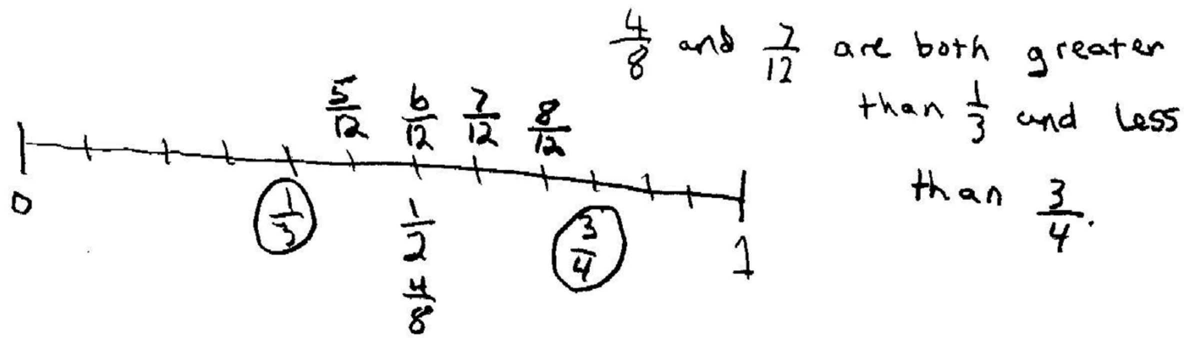
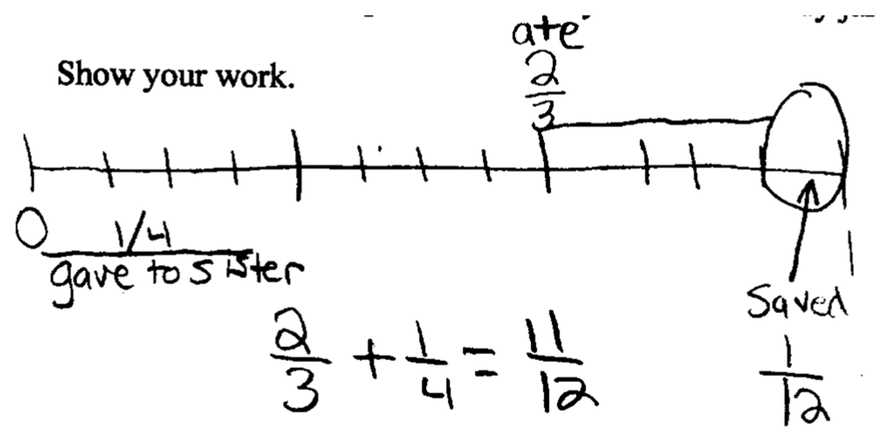
Mining for Evidence

Comparing Fractions

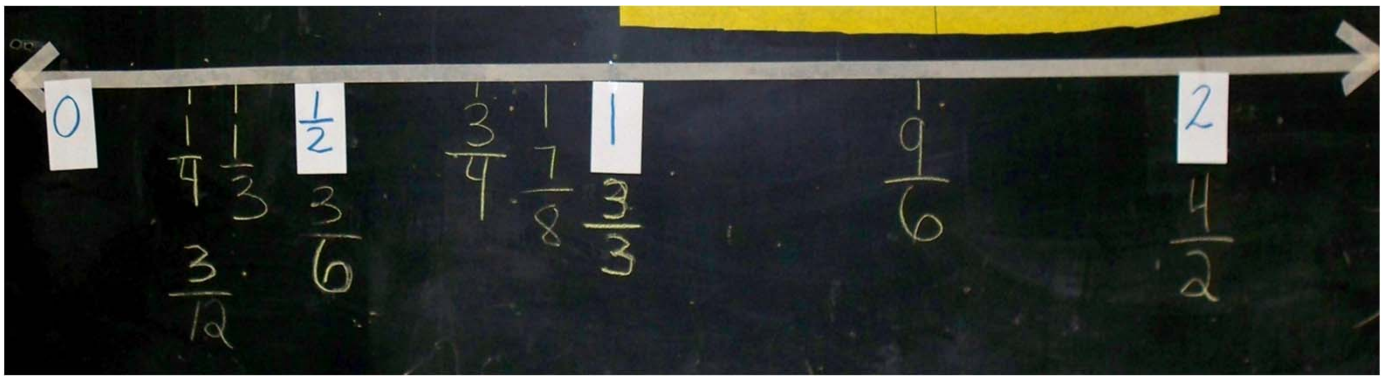
- **What reasoning strategy did students use or attempt to use when solving these problems?**
- **Choose one or two student solutions and answer – What are the implication for the next instructional steps?**

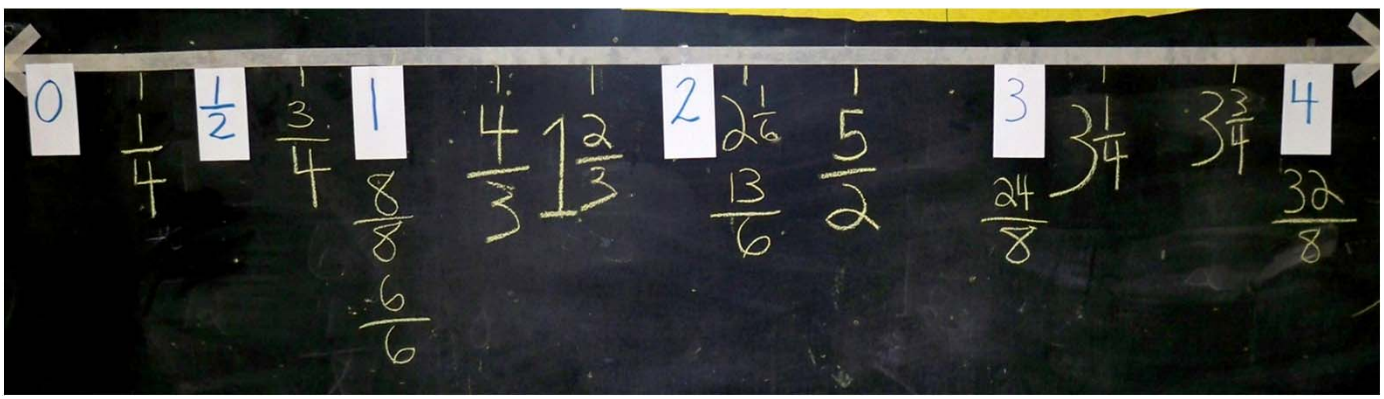


Number Lines



Number lines can help build understanding of equivalence, magnitude, and the density of rational numbers (Behr & Post, 1992; Saxe, Shaughnessey, Shannon, Langer-Osama, Chinn, & Gerhardt, 2007; VMP OGAP, personal communication, 2005, 2006, 2007).





SOME research related to number lines…

Some students have difficulty integrating the visual model (line) and the symbols necessary to define the unit. The symbols and the tick marks that define the units and sub-units can act as distractors

(Behr, Lesh, Post, & Silver, as cited in Bright et al, 1988).

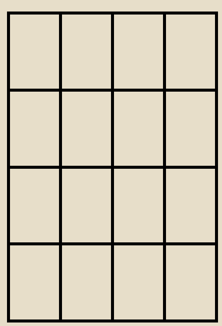
Some students have a difficult time locating fractions on number lines that have been marked to show multiples of the unit or show marks to span from negative numbers to positive numbers (Novillis – Larson, as cited in Behr & Post, 1992; VMP OGAP, 2005).

Students don't always understand that the numbers associated with points on a number line tell how far the points are from 0 (Pettito, 1990). For example, the two points marked 3 and -3 on a number line are both 3 units from 0.

Middle School Fraction Dilemma

- Many students arrive at middle school without the understanding and procedural fluency with fractions necessary to engage in the mathematics required at middle school.
- Many middle school and high school teachers assume that students will naturally make the transition from knowledge ABOUT fractions to application in the new mathematical topics and concepts.

Shade ½ of the figure.



What is the value of $24x - \frac{1}{2}$, when $x = \frac{1}{3}$?

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Fraction Demand

- Fraction Concept Development and Application
  - Foundational Concepts Elementary Grades
  - Development of Understanding and Procedural Fluency Grades 4 - 6
  - Application in a Range of Situations Grades 7 +

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Mapping Fraction Demand

- Identify applications of fraction concepts and skills at the grade level.

	Grade 6	Grade 7	Grade 8
New to grade level (CCSS)	• Divide fractions by fractions • Understand rational number as a fraction on a number line • Understand ordering and absolute value of absolute values	Solve problems involving rational numbers with all operations	No new fraction content
Applied at grade level			

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Bringing OGAP to your school, district, or state involves…

Significant Professional Development by OGAP team and ongoing support system at the school level

- In an understanding of formative assessment
- In the use of OGAP formative assessment materials and processes.
- on the substance of the math education research that is foundational to the OGAP materials and processes.
- Use of the materials "real time" with students with links to mathematics programs.

Tools and Resources to support system

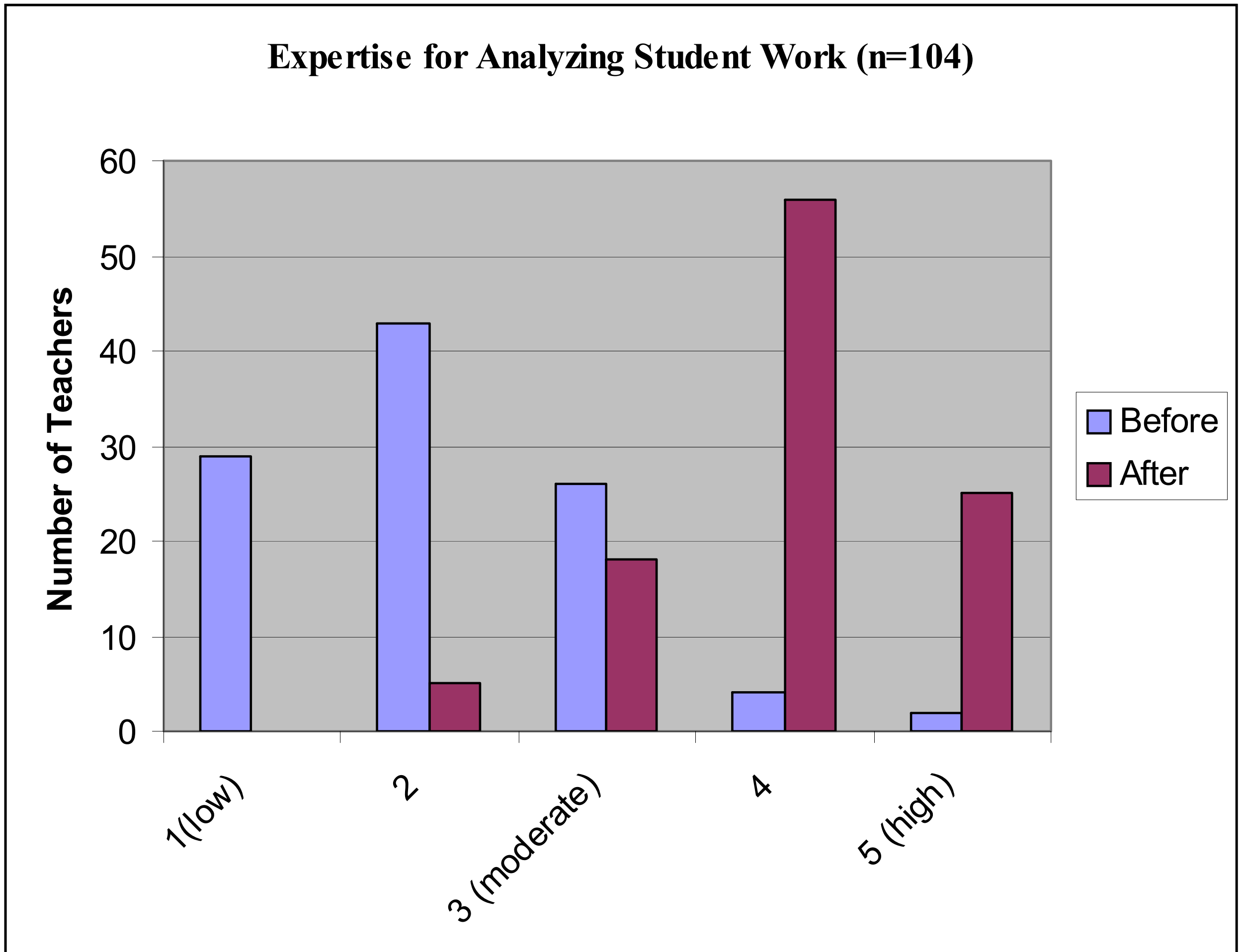
- Some pre-assessments and ongoing items
- Strategies and related tools for analyzing student work and making instructional decisions

What do *teacher leaders and teachers* say about their experience in relationship to the stated goals and the use of OGAP formative assessment system?

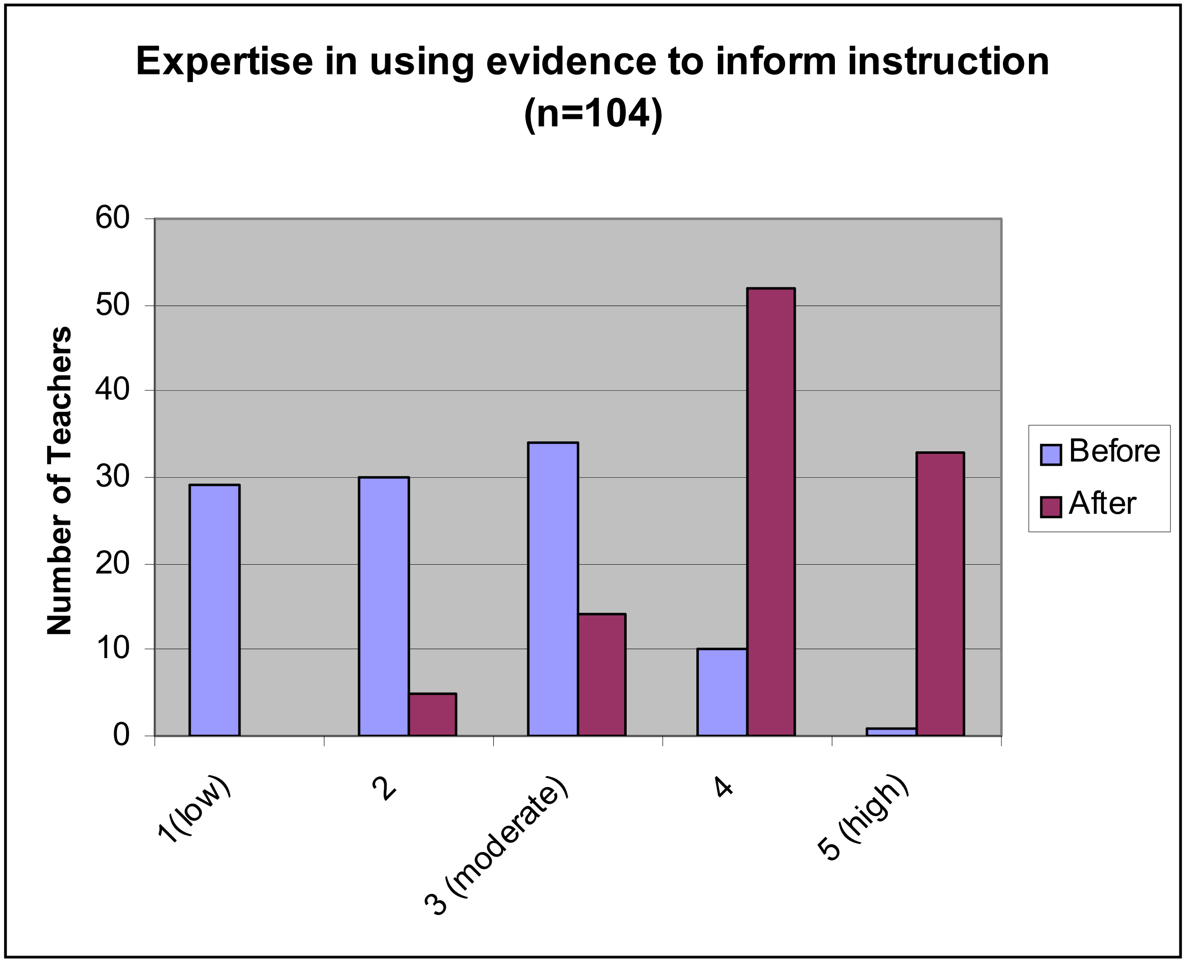
Results based on a spring 2007 online survey

Expertise for analyzing student work (for evidence of developing understanding, common errors and misconceptions)…

Before and After Experience



Expertise in using evidence in student work to inform instruction…



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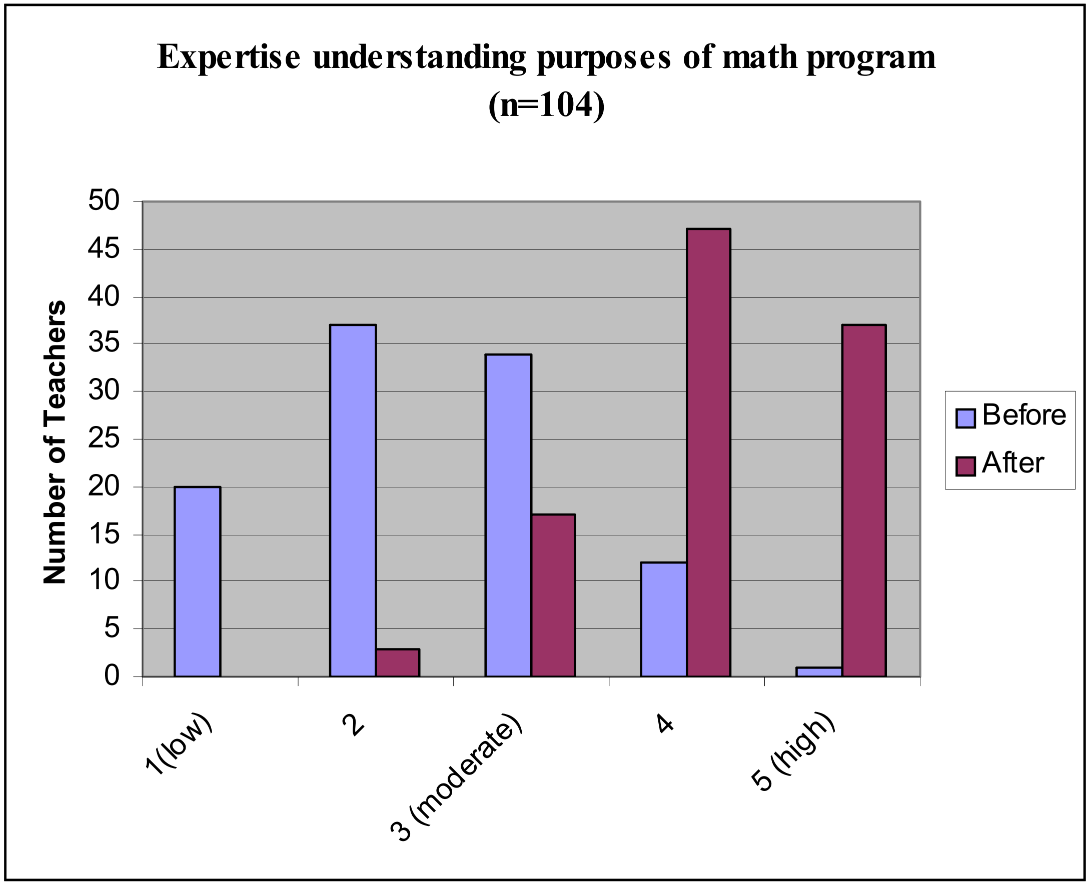
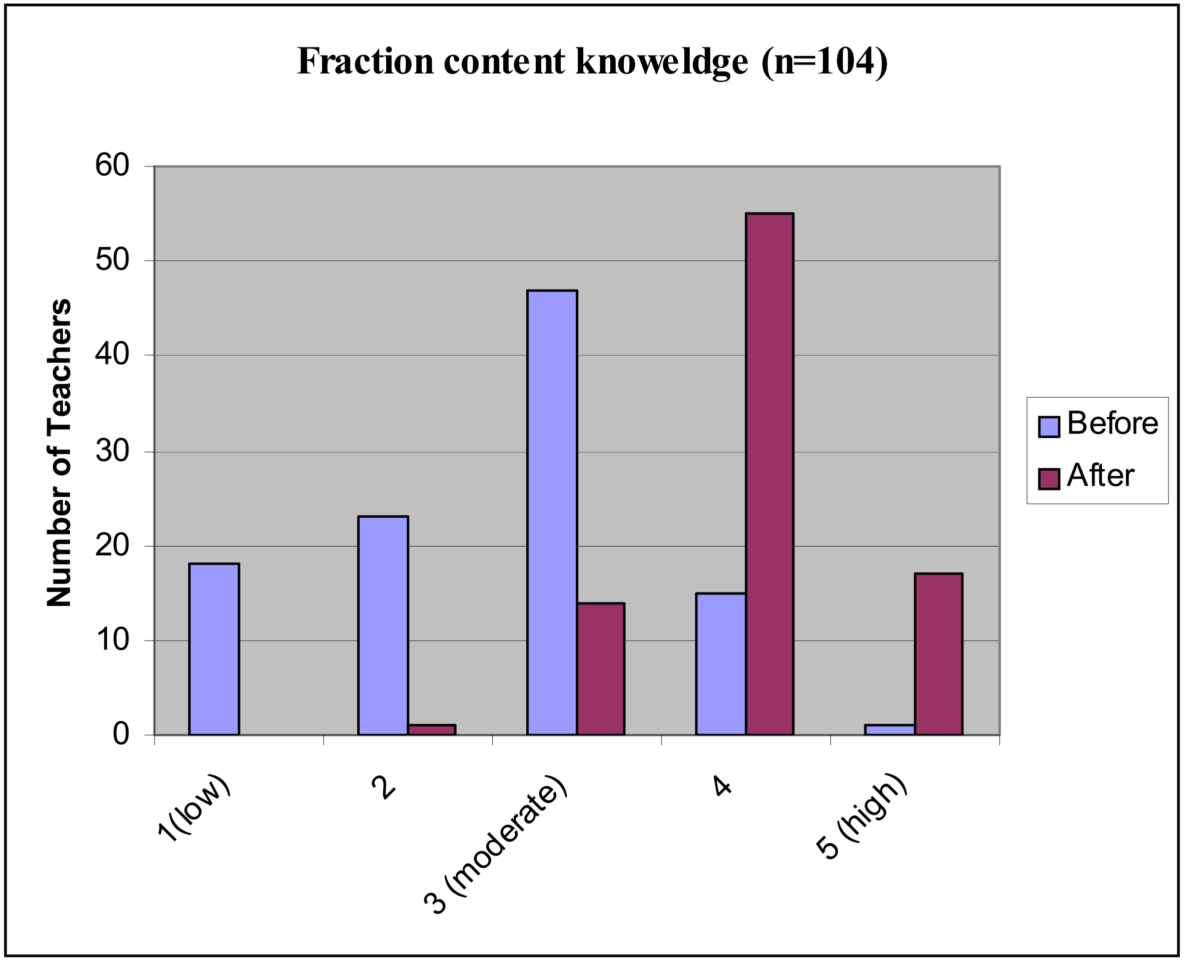
Understanding purposes of activities in mathematics program…

Before and After Experience



Fraction content knowledge…

Before and After Experience



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Pre-post Question – Pilot OGAP Teacher Assessment (2007)

Provide three strategies students can use to solve this problem. Provide examples.

1) Which fraction is closest to 1? Show your work.

$\frac{1}{2}$ $\frac{7}{9}$ $\frac{11}{13}$ $\frac{1}{6}$

Pilot OGAP Teacher Assessment Question

Sample Teacher Responses

Pre-assessment Q1 A

(1) $\frac{1}{2} = \frac{117}{234}$ $\frac{7}{9} = \frac{182}{234}$ $\frac{11}{13} = \frac{198}{234}$

$\frac{1}{6} = \frac{39}{234}$ $\therefore \frac{11}{13}$ is closest to 1

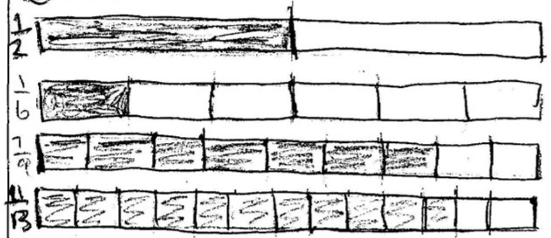
(2) Use fraction bars kit provided, (ninths + thirteenths are in it)

(3)

Post-assessment Q1 A

(1) Unit fractions: $\frac{1}{2}, \frac{1}{6}$
sixths are smaller parts than halves.

(2) Use of area models



(3) Use $\frac{1}{2}$ benchmark.
Using unit fraction reasoning, $\frac{1}{6}$ is smaller than $\frac{1}{2}$.
$\frac{7}{9}$ and $\frac{11}{13}$ are greater than $\frac{1}{2}$.
(continue on back as needed)

$\frac{11}{13}$ is $\frac{2}{13}$ away from 1 whole.
$\frac{7}{9}$ is $\frac{2}{9}$ away from the 1 whole.
Since 13ths are smaller, $\frac{11}{13}$ is closer to 1.

Findings (Petit-Cunningham, 2008)

- Teacher leaders increased the range of strategies that they used pre to post to solve the two problems.

- Mentees also increased the range, but to a lesser degree

Mentors and Mentees Pre - Post Teacher Assessment			
	Pre mean	Post mean	T-test (p-) Significance (p< 0.05)
Mentors (n=25)	6.16	9.8	3.52E-08
Mentees (n= 42)	5.6	7.9	7.73E-06

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Recent Publications:

Petit, Laird, and Marsden (2010), A Focus on Fractions: Brining Research to the Classroom. Routledge, New York and London.

Petit, Laird, & Marsden (September, 2010). They get fractions as pies – but now what?. Mathematics in the Middle School, NCTM, Reston, Virginia.

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Student work samples used with permission of the Vermont Mathematics Partnership funded by the US Department of Education (Award Number S366A020002) and the National Science Foundation (Award Number EHR-0227057)

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- Rebecca Young, Hardwick Schools

Plus about 250 Vermont and Alabama teachers and teachers and about 5000 students who participated in OGAP Exploratory Studies and 2006-2008 scale-up

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OGAP Sites:

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Alabama
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Ohio
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