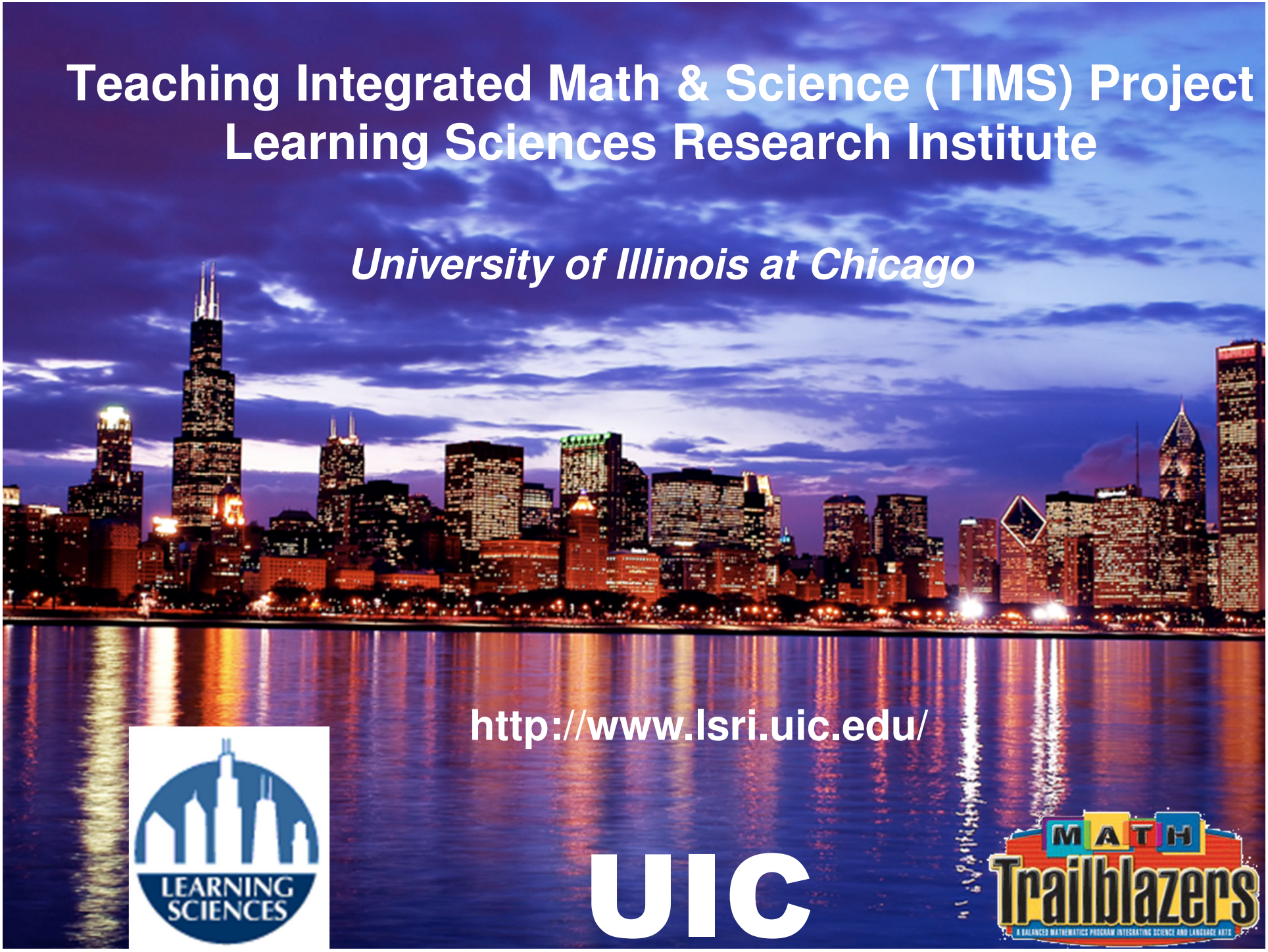
Teaching Integrated Math & Science (TIMS) Project
Learning Sciences Research Institute

University of Illinois at Chicago



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MATH Trailblazers
A BALANCED MATHEMATICS PROGRAM INTEGRATING SCIENCE AND LANGUAGE ARTS

Research to Practice: You Know What They Know, So, Now What?

Jennifer Mundt Leimberer (jleimb1@uic.edu)

Teaching Integrated Mathematics and Science (TIMS) Project

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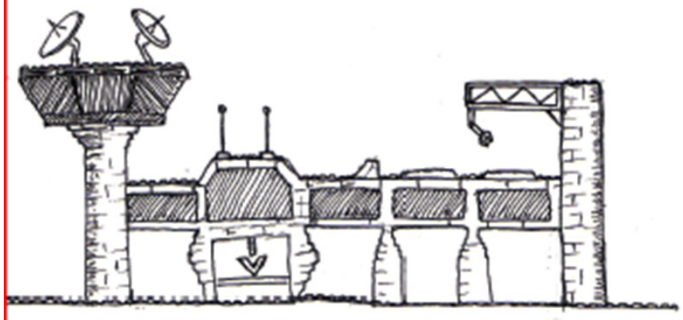
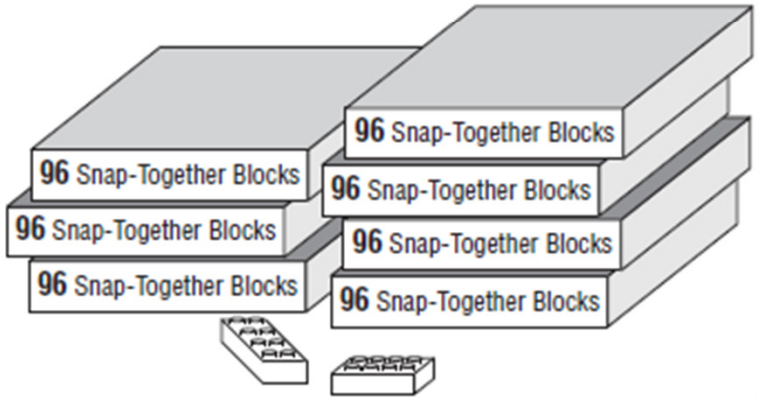
A BALANCED MATHEMATICS PROGRAM INTEGRATING SCIENCE AND LANGUAGE ARTS



Jackie, Jerome, and Maya are building a model city for the moon using snap-together blocks. They ran out of blocks, so they put their money together to buy more. A new box holds 96 blocks. They have enough money to buy 7 boxes.

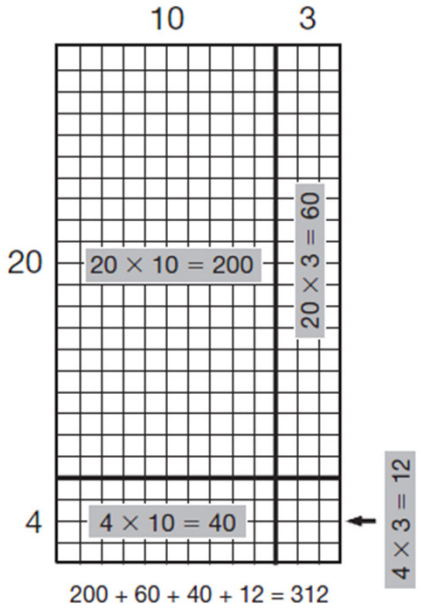
1. Jerome said, "That will give us almost 700 new blocks."
   - **A.** What numbers did Jerome use to estimate?
   - **B.** Why does he think the total will be less than 700 blocks? Why not *about* 700 blocks or *more* than 700 blocks?

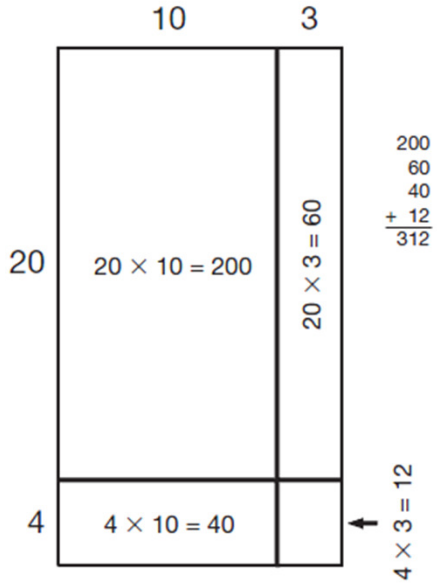


Discussion Question:

What can you learn about what students know and can do from this problem?

Grace drew two rectangles to show how she breaks the numbers in a multiplication problem into tens and ones.





200
60
40
+ 12
312

6. Explain why there are four parts to Grace's rectangles. What does the area of each part represent?
7. **A.** How did Grace break apart 13 and 24?

   B. Which part of Grace's rectangle is the largest? Which is the smallest?
8. If you want to estimate an answer to 13 × 24, which part of the rectangle would be the best to use? Why?

Discussion Question:

What can you learn about what students know and can do from this problem?

Discussion Question:

What can you learn about what students know and can do from this problem?

Name _____ Date _____

2. Linda solved 52 × 24 using expanded form:

$$
\begin{array}{rl}
52 = & 50 + 2 \\
\times\ 24 = & 20 + 4 \\
\hline
 & 100 \leftarrow 50 \times 20 \\
 & 200 \leftarrow 50 \times 4 \\
 & 40 \leftarrow 2 \times 20 \\
 & +\ 8 \leftarrow 2 \times 4 \\
\hline
 & 348
\end{array}
$$

A. Check Linda's answer by solving 52 x 24 using the rectangle method.

B. What mistake did Linda make in solving the problem? Use your rectangle to help explain your answer.

You Know What They Know, So, Now What?

Session Overview

Introduction: A look at multiplication

Part 1: Framework for defining “What they Know”

Part 2: Design rational for “So, Now What” solutions

Math Trailblazers
Research and Revision Study

2003–2006	Research on implementation of 2nd edition in classrooms
2006–2009	Revision and field test of new materials in grades 1–5
2008–2010	Student Achievement Study
2009–2012	Final revision of materials for publication

National Science Foundation
WHERE DISCOVERIES BEGIN

Research Studies

- Whole Number Study–UIC & KSU 2003–2008
- Implementation Study–UIC 2003–2006
- Fractions and Ratios–UMN 2004–2006
- Video Study–UIUC 2003–2006
- Field Test Study–UIC 2006–2010
- Student Achievement Study–UIC 2009–2011
- Embedded Assessment Study–UIC Current

Data Sources*	Number
Teacher Surveys	1500
Classroom Observations	250
Teacher Interviews	190
Student Interviews*	325
Consultant Reviews	3 (math, geometry, assessment[1])
Feedback Meetings	15 meetings over 25 days

Plus student work samples of open-response questions, end-of-year tests, labs, and other activities

[1](Pellegrion and Goldman, 2007)

Assessment Strengths and Weaknesses from CPS Review and Analysis

Strengths

- Reflects content in NCTM's *Principles and Standards*
- Provides a balance of assessment types (e.g. tests, quizzes, projects (labs), open-response problems, and observations)
- Assesses students on what they were taught and represents activities in the curriculum
- The Assessment Indicators provide a developmental framework that can be used to evaluate student progress
- Answer keys or sample student work often provide performance criteria

Assessment Strengths and Weaknesses from CPS Review and Analysis

Weaknesses

- Few assessments require student to make mathematical connections within the assessment
- Does not provide many opportunities for self- or peer-assessment
- Only open-response questions include scoring guides and procedures
- None of the assessments make specific recommendations to support special needs students
- More structure is needed to assist teachers in making decisions based on the results of assessments

(Pellegrino & Goldman, 2004a, 2004b, 2007)

Context of Response to Intervention (RtI)

- Tier 1 occurs at the whole-class level and relies on the differentiation of instruction in the general curriculum.
- Tier 2 occurs in smaller groups and tailors instruction based on targeted student needs.
- Tier 3 occurs with students requiring individualized intervention..

Appropriate Use of Assessment: What to Avoid

"Today was definitely a day of inappropriate assessment in the school where I teach. We are starting [a Response to Intervention] program this year, and we all had to give our first round of Curriculum-Based Measures, which was a timed test with 35 straight-up computation problems—addition, subtraction, multiplication, division. We're supposed to use our students' scores to place them in groups for doing problem solving with fractions. So my question is, what does zipping through 35 basic computations have to do with solving problems? That test did not tell me anything about what my kids can do with problem solving, or fractions, or what they need to do to get better at either of those things."

What Is Needed to Inform Instruction

- What does it look like to “Get it”?
- What has happened to help students “Get it”?
- So, now what should happen?

What Is Needed to Inform Instruction

What does it look like to "Get it"?

- Create explicit expectations;
- Assess those expectations;
  - Provide tasks that "force the issue" and attends to interconnections
  - Provide tools and routines for feedback

A more in-depth analysis asks teachers to be aware of the "big ideas" in mathematics and then to connect the identified standards to these ideas.

-- Dacey & Lynch((2007)

Identify Key Ideas

Grade 4 Unit 11
Multiplication with Larger Numbers

Expectations

Use this list of expectations to assess students on the key concepts and skills in this unit.
Students are able to do the following:

- E1.* Demonstrate understanding of the place-value concepts and mathematical properties involved in multiplication of 2-digit by 2-digit numbers (e.g., use the distributive property to multiply). [Algebra 4]
- E2.* Show connections between models and strategies for multiplying 2-digit by 2-digit numbers (e.g., demonstrate partial products using a rectangle model for multiplication).
- E3.* Estimate products of multidigit numbers.
- E4.* Multiply 2-digit by 2-digit numbers using mental math strategies and paper-and-pencil methods (e.g., expanded form, all-partials).
- E5. Multiply 2-digit by 2-digit numbers using the compact method.
- E6. Choose appropriately from among estimation, mental math strategies, and paper-and-pencil methods to multiply multidigit numbers.
- E7.* Demonstrate fluency with the division facts for the last six facts.
- E8.* Write the number sentences in the fact families for the last six facts.

*** Denotes Benchmark Expectation**

Unit 11 Key Assessment Opportunities Chart

Content: Key Ideas in Unit 11 / Unit 11 Expectations	L1 DAB *Solving Multiplication Problems* Check-In Q# 1–4	L2 URG *Multiplication Quiz 1***	L2 SG *All-Partials Revisited* Check-In Q# 18	L3 SG *Multiplication with 2-digit Numbers* Check-In Q# 11	L3 DAB *Connecting Multiplication Methods* Check-In Q# 3	L4 URG *Multiplication Quiz 2***	L4 SG *Compact Multiplication Revisited* Check-In Q# 5	L4 SG *Compact Multiplication Revisited* Check-In Q# 19–20	L5 SG *Choosing Strategies to Multiply* Check-In Q# 23–27	L5 DAB *Dancing in TIMSville***
Number 2 **Operations: Understand the meaning of numerical operations and their application for solving problems.**										
E1 * Demonstrate understanding of the place-value concepts and mathematical properties involved in multiplication of 2-digit by 2-digit numbers (e.g., use the distributive property to multiply). [Algebra 4]	X	X			X	X	X			
E2 * Show connections between models and strategies for multiplying 2-digit by 2-digit numbers (e.g., demonstrate partial products using a rectangle model for multiplication).	X		X		X	X				
Number 3 **Computation and Estimation: Use efficient and flexible procedures to compute accurately and make reasonable estimates.**										
E3 * Estimate products of multidigit numbers.	X	X		X		X		X	X	
E4 * Multiply 2-digit by 2-digit numbers using mental math strategies and paper-and-pencil methods (e.g., expanded form, all-partials).	X	X	X	X		X		X	X	
E5 Multiply 2-digit by 2-digit numbers using the compact method.						X		X	X	
E6 Choose appropriately from among estimation, mental math strategies, and paper-and-pencil methods to multiply multidigit numbers.	X	X				X			X	X

* Denotes Benchmark Expectation
** Includes a Feedback Box

Math Facts

		L1 DPP item A *The Last Six Facts*	L4 DPP item M *Fact Family Quiz: Last Six Facts*	L5 DPP item O *Facts Quiz: Last Six Facts*
Number 3	**Computation and Estimation: Use efficient and flexible procedures to compute accurately and make reasonable estimates.**			
E7*	Demonstrate fluency with the division facts for the last six facts.	×		×
E8*	Write the number sentences in the fact families for the last six facts.		×	

Problem Solving and Communication

		L2 SG *All Portals Revisited* Check-In Q# 18**	L5 DAB *Dancing in TIMSville* **
Problem Solving			
PSE1	**Know the problem.** I read the problem carefully. I know the questions to answer and what information is important.	×	
PSE2	**Find a strategy.** I choose a good and efficient strategy for solving the problem.		×
PSE3	**Check for reasonableness.** I look back at my solution to see if my answer makes sense. If it does not, I try again.	×	×
PSE4	**Check my calculations.** If I make mistakes, I correct them.		
PSE5	**Show my work.** I show or tell how I arrived at my answer so that someone else can understand my thinking.	×	×
PSE6	**Use labels.** I use labels to show what numbers mean (such as 15 boys or 6 inches).		

* Denotes Benchmark Expectation
** Includes a Feedback Box

What Is Needed to Inform Instruction

What does it look like to “Get it”?

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  Provide tasks that “force the issue” and attends to interconnections

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A more in-depth analysis asks teachers to be aware of the “big ideas” in mathematics and then to connect the identified standards to these ideas.

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Unit 11 Key Assessment Opportunities Chart

Content

Key Ideas in Unit 11 / Unit 11 Expectations		L1 DAB *Solving Multiplication Problems* Check-In Q# 1–4	L2 URG *Multiplication Quiz 1***	L2 SG *All-Partials Revisited* Check-In Q# 18	L3 SG *Multiplication with 2-digit Numbers* Check-In Q# 11	L3 DAB *Connecting Multiplication Methods* Check-In Q# 3	L4 URG *Multiplication Quiz 2***	L4 SG *Compact Multiplication Revisited* Check-In Q# 5	L4 SG *Compact Multiplication Revisited* Check-In Q# 19–20	L5 SG *Choosing Strategies to Multiply* Check-In Q# 23–27	L5 DAB *Dancing in TIMSville***
Number 2	**Operations: Understand the meaning of numerical operations and their application for solving problems.**										
E1 *	Demonstrate understanding of the place-value concepts and mathematical properties involved in multiplication of 2-digit by 2-digit numbers (e.g., use the distributive property to multiply). (Algebra 4)	X	X			X	X	X			
E2 *	Show connections between models and strategies for multiplying 2-digit by 2-digit numbers (e.g., demonstrate partial products using a rectangle model for multiplication).	X		X		X	X				
Number 3	**Computation and Estimation: Use efficient and flexible procedures to compute accurately and make reasonable estimates.**										
E3 *	Estimate products of multidigit numbers.	X	X		X		X		X	X	
E4 *	Multiply 2-digit by 2-digit numbers using mental math strategies and paper-and-pencil methods (e.g., expanded form, all-partials).	X	X	X	X		X		X	X	
E5	Multiply 2-digit by 2-digit numbers using the compact method.						X		X	X	
E6	Choose appropriately from among estimation, mental math strategies, and paper-and-pencil methods to multiply multidigit numbers.	X	X				X			X	X

* Denotes Benchmark Expectation
** Includes a Feedback Box

Discussion Question:

What can you learn about what students know and can do from this problem?

Name _____ Date _____

2. Linda solved 52 × 24 using expanded form:

$$
\begin{array}{rl}
52 = & 50 + 2 \\
\times\ 24 = & 20 + 4 \\
\hline
 & 100 \leftarrow 50 \times 20 \\
 & 200 \leftarrow 50 \times 4 \\
 & 40 \leftarrow 2 \times 20 \\
 & +\ 8 \leftarrow 2 \times 4 \\
\hline
 & 348
\end{array}
$$

A. Check Linda's answer by solving 52 x 24 using the rectangle method.

B. What mistake did Linda make in solving the problem? Use your rectangle to help explain your answer.

The Confused Contessa solved the problems in Questions 7, 8, and 9. Estimate to see if her answers are reasonable. Find her mistakes and draw a circle around the incorrect part. Then, solve the problems correctly using her method.

Contessa's Work | Your Estimate | Your Solution

7.

$$\begin{array}{rl} 63 = & 60 + 3 \\ \times\ 27 & 20 + 7 \\ \hline & 21 \leftarrow \boxed{7 \times 3} \\ & 420 \leftarrow \boxed{7 \times 60} \\ & 140 \leftarrow \boxed{20 \times 7} \\ & +\ 60 \leftarrow 20 \times 3 \\ \hline & 641 \end{array}$$

Contessa's Work | Your Estimate | Your Solution

8.

$$\begin{array}{r} 95 \\ \times\ 31 \\ \hline \end{array}$$

	90	5	
30	30 x 90 = 270	30 x 5 = 150	270 150 90
1	1 x 90 = 90	1 x 5 = 5	+ 5 515

Multiplication Quiz 1 Feedback Box

	Expec-tation	Check In	Comments
Show how to use place value in multiplication [Q#7,8].	E1		
Estimate products [Q#6A].	E3		
Multiply multidigit numbers [Q#1–12]. • Using mental math [Q#6B] • Using rectangles [Q#8] • Using expanded form [Q#7] • Using the all-partials method [Q#9]	E4		
Choose appropriately from among mental math strategies and paper-and-pencil methods to multiply multidigit numbers [Q#1–12].	E6		

What Is Needed to Inform Instruction

What does it look like to “Get it”?

- Create explicit expectations;
- Assess those expectations;

  Provide tasks that “force the issue” and attends to interconnections

  Provide tools and routines for feedback

A more in-depth analysis asks teachers to be aware of the “big ideas” in mathematics and then to connect the identified standards to these ideas.

-- Dacey & Lynch((2007)

What Is Needed to Inform Instruction

- What does it look like to “Get it”?
- What has happened to help students “Get it”?
- So, now what should happen?

What Is Needed to Inform Instruction

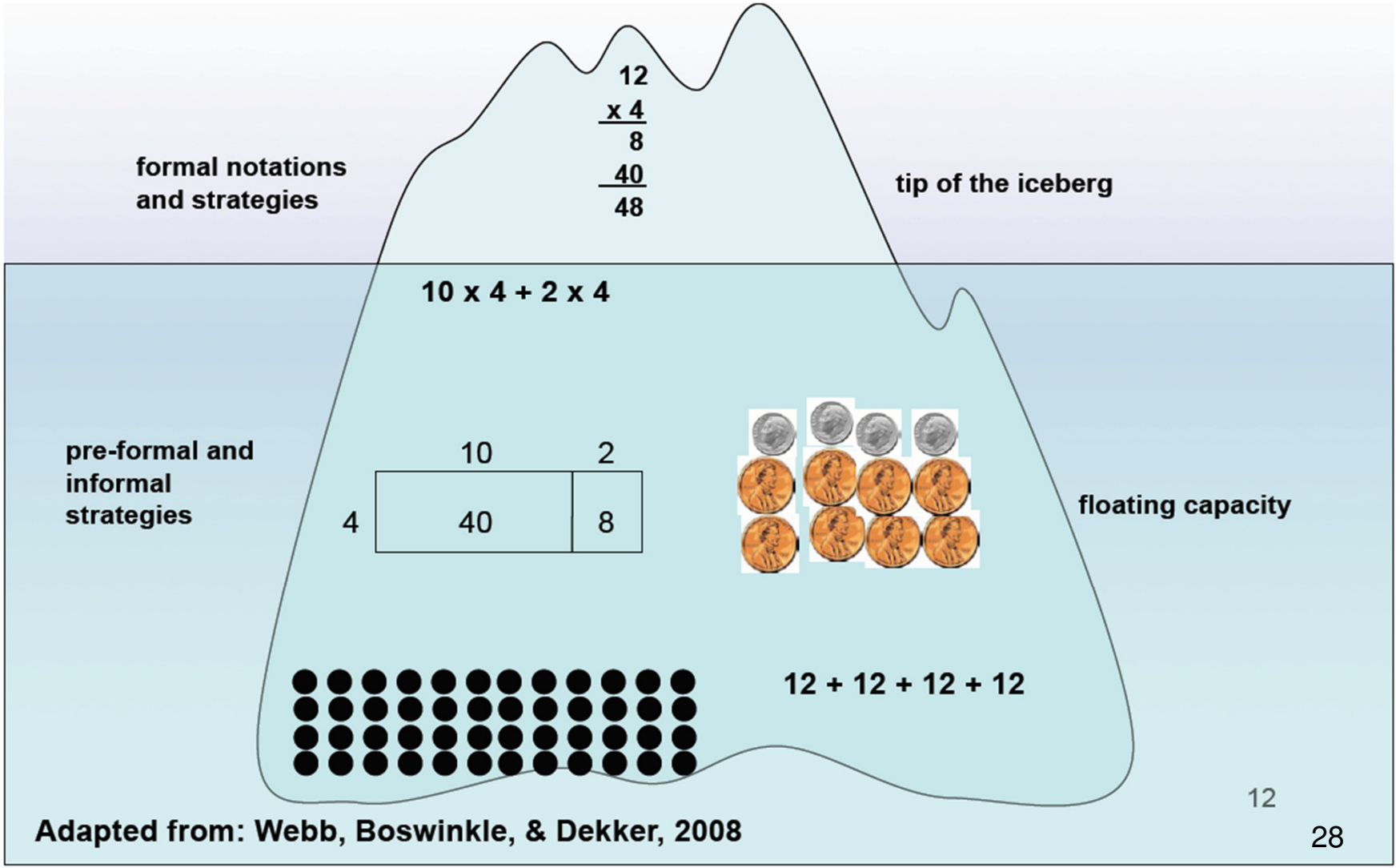
What has happened to help students "Get it"?

What representations have they been using?

What have they been asked to do with them?

How did students "enter" this next string of content?

The Iceberg Model



"The iceberg model supports teacher thinking about learning processes and strategies used by students (Boswinkle & Morelands, 2001). This model has proven to be a powerful metaphor for illustrating how students need to experience a broad range of mathematical models to make sense of formal mathematical representations."

- Webb, Boswinkle, & Dekker, 2008

"To find one's way around the mathematical terrain, it is important to see how the various representations connect with each other, how they are similar, and how they are different. The degree of students' conceptual understanding is related to the richness and extent of the connections they have made."

- National Research Council, 2001

Multiplication Strategies Menu

Breaking into tens and ones

Using Expanded Form

$$\begin{array}{r} 23 \\ \times\ 6 \\ \hline \end{array} = \begin{array}{r} 20 + 3 \\ \times\ 6 \\ \hline 120 + 18 = 138 \end{array}$$

or

	20	3
6	6 × 20 = 120	6 × 3 = 18

$$\begin{array}{r} 120 \\ +\ 18 \\ \hline 138 \end{array}$$

Using All-Partials

$$\begin{array}{r} 23 \\ \times\ 6 \\ \hline 18 \\ +\ 120 \\ \hline 138 \end{array} \quad \textit{or} \quad \begin{array}{r} 23 \\ \times\ 6 \\ \hline 120 \\ +\ 18 \\ \hline 138 \end{array}$$

Compact Method

$$\begin{array}{r} {\scriptstyle 1}\,23 \\ \times\ 6 \\ \hline 138 \end{array}$$

Other ways to use simpler problems

Thinking about money

27 × 4

$27 \times 4 = 25 \times 4 + 2 \times 4$
$= 100 + 8$
$= 108$

Using Simpler Numbers

48 × 6

I know 48 + 2 = 50.
So, 50 X 6 = 300 and 2 x 6 = 12.
Then I subtracted 300 – 12 = 288.

Another Strategy: _____

Multiplication Strategies Menu for Larger Numbers

Using Rectangles

6 × 623

6	600	20	3
	6 × 600 = 3600	6 × 20 = 120	6 × 3 = 18

3600
120
+ 18
3738

Using Expanded Form

623 = 600 + 20 + 3
× 6 × 6
3600 + 120 + 18 = 3738

Using All-Partials

623
× 6
18
120
+ 3600
3738

or

623
× 6
3600
120
+ 18
3738

Using the Compact Method

1 1
623
× 6
3738

Thinking About Money

127 × 4

127 = 100 + 25 + 2
I think: 4 dollars + 4 quarters
+ (2 × 4) pennies
= 400 + 100 + 8
= 508

Using Simpler Numbers

298 × 4

I know 298 + 2 = 300.
So, 300 × 4 = 1200 and 2 × 4 = 8.
Then I subtracted 1200 − 8 = 1192.

Halving and Doubling

264 × 5

I know multiplying by 10 is easier than multiplying by 5. I double 5 to 10 and I take $\frac{1}{2}$ of 264, which is 132. 132 × 10 = 1320. Or I could multiply 264 × 10 = 2640 and take half of that: 1320.

Multiplication Strategies Menu

Paper-and-Pencil Methods

Using Rectangles

7 × 326

	300	20	6
7	7 × 300 = 2100	7 × 20 = 140	7 × 6 = 42

2100
140
+ 42
2282

28 × 63

	60	3
20	20 × 60 = 1200	20 × 3 = 60
8	8 × 60 = 480	8 × 3 = 24

1200
480
60
+ 24
1764

All Partials

326
× 7
2100 ← 300 × 7
140 ← 20 × 7
+ 42 ← 6 × 7
2282

63
× 28
1200 ← 60 × 20
480 ← 60 × 8
60 ← 3 × 20
+ 24 ← 3 × 8
1764

Expanded Form

63 = 60 + 3
× 28 = 20 + 8
1200
480
60
24
1764

326 = 300 + 20 + 6
× 7 × 7
2100 + 140 + 42 = 2282

Compact

1 4
326
× 7
2282

2
63
× 28
504
1260
1764

Combination

63
× 28

63 × 20 = 1260 + 2 63 × 8 = 504 = 1764

Multiplication Strategies Menu

Mental Math and Estimation

Using Simpler Numbers

$205 \times 8 = (200 \times 8) + (5 \times 8)$

$= 1600 + 40$

$= 1640$

$72 \times 99 = (72 \times 100) - 72$

$= 7200 - 72$

$= 7128$

Using Convenient Numbers to Estimate

63×28

$60 \times 30 = 1800$

1800 is a reasonable estimate.

Halving and Doubling

264×5

I know multiplying by 10 is easier than multiplying by 5. I double 5 to 10 and I take $\frac{1}{2}$ of 264 which is 132. $132 \times 10 = 1320$. Or I could multiply $264 \times 10 = 2640$ and take half of that: 1320.

Finding a Range

45×65

$40 \times 60 = 2400$

$50 \times 70 = 3500$

The exact answer will be between 2400 and 3500, or about 3000.

What Is Needed to Inform Instruction

- What does it look like to “Get it”?
- What has happened to help students “Get it”?
- **So, now what should happen?**

What are typical “so, now what” strategies?

- Lots of practice of dd x dd problems
- “Going through the same door” again and again and still not getting it. . .
- Grouping students by general ability
- Nothing . . .teachers just keep going
- Messages to teachers at the end of the unit in little boxes

What does research say about "now what" ?

- Provide instruction that explicitly helps connect models, representations, and concepts. (Fuson, 1992; Ginsburg,1997)

- Provide (but guide) student choices of strategies. (Gersten and Chard, 2001)

What does research say about "now what" ?

- Practice on computational procedures should be designed to build on and extend understanding. (National Research Council, 2001)
- Develop shorter, carefully constructed problem sets because they may be more effective in helping develop fluency in facts and procedures. (Diezman et al, 2003)

What does research say about "now what" ?

- Emphasis on thinking strategies (National Research Council, 2001)
- Develop shorter, carefully constructed problem sets because they may be more effective in helping develop fluency in facts and procedures. (Diezman et al, 2003)

What does research say about "now what" ?

- Facilitate opportunities that provoke a stumble due to superficial understanding
- Ask students to analyze other's solutions
- Ask students to reason with tools
- Scaffold opportunities that help students uncover mathematical ideas
- Need to enter content through a different door

Our “so, now what” design goals

- Put the practice and intervention where it is needed
- Build time right in (rather than Part 3 or in the boxes)
- Create models/tasks that are transportable
- Include “bottomless pits” of practice
- Something for everyone
- Create Targeted Practice based on expectations

Grade 4 Unit 11 Outline
Multiplication with Larger Numbers

	Sessions	Description	Supplies
LESSON 1 **From the Fish Hatchery** SG pages 404–407 DAB pages 581–592 HP Parts 1–2 page 574 URG pages 33–55 DPP A–D	2	**ACTIVITY:** Student extend their understanding of multiplication concepts to multiplication of 2-digit by 2-digit numbers. They use mental math, rectangles, and expanded form to find solutions. **EXPECTATIONS:** E1, E2, E3, E4, E6, E7	
LESSON 2 **All-Partials Revisited** SG pages 408–413 DAB page 593 URG pages 56–76 DPP E–H	2	**ACTIVITY:** Students explore another paper-and-pencil method for multiplication of 2-digit by 2-digit numbers, the all-partials method. They compare this method to using the rectangle model and the expanded form. **EXPECTATIONS:** E1, E2, E3, E4, E6, PSE1, PSE3, PSE5 **ASSESSMENT PAGES:** *Multiplication Quiz 1*, Unit Resource Guide, Pages 69–71	
LESSON 3 **Workshop: Multiplication with Two-Digit Numbers** SG pages 414–416 DAB pages 595–610 HP Parts 3–7 pages 575–576 URG pages 77–95 DPP I–J	1–2	**ACTIVITY:** Students practice solving multiplication problems involving 2-digit numbers. This Workshop allows students to choose from several problem sets and a game to review multiplication concepts and practice multiplication methods. **EXPECTATIONS:** E1, E2, E3, E4	• *Multiplication Strategies Menu* created in Lesson 2

	Sessions	Description	Supplies
LESSON 4 **Compact Multiplication Revisited** SG pages 417–423 URG pages 96–116 DPP K–N	2–3	**ACTIVITY:** Students review the compact method of multiplication and then use it to multiply by two-digit numbers. They compare all paper-and-pencil methods and practice multiplication. **EXPECTATIONS:** E1, E2, E3, E4, E5, E6, E8 **ASSESSMENT PAGE:** *Multiplication Quiz 2*, Unit Resource Guide, Pages 107–110	• *Multiplication Strategies Menu* created in Lesson 2
LESSON 5 **Choosing Strategies to Multiply** AB pages 57–70 SG pages 424–428 DAB pages 611–615 HP Parts 8–10 pages 577–578 URG pages 117–141 DPP O–R	2	**ACTIVITY:** Students apply strategies involving estimation, mental math, and paper-and-pencil methods to solve problems involving two-digit multiplication. This lesson focuses on choosing and applying strategies flexibly to match the context of problems. Students also use these strategies flexibly to solve an extended, open-response problem involving two-digit multiplication. **EXPECTATIONS:** E3, E4, E5, E6, E7, PSE2, PSE3, PSE5	• *Multiplication Strategies Menu* created in Lesson 2 • rulers • *Division Facts I Know* charts

CONNECTIONS

A current list of connections is available at www.mathtrailblazers.com.

Literature

Suggested Titles

- Haskins, Jim. *Count Your Way Through the Arab World*, illustrated by Dana Gustafson. Carolrhoda Books, Minneapolis, 1987.
- Ifrah, Georges. *The Universal History of Numbers: From Prehistory to the Invention of the Computer.* John Wiley & Sons, New York, 2000.
- St. John, Glory. *How to Count Like a Martian.* Henry Z. Walch, New York, 1975.
- Schmandt-Besserat, Denise. *The History of Counting*, illustrated by Michael Hays. Morrow Junior Books, New York, 1999.
- Sitomer, M., and H. Sitomer. *How Did Numbers Begin?*, illustrated by Richard Cuffari. Thomas J. Crowell, New York, 1976.

Targeted Practice

All students need…

- Attention to conceptual underpinnings of the content and skills, and
- Opportunities to articulate concepts and communicate solutions.

Targeted Practice

1) Students who are "working on it" and need some extra help should circle the problem set marked with a star (★). These problems provide scaffolded support for developing the essential underlying concepts as well as some opportunities for practice.
2) Students who are "getting it" and just need more practice should circle the problem set marked with a circle (●). These problems mainly provide opportunities to practice with some concept reinforcement and some opportunities for extension.
3) Students who have "got it" and are ready for a challenge or extension should circle problems marked with a square (■). These problems provide some practice and then move into opportunities for extension.

Pretend you are a student learning this content, choose your practice.

Multiplication with 2-Digit Numbers Workshop Menu

- Look at each row in the Can I Do This? column.
- For each row, decide whether you are "Working On It", whether you are "Getting It", or whether you already know it (Got It).
- Remember, you may feel you are "Working On It" for one row, but for another row, you know you have already "Got It."
- On this page, draw a circle around each set of problems you decide to do.
- If one set of problems seems too easy or too hard, choose a different set from the same row.

Can I Do This?	Working On It!	Getting It!	Got It!
Show that I **understand multiplication methods** **Make connections** between methods	*Discovery Assignment Book* ★ Q# 1–3	*Discovery Assignment Book* ● Q# 2–4	*Discovery Assignment Book* ■ Q# 3–4
Estimate products	*Discovery Assignment Book* ★ Q# 12, 13, 21–24	*Discovery Assignment Book* ● Q# 12, 13, 21–26	*Discovery Assignment Book* ■ Q# 13, 21, 25–27
Use different methods to **multiply 2-digit numbers**	*Student Guide* ★ Q# 1–6, 8, 11 *Discovery Assignment Book* ★ Q# 5–8, 12–15, 20	*Student Guide* ● Q# 4, 5, 7, 8, 11 *Discovery Assignment Book* ● Q# 7–9, 11–18, 20	*Student Guide* ■ Q# 4, 6, 7, 9–11 *Discovery Assignment Book* ■ Q# 7–11, 13, 16–20

Show that I **understand multiplication methods**	*Discovery Assignment Book* Play *Multiplication Digits Game* ★ ● ■

★1. Irma solved 12 x 52 using rectangles.

	50	2
10	$50 \times 10 = 500$	$10 \times 2 = 20$
2	$50 \times 2 = 100$	4

$$\begin{array}{r} 500 \\ 100 \\ 20 \\ +\ \ 4 \\ \hline 624 \end{array}$$

A. Use Irma's rectangle to fill in the blank boxes for the same problem using the all-partials method.

$$\begin{array}{r} 12 \\ \times\ 52 \\ \hline \square \\ \square \\ 20 \\ +\ \ 4 \\ \hline 624 \end{array}$$

B. What numbers did Irma multiply in both problems to get 500?

●■ 4. Professor Peabody's cat had muddy feet and walked across some problems. Fill in the missing spots to show each solution correctly.

A. 156 × 42

	100	50	6
40	40 × 30 = 4000	40 × 50 =	
2		2 ×	12

4000 + 6552

B. 64 × 18

64 × 18 = 64 × (10 + 8)

600

40

32

1152

C.

89 × 46

360

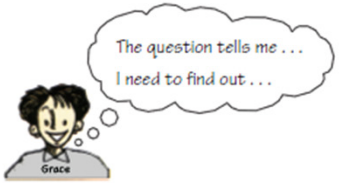
480

+ 54

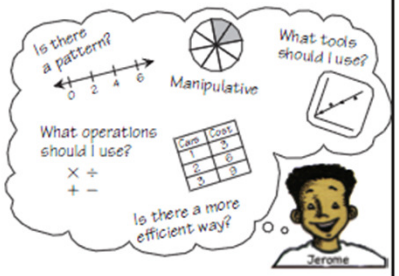
Math Practices

Solving a problem:

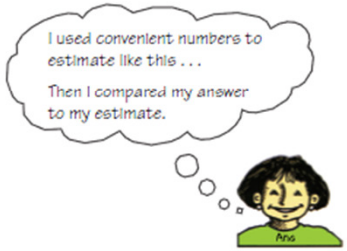
1. Know the problem. I read the problem carefully. I know the questions to answer and what information is important.



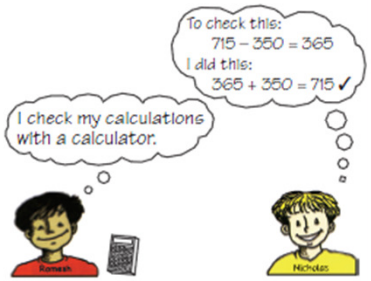
2. Find a strategy. I choose good tools and an efficient strategy for solving the problem.



3. Check for reasonableness. I look back at my solution to see if my answer makes sense. If it does not, I try again.

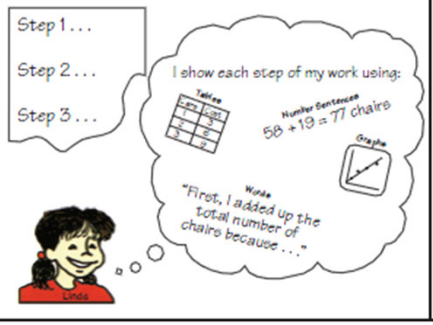


4. Check my calculations. If I make mistakes, I correct them.

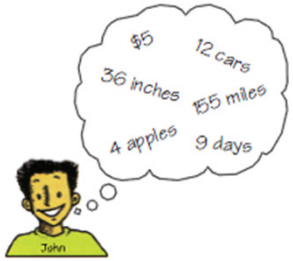


Showing or telling how I solve a problem:

5. Show my work. I show or tell how I arrived at my answer so that someone else can understand my thinking.



6. Use labels. I use labels to show what numbers mean.



Hour Walk

If you walked steadily for an hour, about how many steps would you take?

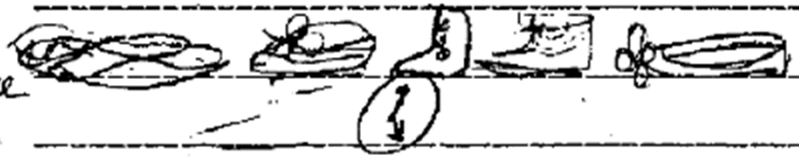
A. 500 **B.** 1000 **C.** 5000 **D.** 10,000 **E.** 50,000 **F.** 100,000

Make an estimate without walking for one hour. Explain how you made your estimate. Show all your work.

Michael's Work

We decided to multiply 100 by 60 because we took 100 steps and there are 60 minutes in an hour.

We picked 5000 steps because all three of us in our group walk about 100 steps per minute for 60 minutes. 100 × 60 = 6,000 and 5,000 is the closest to 6,000 steps.



Name	Number of steps in a minute
Michael	112
Roberto	97
Jessie	108

Keenya's Work

6,000 is closest to our answer

1,059 steps 105 steps
10:00 min. 1:00 min.

My partner and I walked for 10 min. We both got exactly 1,059 steps since we both have the same answer we don't have to averge out our work 10 x 6 is 60 and thiers are 60 min. In an hour we multiply 10.59 x 6 = 6,354 step thats our answer if we walked 1 hour

We also walked for 1 min. Again we got the same answer. It was 105 steps. 1 x 60 = 60 so we multiplyed 105 x 60 We got 6,350 steps that's our other answer for how many steps it would take for 1 hour. We know we our close because the our answers are only 4 steps away We can find a better answer if we average out our answers than we divided 12,704 ÷ 2 = 6,352 This is the best answer we can come up with.

Time	How many steps	answer
10 min	1059	6,354
1 min.	105	6,350

First we added

6354
+6350
12,704

Research to Practice: You Know What They Know, So, Now What?

Jennifer Mundt Leimberer (jleimb1@uic.edu)

Teaching Integrated Mathematics and Science (TIMS) Project

Learning Sciences Research Institute, University of Illinois at Chicago

www.lsri.uic.edu go to Projects then go to

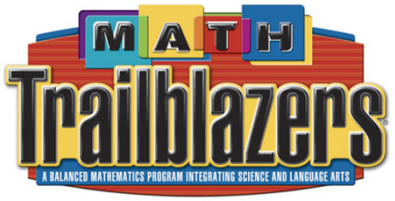
Math Trailblazers Research and Revision Study

43rd NCSM Annual Conference

Indianapolis, Indiana

April 12, 2011

University of Illinois at Chicago
Learning Sciences Research Institute



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