NCTM's Reasoning and Sense Making Initiative: Current Progress and a Look Ahead

A Presentation at the Annual NCSM Conference, Indianapolis, Indiana 2011

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Goals of Presentation

- Background that led to NCTM's Reasoning and Sense Making Initiative for Secondary Mathematics
- Info on the publications, and some sample Reasoning Tasks
- Ongoing events in the Initiative: What's happening now, and in the future

History of Reasoning and Sense Making in NCTM's Principal Position Documents

2000 - *Principles and Standards for School Mathematics*

- Updated the 1989 standards, and incorporated the messages in
  - *Professional Standards for Teaching Mathematics* (1991)
  - *Assessment Standards for School Mathematics (1995)*

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NCTM Process Standards

- *Problem Solving* is an integral part of mathematics learning.
- *Reasoning and Proof* are ways of expressing justification.
- *Communication* is an essential part of mathematics education.
- *Connections* are critical in mathematics, both across mathematical topics and to contexts outside mathematics.
- *Representations* of mathematical ideas are fundamental to enhancing mathematical understanding.

NCTM's Recent Position Documents

2006 - *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence*

- Outlined the most important mathematical topics for each grade level; based on *Principles and Standards;* importance of reasoning and conceptual understanding emphasized

But what about high school mathematics?

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NCTM's Series on Reasoning & Sense Making

The Council has published its official position on the role and importance of focusing on students' reasoning in the teaching of mathematics in the new publication

Focus in High School Mathematics: Reasoning & Sense Making

(NCTM, 2009)

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Goal of FHSM

Focus in High School Mathematics: Reasoning and Sense Making underscores the critical role of the **Process Standards** outlined in NCTM's *Principles and Standards*.

It provides a foundation for achieving the principal goals for the mathematical experiences of all secondary students.

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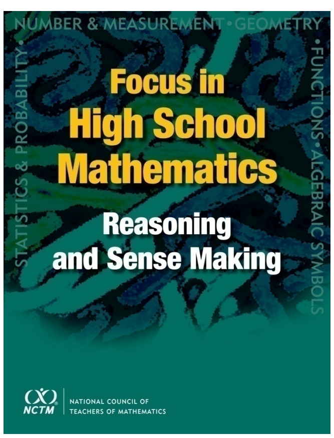

Focus in High School Mathematics

Putting Reasoning and Sense Making at the Center

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Critical Need for the Reasoning and Sense Making Effort

Students need opportunities to:

- Make conjectures
- Share their representations of problems
- Question their peers
- Justify their reasoning
- Pose further questions

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Focus on Reasoning

Reasoning

"The process of drawing conclusions on the basis of evidence or stated assumptions." (FHSM)

- *Justification*
- *Generalization*
- *Building towards proof*

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Focus on Sense Making

Sense making:

"Developing understanding of a situation, context, or concept by connecting it with existing knowledge." (FHSM)

- *Communicating your thinking*
- *Listening to the thinking of others*
- *Connecting the mathematics and the context*
- *Connecting across mathematical ideas*

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From FHSM preface:

"Currently, many students have difficulty because they find mathematics meaningless....

With purposeful attention and planning, teachers can hold all students in every high school mathematics classroom accountable for personally engaging in reasoning and sense making, and thus lead students to experience reasoning for themselves rather than merely observe it." (NCTM 2009, pp. 5–6)

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The role of RSM in School Mathematics

"Reasoning and sense making should occur in every mathematics classroom every day.

In such an environment, teachers and students ask and answer such questions as 'What's going on here?' and 'Why do you think that?'"

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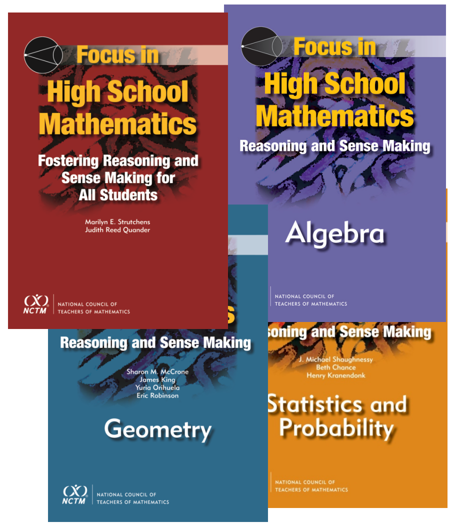
In other words,

"Mathematics is not a spectator sport!"

Focus in High School Mathematics: Reasoning and Sense Making

- **Topic Books**
  - Statistics and Probability
  - Algebra
  - Geometry
  - Equity
  - Technology (2011)



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The High School RSM series

- Each volume in the Reasoning and Sense Making in Mathematics series presents detailed examples of mathematical tasks, along with samples of student reasoning and follow-up discussion.

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Goal: To Nurture and Establish Mathematical Reasoning Habits within our Students

- Analyzing a Problem
- Implementing a Strategy
- Monitoring one's Progress
- Seeking and using Connections
- Reflecting on one's Solutions

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Sample Tasks

Suppose you are a "typical" Algebra II student…

- Find the distance between the points (-2, 5) and (8, -1).

Typical responses:

- I know there's a formula for that. I can't quite remember what it is.
- I think it's x1 plus x2 squared... Or maybe we need to subtract?
- There's a big square root, right?
- Wait, I think we have to divide by 2! Or is that the slope formula?

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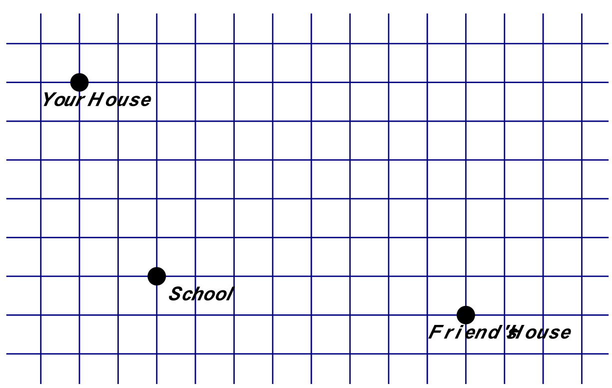
- Chapter 1, FHSM

A Different Approach



1. How many blocks would we have to drive to get from your house to your friend's house? Draw a path you would have to drive and calculate the distance.

2. What if you could use a helicopter to fly straight from your house to your friend's house? Draw the path you would take. How could we find the distance "as the crow flies"?

3. Establish a coordinate-axis system, using the school as the origin. What would the coordinates be for your house? For your friend's house?

4. How could you use the coordinates to calculate the distance from your home to your friend's house?

Eruptions of the Old Faithful Geyser—Becoming a Data Detective

- Data on wait times between successive eruptions (blasts) of geysers were first collected by the National Park Service and the U.S. Geological Survey in Yellowstone National Park.
- The data were collected to establish some baseline information that could then be used to track and compare long-term behavior of geysers.

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Becoming a Data Detective

- In this investigation, you will put on a 'data detective' hat, investigate some data on Old Faithful, and conjecture about the time that someone might expect to wait for Old Faithful to erupt.

—Chapter 2 FHSM: Statistics & Probability

Old Faithful --Minutes Between Blasts

–each row represents about 1 days' data

1) 86 71 57 80 75 77 60 86 77 56 81 50 89 54 90 73 60 83
2) 65 82 84 54 85 58 79 57 88 68 76 78 74 85 75 65 76 58
3) 91 50 87 48 93 54 86 53 78 52 83 60 87 49 80 60 92 43
4) 89 60 84 69 74 71 108 50 77 57 80 61 82 48 81 73 62 79
5) 54 80 73 81 62 81 71 79 81 74 59 81 66 87 53 80 50 87
6) 51 82 58 81 49 92 50 88 62 93 56 89 51 79 58 82 52 88
7) 52 78 69 75 77 53 80 55 87 53 85 61 93 54 76 80 81 59
8) 86 78 71 77 76 94 75 50 83 82 72 77 75 65 79 72 78 77
9) 79 75 78 64 80 49 88 54 85 51 96 50 80 78 81 72 75 78
10) 87 69 55 83 49 82 57 84 57 84 73 78 57 79 57 90 62 87
11) 78 52 98 48 78 79 65 84 50 83 60 80 50 88 50 84 74 76
12) 65 89 49 88 51 78 85 65 75 77 69 92 68 87 61 81 55 93
13) 53 84 70 73 93 50 87 77 74 72 82 74 80 49 91 53 86 49
14) 79 89 87 76 59 80 89 45 93 72 71 54 79 74 65 78 57 87
15) 72 84 47 84 57 87 68 86 75 73 53 82 93 77 54 96 48 89
16) 63 84 76 62 83 50 85 78 78 81 78 76 74 81 66 84 48 93

Task set up

- Pick any row of these wait times so that your ‘group’ has a sample of a day of Old Faithful wait times
- Look over the data. Is there anything that you notice, or anything that you wonder about in your sample of data? Jot down some “*notices*” and “*wonders*.”

Graphical representations

- Create at least one type of visual or graphical representation for that row of data to help to visualize any patterns in the wait times.
- Continue to jot down any additional “*notices*” and “*wonders*” that occur to you.

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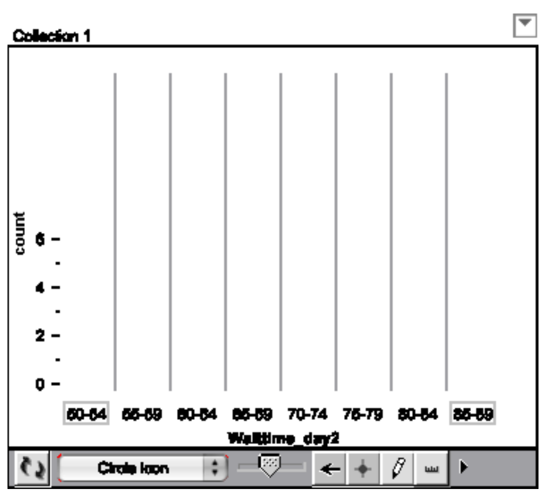
Group C Reasoning 1

- "On the basis of our first frequency graph we'd expect to wait about 75 minutes, because it shows ***most*** wait times for the eruption in the 75 to 79 minute range.

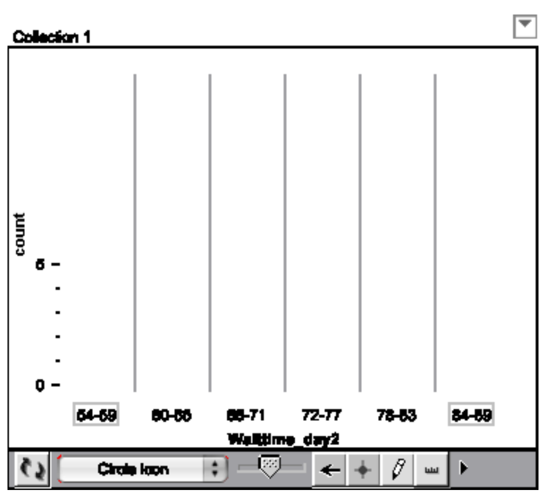
Group C Graph 1



Group C Reasoning 2

- But then we saw that if we chose our intervals in another way we obtained something different. There is no obvious pattern here, and we thought that a person could just as easily wait about 55, or 75, or 85 minutes, because all three of those times were equally frequent in this (second) graph, each occurring 4 times."

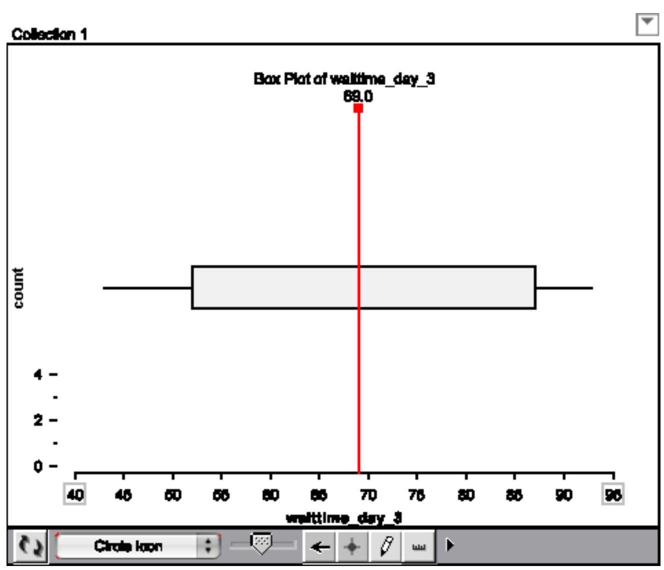
Group C Graph 2



Group D Reasoning

- “The middle 50 percent goes from 65 minutes to 82 minutes for day 2, and from about 53 minutes to 87 minutes for day 3.
- So, overall from the two days combined we concluded that 50 percent of the time you’d probably have to wait at least an hour, and perhaps as much as an hour and 20 minutes.”

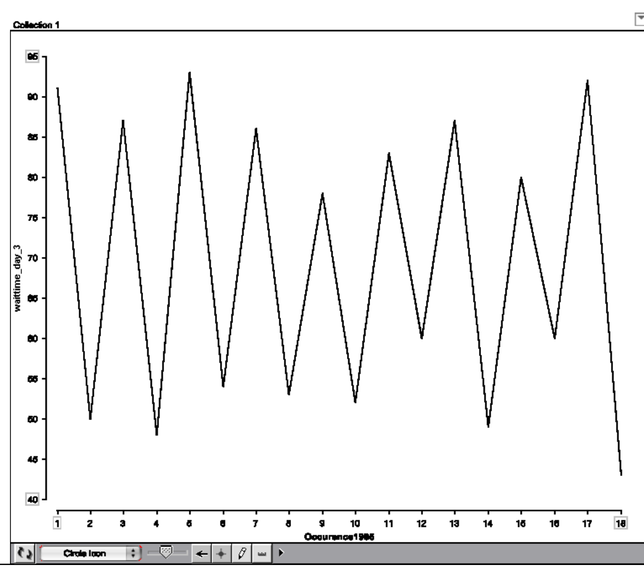
Group D Graph



Group E Reasoning

- “We think we see a pattern in the data. There seems to be an up-down pattern in the wait times in day 3. It was easier to see when we connected the dots in our plot.
- Then we did the same thing for day 2, and the up-down pattern in wait times appears there, too. It’s not always perfect, but a long wait time is usually followed by a short time, and a short one by a long one.”

Group E day 3 graph



Horseshoes in Flight

The height of the horseshoe (measured in feet) as a function of time (measured in seconds and represented by the variable t) from the instant of release is

$$1\,{}^{3}/_{16} + 18t - 16t^2.$$

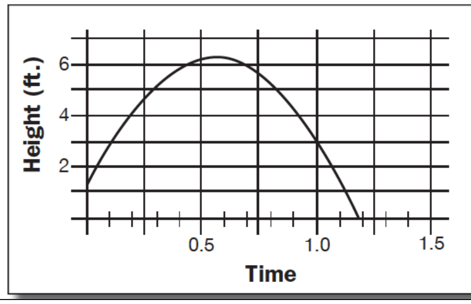
The expressions (a)–(d) below are equivalent:

(a) $1\,{}^{3}/_{16} + 18t - 16t^2$

(b) $-16(t - {}^{9}/_{16})(t + {}^{1}/_{16})$

(c) ${}^{1}/_{16}(19 - 16t)(16t + 1)$

(d) $-16(t - {}^{9}/_{16})^2 + {}^{100}/_{16}$



Update on NCTM's High School RSM Project

- Resources on Web
- Task Development work
- Video task force
- Summer Interactive Conference

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Resources on the Web

- www.nctm.org/hsfocus
- Includes:
  - PowerPoint and speaker notes
  - Executive summary
  - Summaries for various audiences (teachers, students, families, administrators)
  - Sample tasks
- Goal: To build this to become a RSM portal

Task Writing Task Force

- Goal: To create a library of tasks that teachers could use in their classrooms
- A set of five initial tasks is being revised by the task force to:
  - Be more user-friendly
  - Better highlight connections with RSM
- An additional set is being created.

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Video Task Force

- Goal: To create video examples of RSM in the high school mathematics classroom
- Video accompanied by student work with light scribe pens is being assembled
- Pieces of the first prototype have been developed and will be pulled together into a finished example this year

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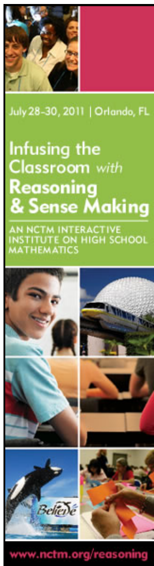
July 28–30, 2011 | Orlando, Florida

Infusing the Classroom *with* **Reasoning & Sense Making**

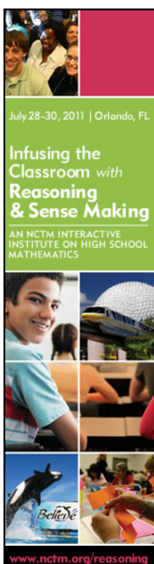
AN NCTM INTERACTIVE INSTITUTE ON HIGH SCHOOL MATHEMATICS

It's not your typical NCTM conference!



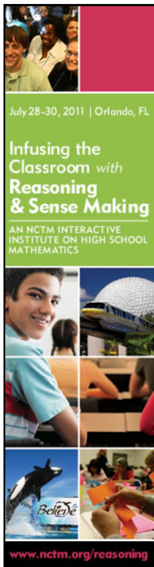
What, When, Where?

- 2.5 day interactive institute
- Orlando, FL   July 28-30, 2011
- Session types
  - Keynotes, workshops, task groups, discussion groups
- Strands: beginning algebra, geometry, intermediate algebra, prob & stats



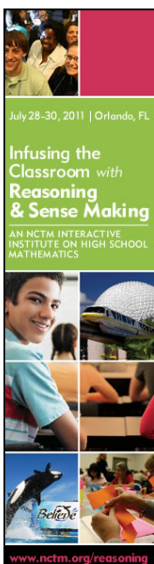
Session Types

- **Keynote Sessions** – Well known mathematics leaders will address critical topics related to and supporting reasoning and sense making.
- **Workshops** – Practitioners engage participants in hands-on activities that promote reasoning and sense making.
- **Task Groups** – Participants register for a strand (beginning algebra, intermediate algebra, geometry, probability and statistics) and experience working through a task as a new learner. In the third Task Group Session participants will develop or modify a task using a template provided.
- **Discussion Groups** - Facilitated discussion among small groups of role-alike or interest-alike people. Discussion will help guide participants in the development of their action plan



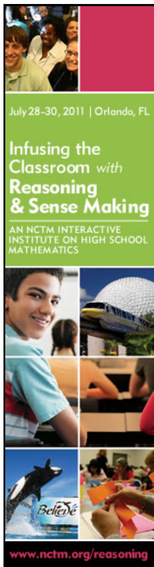
Outcomes

- understand that reasoning and sense making are integral to the teaching of the *Common Core State Standards for Mathematics*, state standards, and provincial guidelines.
- learn strategies for creating and maintaining a high school classroom infused with reasoning and sense making.
- learn how to use, select and develop mathematical tasks to reveal student thinking and engage students in reasoning and sense making.
- create an action plan for their school or practice informed by institute experiences.



Location, Location, Location

Renaissance Orlando at Sea World



Who?

- Classroom teachers
- Supervisors, Leaders, Administrators
- Mathematics Teachers Educators
- PD providers
- Pre-service teachers

YOU!

NCTM's new Reasoning and Sense Making Initiative--Summary

- Special Conference this summer! --An Interactive Institute with a Focus on RSM for Secondary Teachers
- Creation of Video Clips and supporting materials—examples of secondary students engaged in RSM in classrooms
- Development and Posting of examples of RSM tasks on the NCTM website—there are five up there now, more on the way!

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Reasoning and Sense Making has long been a goal of instruction for NCTM

Reasoning and sense making is an evolution of NCTM's longstanding position that problem solving should be the emphasis of mathematics teaching and learning.

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"The processes of mathematics—Problem Solving, Reasoning and Proof, Connections, Communication, and Representation—are all manifestations of the act of making sense and of reasoning …"

National Council of Teachers of Mathematics (2009). *Focus in High School Mathematics: Reasoning and Sense Making*. Reston, VA, p.5

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Reasoning and Sense Making

"A focus on **reasoning and sense making**, when developed in the context of important content, will ensure that students can accurately carry out mathematical procedures, understand why those procedures work, and know how they might be used and their results interpreted."

----goals of Reasoning and Sense Making

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Discussion Questions

- What impact might the High School RSM project have on your work as a teacher leader?
- What questions or suggestions do you have?

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Focus on Reasoning and Sense Making

"Reasoning and sense making is a way to approach instruction no matter what content you are teaching." (FHSM)

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THANK YOU FOR COMING

Now, go forth, and REASON!

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HORSESHOES IN FLIGHT

Example 5 from *Focus in High School Mathematics: Reasoning and Sense Making*, NCTM

Purpose	Students analyze the structure of algebraic expressions and a graph to determine what information each expression readily contributes about the flight of a horseshoe. This task is particularly relevant to students who are studying (or have studied) various quadratic expressions (or functions). The task also illustrates a step in the mathematical modeling process that involves interpreting mathematical results in a real-world context.	
Focus on Reasoning and Sense Making	***FHSM* Reasoning Habits** Analyzing a problem—identifying relevant representations; looking for hidden structure Reflecting on a solution—interpreting a solution; justifying or validating a solution ***PSSM* Process Standards** Reasoning and proof—recognize reasoning and proof as fundamental aspects of mathematics Connections—recognize and apply mathematics in contexts outside of mathematics Representation—select, apply, and translate among mathematical representations to solve problems	***CCSS* Mathematical Practices** 4. Model with mathematics. 7. Look for and make use of structure.
Focus on Mathematical Content	***FHSM* Key Elements** Reasoning with algebraic symbols—meaningful use of symbols; linking expressions and functions	***CCSS* Content Standards** A-SSE-3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
Materials and Technology	Horseshoes in Flight activity sheet	

In the Classroom

Task:

Derive information about the flight of a horseshoe from a graph and four equivalent expressions that describe its flight.
(See the attached activity sheet.)

Use in the Classroom

After introducing the task and distributing the activity sheet, you might begin by having students work individually or in groups to come up with their own conclusions.

If students have difficulty getting started on the task, you might ask, "Just by looking at the four expressions, (a)–(d), without doing any computation, can you tell me one thing that you know about the flight of the horseshoe?"

A class discussion of possible responses might follow, encompassing the questions on the activity sheet. Question 1 asks which expression is most useful for finding the maximum height of the horseshoe. You might have different students present their solutions and the reasoning behind them. Question 2 probes other conclusions that students can draw about the horseshoe's flight from the other expressions. If the students are having trouble with question 1, discussing question 2 first might be easier and more productive.

After the students have discussed what they can conclude from the other expressions, ask questions to focus their attention on the relationships among their conclusions about the values of the expressions, where those values show up on the graph, and what those values mean in the context.

You might conclude by having the class observe that although these expressions are equivalent, each one provides different insights into the situation—an observation that will naturally raise the question, Are there other situations like this one?

Asking these types of questions may assist students in realizing that they need to analyze the problem by looking at the structure and various parts of the algebraic expressions.

Question 1 requires students to choose an appropriate representation and justify their choice.

In responding to question 2, students move back and forth between the parts of the mathematical models and the real-world context as they analyze expressions, draw mathematical conclusions, and interpret the results in context.

Reflecting on the solutions can suggest or strengthen students' belief in the potential of a mathematical reasoning strategy that employs multiple mathematical representations in other situations.

Focus on Student Thinking

In responding to question 1, students might note that since the term $-16(t - 9/16)^2$ is always negative or zero, expression (d) makes it clear immediately that the height can never go above 100/16, or 6 1/4, feet, and it reaches this height at $t = 9/16$ seconds, which is when that first term has the value 0.

Alternatively, if students look at the graph, they might notice that the maximum height is a little more than 6 feet and is reached after just slightly more than 0.5 seconds have elapsed. They could then use this graphical information to help narrow the group of expressions to determine which one would be most useful for finding the maximum height.

In considering question 2, students might note that substituting in $t = 0$ in expression (a) reveals that the initial height of the horseshoe is 1 3/16 feet, which would be where the thrower held the horseshoe at the start of the toss. This is also the y-intercept of the graph.

Students might note that expressions (b) and (c) are essentially the same, since they are factored forms of the original expression. Such forms are useful in finding the zeros, or x-intercepts, which in this case are – 1/16 and 19/16. The negative zero isn't useful in considering the flight of the horseshoe, since the flight begins at $t = 0$. However, the positive zero tells when the height of the horseshoe returns to 0 feet, which is when it hits the ground. In looking at the graph, this will be at about 1.2 seconds, which is close to 19/16 seconds.

In looking at the factored forms, (b) and (c), students might also note that since the graph is a parabola and parabolas are symmetric, the maximum height should occur at the midpoint of the two zeros, which would be 19/16 seconds, the same value as shown in expression (d).

Assessment

To gain insight into issues that you might need to address in the discussion, walk around and observe what the students initially do with the task.

For homework or as a summative assessment, you might assign one of the following tasks:

- Ask students to write up their conclusions about the task. Alternatively, give them an expression with different coefficients and ask them to use that expression to analyze the path of the horseshoe (or some other object).
- Ask students to write an expression giving the profit that Crumbly Cookies will make in the situation described below and then analyze what they can conclude from the original expression or equivalent expressions:

  A survey showed that if Crumbly Cookies sells a dozen cookies for x dollars per dozen, it will sell $1000 – 200x$ dozens. Their cost to make a dozen cookies is about $1.
- Give the students an expression and ask them first to come up with a situation that that it might describe and then to find out what they can about that situation by analyzing the expression and equivalent forms of it.

Source

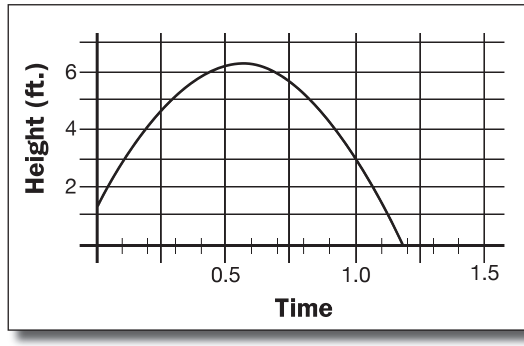
National Council of Teachers of Mathematics (NCTM). *Focus in High School Mathematics: Reasoning and Sense Making*. Reston, Va.: NCTM, 2009. Example 5, pp. 32–33.

HORSESHOES IN FLIGHT

Student Activity Sheet

As shown in the graph, the height of a thrown horseshoe depends on the time that has elapsed since its release. (Note that this graph of the horseshoe's height is parabolic, but that it is not the same as the graph of the horseshoe's flight path.)

The height of the horseshoe (measured in feet) as a function of time (measured in seconds and represented by the variable t) from the instant of release is



$$1\,{}^{3}\!/_{16} + 18t - 16t^2.$$

The expressions (a)–(d) below are equivalent:

(a) $1\,{}^{3}\!/_{16} + 18t - 16t^2$

(b) $-16(t - {}^{9}\!/_{16})(t + {}^{1}\!/_{16})$

(c) ${}^{1}\!/_{16}(19 - 16t)(16t + 1)$

(d) $-16(t - {}^{9}\!/_{16})^2 + {}^{100}\!/_{16}$

1. Which expression is the most useful for finding the maximum height of the horseshoe, and why is it the most useful expression?

2. What information can you conclude about the horseshoe's flight from other equivalent expressions? Explain your answers.