USING A TRANSFORMATIONAL APPROACH TO SIMILARITY

Supporting teachers to implement the Common Core State Standards with their students

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SESSION GOALS

Use a developing set of PD materials to:

- Provide insight into the Common Core State Standards and students' developing conceptions of similarity
- Examine static and transformational views of similarity through the lens of three students
- Provoke thought about the implications for supporting teachers and teacher leaders

COMMON CORE STATE STANDARDS

The Common Core State Standards for 8th Grade Geometry support a transformational approach to understanding congruence and similarity:

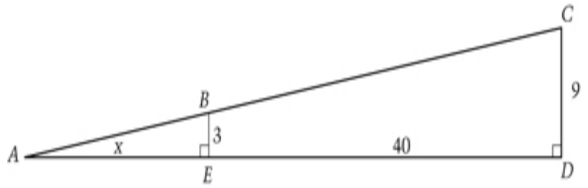
- Understand that a two-dimensional figure is **congruent** to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
- Understand that a two-dimensional figure is **similar** to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, **and dilations**; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

WHY USE A TRANSFORMATIONAL APPROACH

- Many teachers and textbooks treat congruence as “same size, same shape” and similarity as “same shape, different size”. This is not sufficient nor precise enough to transition from middle school to high school geometry.
- The key to grade specific rigor (informal to increased formalism) in CCSS is the transformational approach.
- Transformations (rigid motions + dilation) as the basis for both congruence and similarity are not well represented in current materials.

NAEP ITEM

The 2007 8th grade NAEP item below was classified as "Use similarity of right triangles to solve the problem."

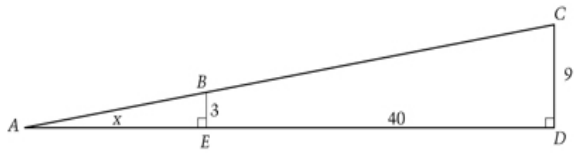


16. The figure above shows two right angles. The length of AE is x and the length of DE is 40.

Show all of the steps that lead to finding the value of x. Your last step should give the value of x.

WHY IS THIS ITEM SO DIFFICULT?

The 2007 8th grade NAEP item below was classified as "Use similarity of right triangles to solve the problem."

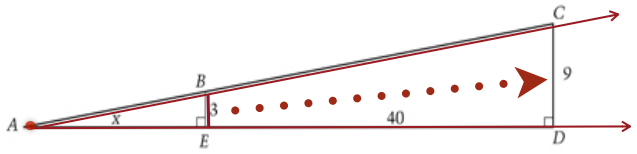


16. The figure above shows two right angles. The length of AE is x and the length of DE is 40. Show all of the steps that lead to finding the value of x. Your last step should give the value of x.

Only 1% of students answered this item correctly.

USING TRANSFORMATION

The 2007 8th grade NAEP item below was classified as "Use similarity of right triangles to solve the problem."



16. The figure above shows two right angles. The length of AE is x and the length of DE is 40.
Show all of the steps that lead to finding the value of x. Your last step should give the value of x.

47 % of the teachers got the item correct (n=45)

None of them used a transformational approach

A transformational conception of similarity would enable a student to determine correspondence and set up a correct proportion. If a student understood that two figures are similar if one is congruent to a dilation of the other, then they could easily determine that a 3-dilation of the smaller triangle (ABE), centered at point A, yields the larger triangle (ACD). The student would know the scale factor is 3 because the side of 3 units is enlarged to a corresponding side of 9 units.

LEARNING AND TEACHING GEOMETRY PROJECT

- NSF project developing modular video case-based, PD materials
  - 1 Foundation Module (10, sequenced 3-hour sessions)
  - 4 Extension Modules (2, 3-hour sessions each)
- **Staff:** Nanette Seago (PI), Mark Driscoll (Co-PI), Jennifer Jacobs, Johannah Nikula, Patrick Callahan, Hilda Borko
- **Advisory Board:**Harold Asturias, Tom Banchoff, Phil Daro, Megan Franke, Karen Koellner, Glenda Lappan, Hung-Hsi Wu
- **Evaluation Team:** [Horizon Research, Inc.] Dan Heck, Kristen Malzahn, Courtney Nelson

LTG MATERIALS

- Built around authentic video clips from grades 6-8 classrooms
- Focus on similarity and its mathematical use in teaching
- Modular in design--coherent, sequenced set of video case professional development sessions
- Well-specified facilitator support materials:
  - Detailed sample agendas designed with an eye toward making the goals, design and underlying core principles explicit

LTG FOUNDATION MODULE

Foundation Module Map

10, 3-hr Sessions

Session 1	Session 2	Session 3	Session 4	Session 5	Session 6	Session 7	Session 8	Session 9	Session 10
A dynamic, transformational view of congruence	*A dynamic, transformational view of similarity*	*Relationship between dilation and similarity*	*Preservation of angles through dilation*	*Preservation of angles & proportional lengths through dilation*	*Ratios within & across similar figures*	*Ratios within & across similar figures, part 2*	*Connections between similarity and slope & linearity*	*Area of similar figures*	*Closure and re-capping of big ideas*
Defining Congruence and Similarity			**Relationships and Attributes of Similar Figures**				**Connections**		**Closure**

FOUNDATION MODULE GOALS:

- To examine a dynamic, transformational view of similarity, and geometry in general
- To gain insight into students developing conceptions of similarity
- To collectively enhance *specialized content knowledge* of similarity

SORTING RECTANGLES TASK

A class of 6th grade students is given bags of rectangles. The teacher asks them to decide which rectangle doesn't belong. She defines "doesn't belong" as not an enlargement or reduction.

Bag B

What attributes of the rectangles do you predict students will attend to in determining which one doesn't belong?

MAKAYLA AND VICTORIA

What relationships are Makayla & Victoria attending to?

MAKAYLA & VICTORIA

Makayla	Victoria
The 3x2 doesn't belong because from a 1x2 to a 2x4 you are adding 1 to the width and 2 to the length. It's the same from a 2x4 to a 3x6.	The 3x2 doesn't belong because notice the other numbers: 2x1 (1 is half of 2), 6x3 (3 is half of 6), 4x2 (2 is half of 4). 2 is not half of 3.

*Both Makayla and Victoria are looking at the rectangles as **discrete, static figures**. Their focus is on comparing the numerical relationships between the corresponding parts of the rectangles.*

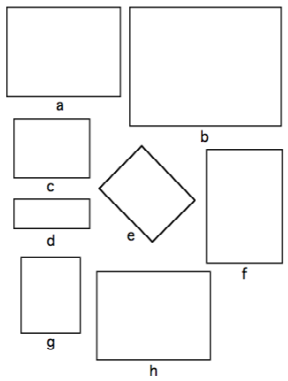
*Victoria notices that there is a pattern **within** each rectangle (one side is half of the other side),*

*Makayla notices a pattern **across** the corresponding sides of the rectangles (add 1 to the width and 2 to the length).*

SIMILAR RECTANGLES

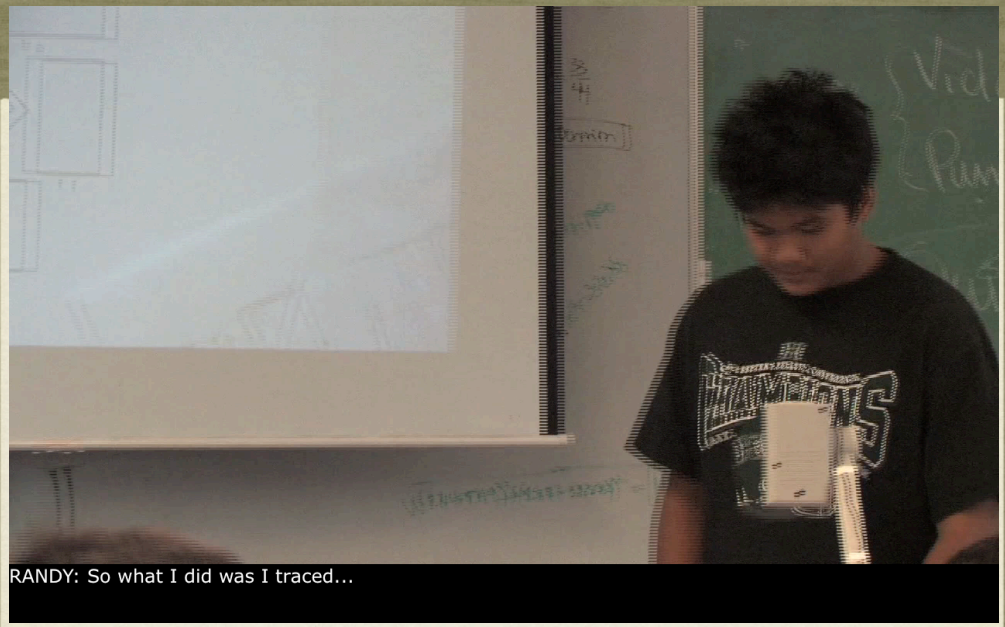
A class of 8th grade students is given the task below.

2. Which rectangles are similar to rectangle **a**? Explain the method you used to decide.



What attributes of the rectangles do you predict students will attend to in determining which rectangles are similar?

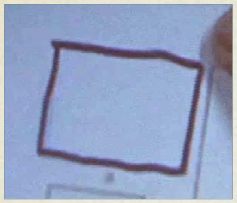
RANDY



How is Randy solving the problem? What relationships is he attending to?

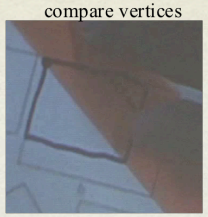
RANDY

Tracing rectangle a using tracing paper

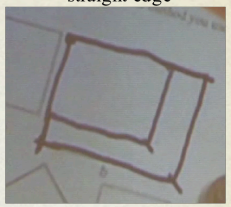


Superimposing rectangle a onto Rectangle b

Using a straight edge to compare vertices



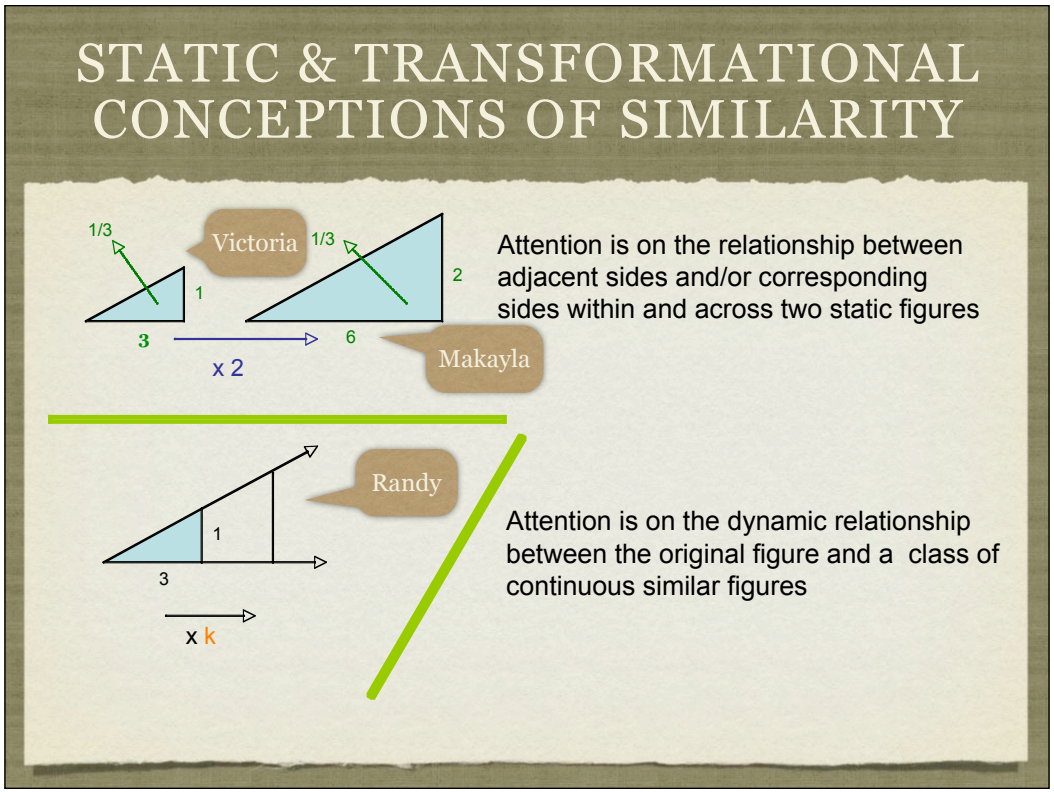
Line up all sides with straight edge



Randy draws on an understanding that geometric transformations--in this case dilation--can be used to compare figures and determine whether they are similar.

Randy appears to be using the geometry of the problem and the motion of dilation as a means of proportionally scaling the rectangles.

Randy traces rectangle a and then superimpose it onto rectangle b. He uses a straight edge and draws lines through the corresponding vertices. Randy tells the class that he used the upper left vertex as the “center of dilation” and he noticed that the sides of the two rectangles “lined up” and they shared a common diagonal. He concludes that the rectangles a and b are similar. He then demonstrates that rectangle d is not similar to rectangle a because they do not share a common diagonal.



Victoria and Makayla's approaches can be labeled as static. They are attending to the numeric relationships within or across discrete figures. Victoria is focused on the relationship between the adjacent sides WITHIN each figure, noticing whether the two sides are in the same proportion. Makayla attends to the relationships across the figures corresponding sides, noticing whether they are growing proportionately (she might notice that as the height goes up by one, the base goes up by three or that the corresponding sides increase proportionately by a scale factor of 2).

Randy's approach is transformational in that he is attending to the dynamic relationship between the original figure and a family of similar figures. Randy's approach does not involve any sort of measurent and rather relies on a visual inspection of accurate scaling.

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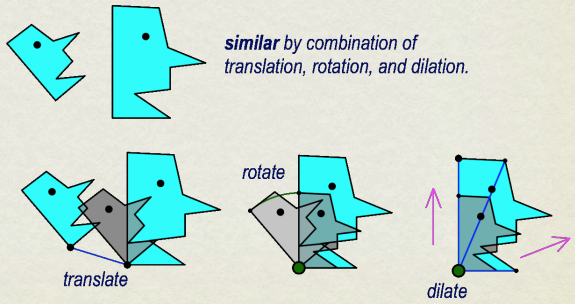

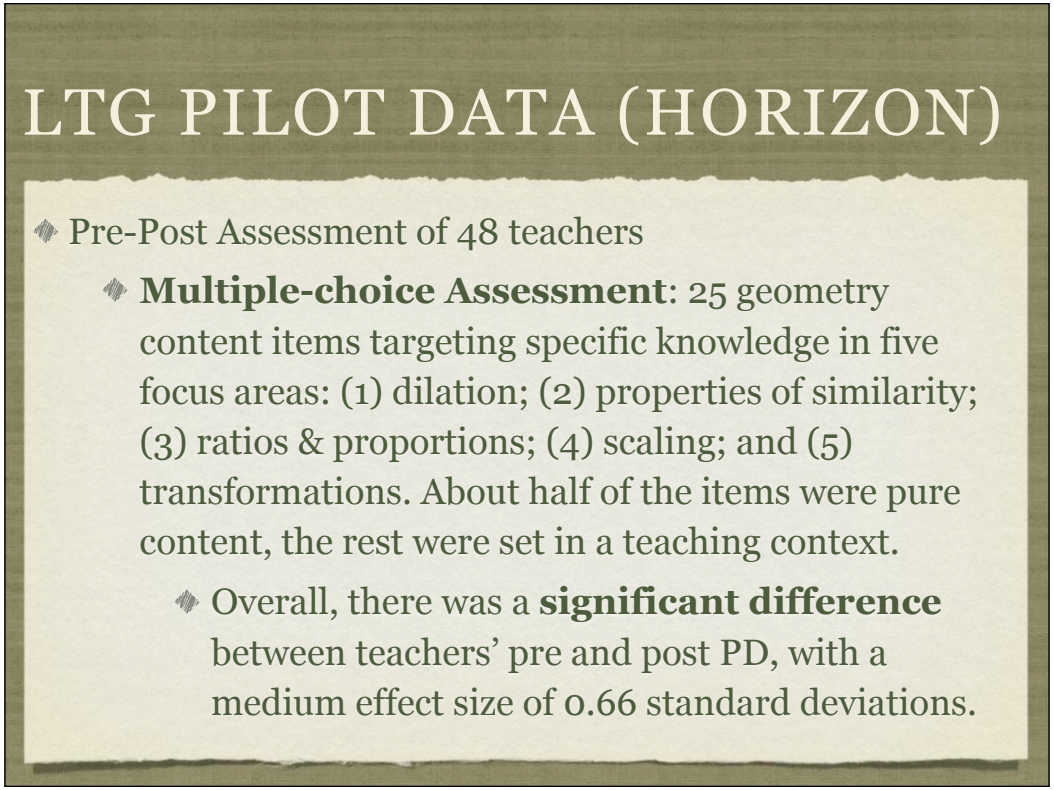

We have found that there is reason for optimism in our project's pilot data. Given the opportunity to learn a transformational approach to similarity through an experience with the LTG foundation module, there was a significant difference between teachers pre and post multiple choice assessments.

In addition, pre-post scores significantly increased on an embedded math assessment based on the sorting rectangles problem that Makayla and Victoria worked on.

LTG TIMELINE

Summer 2012 a preliminary version of the materials will be ready.

A possible facilitator institute to support the use of the materials may accompany the materials release next summer.

Questions? Contact nseago@wested.org