Mathematics
Common Core State Standards

The user has control

- Sometimes a tool is just right for the wrong use.

Old Boxes

- People are the next step
- If people just swap out the old standards and put the new CCSS in the old boxes
  - into old systems and procedures
  - into the old relationships
  - Into old instructional materials formats
  - Into old assessment tools,
- Then nothing will change, and perhaps nothing will

Standards are a platform for instructional systems

- This is a new platform for better instructional systems and better ways of managing instruction
- Builds on achievements of last 2 decades
- Builds on lessons learned in last 2 decades
  - Lessons about time and teachers

Grain size is a major issue

- Mathematics is simplest at the right grain size.
- “Strands” are too big, vague e.g. “number”
- Lessons are too small: too many small pieces scattered over the floor, what if some are missing or broken?
- Units or chapters are about the right size (8-12 per year)
- STOP managing lessons,
- START managing units

What mathematics do we want students to walk away with from this chapter?

- Content Focus of professional learning communities should be at the chapter level
- When working with standards, focus on clusters. Standards are ingredients of clusters. Coherence exists at the cluster level across grades
- Each lesson within a chapter or unit has the same objectives….the chapter objectives

Social Justice

- Main motive for standards
- Get good curriculum to all students
- Start each unit with the variety of thinking and knowledge students bring to it
- Close each unit with on-grade learning in the cluster of standards

Why do students have to do math problems?

1. to get answers because Homeland Security needs them, pronto
2. I had to, why shouldn't they?
3. so they will listen in class
4. to learn mathematics

Why give students problems to solve?

To learn mathematics.

Answers are part of the process, they are not the product.

- The product is the student's mathematical knowledge and know-how.
- The 'correctness' of answers is also part of the process. Yes, an important part.

Wrong Answers

- Are part of the process, too
- What was the student thinking?
- Was it an error of haste or a stubborn misconception?

Three Responses to a Math Problem

1. Answer getting
2. Making sense of the problem situation
3. Making sense of the mathematics you can learn from working on the problem

Answers are a black hole: hard to escape the pull

- Answer getting short circuits mathematics, making mathematical sense
- Very habituated in US teachers versus Japanese teachers
- Devised methods for slowing down, postponing answer getting

Answer getting vs. learning mathematics

- USA:

How can I teach my kids to get the answer to this problem?

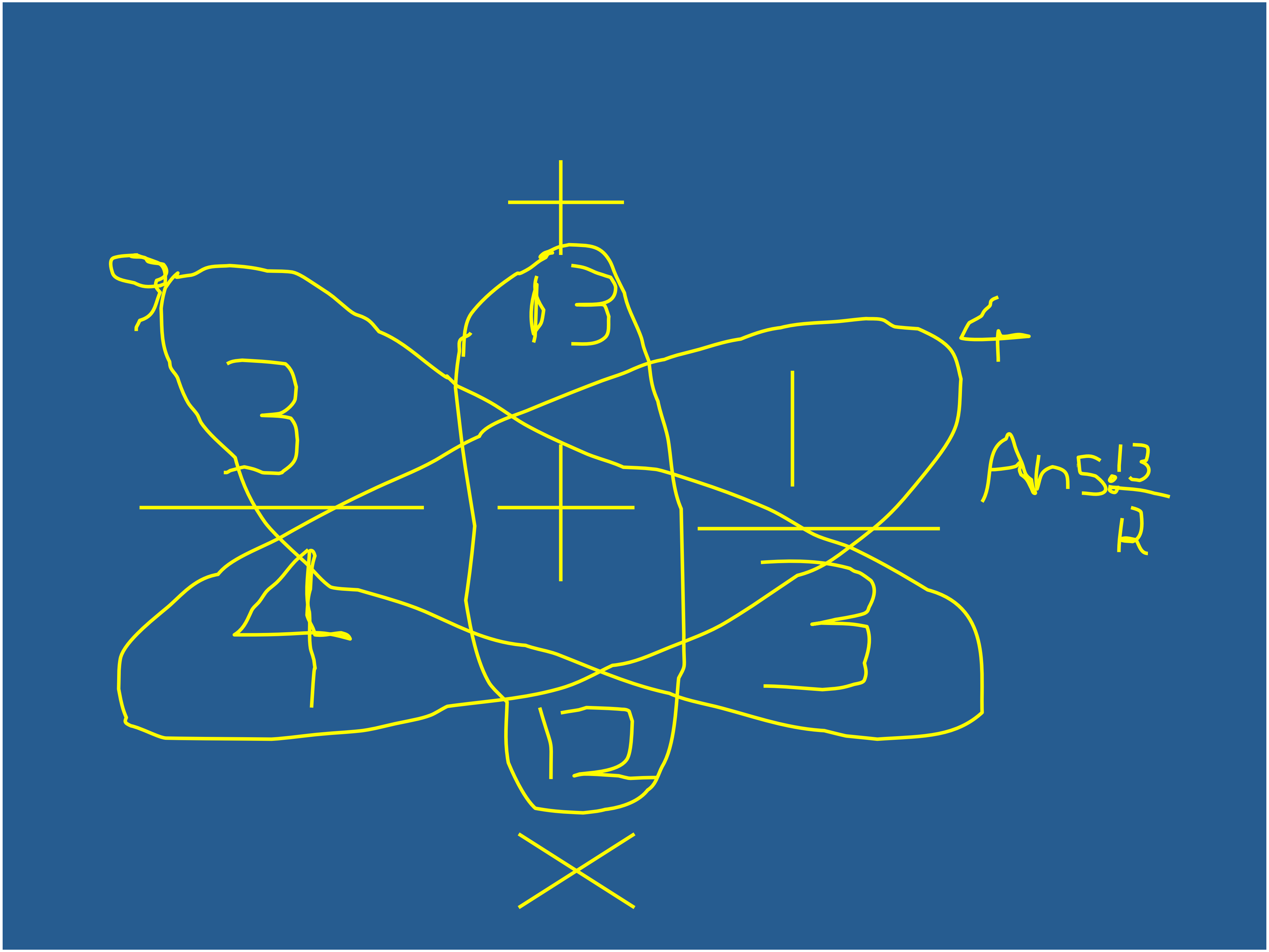
Use mathematics they already know. Easy, reliable, works with bottom half, good for classroom management.

- Japanese:

How can I use this problem to teach the mathematics of this unit?

Butterfly method

$$\frac{3}{4} + \frac{1}{3}$$


13
3
4
1
3
4
12
Ans: 13/12

Use butterflies on this TIMSS item

$1/2 + 1/3 + 1/4 =$

Set up

- Not:
  - “set up a proportion and cross multiply”
- But:
  - Set up an equation and solve
- Prepare for algebra, not just next week’s quiz.

Foil FOIL

- Use the distributive property
- It works for trinomials and polynomials in general
- What is a polynomial?
- Sum of products = product of sums
- This IS the distributive property when “a” is a sum

Canceling

$x^5/x^2 = x \cdot x \cdot x \cdot x \cdot x / x \cdot x$

$x^5/x^5 = x \cdot x \cdot x \cdot x \cdot x / x \cdot x \cdot x \cdot x \cdot x$

Standards are a peculiar genre

1. We write as though students have learned approximately 100% of what is in preceding standards. This is never even approximately true anywhere in the world.
2. Variety among students in what they bring to each day's lesson is the condition of teaching, not a breakdown in the system. We need to teach accordingly.
3. Tools for teachers…instructional and assessment…should help them manage the variety

Differences among students

- The first response, in the classroom: make different ways of thinking students' bring to the lesson visible to all
- Use 3 or 4 different ways of thinking that students bring as starting points for paths to grade level mathematics target
- All students travel all paths: robust, clarifying

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COMMON CORE STATE STANDARDS FOR

Mathematics

COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

Mathematical Practices Standards

1. Make sense of complex problems and persevere in solving them.
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

College and Career Readiness Standards for Mathematics

Expertise and Character

- Development of expertise from novice to apprentice to expert
  - Schoolwide enterprise: school leadership
  - Department wide enterprise: department taking responsibility
- The Content of their mathematical Character
  - Develop character

Two major design principles, based on evidence:

- **Focus**
- **Coherence**

The Importance of **Focus**

- TIMSS and other international comparisons suggest that the U.S. curriculum is 'a mile wide and an inch deep.'

- "On average, the U.S. curriculum omits only 17 percent of the TIMSS grade 4 topics compared with an average omission rate of 40 percent for the 11 comparison countries. The United States covers all but 2 percent of the TIMSS topics through grade 8 compared with a 25 percent non coverage rate in the other countries. **High-scoring Hong Kong's curriculum omits 48 percent of the TIMSS items through grade 4, and 18 percent through grade 8. Less topic coverage can be associated with higher scores on those topics covered because students have more time to master the content that is taught."**
- Ginsburg et al., 2005

U.S. standards organization

[Grade 1]

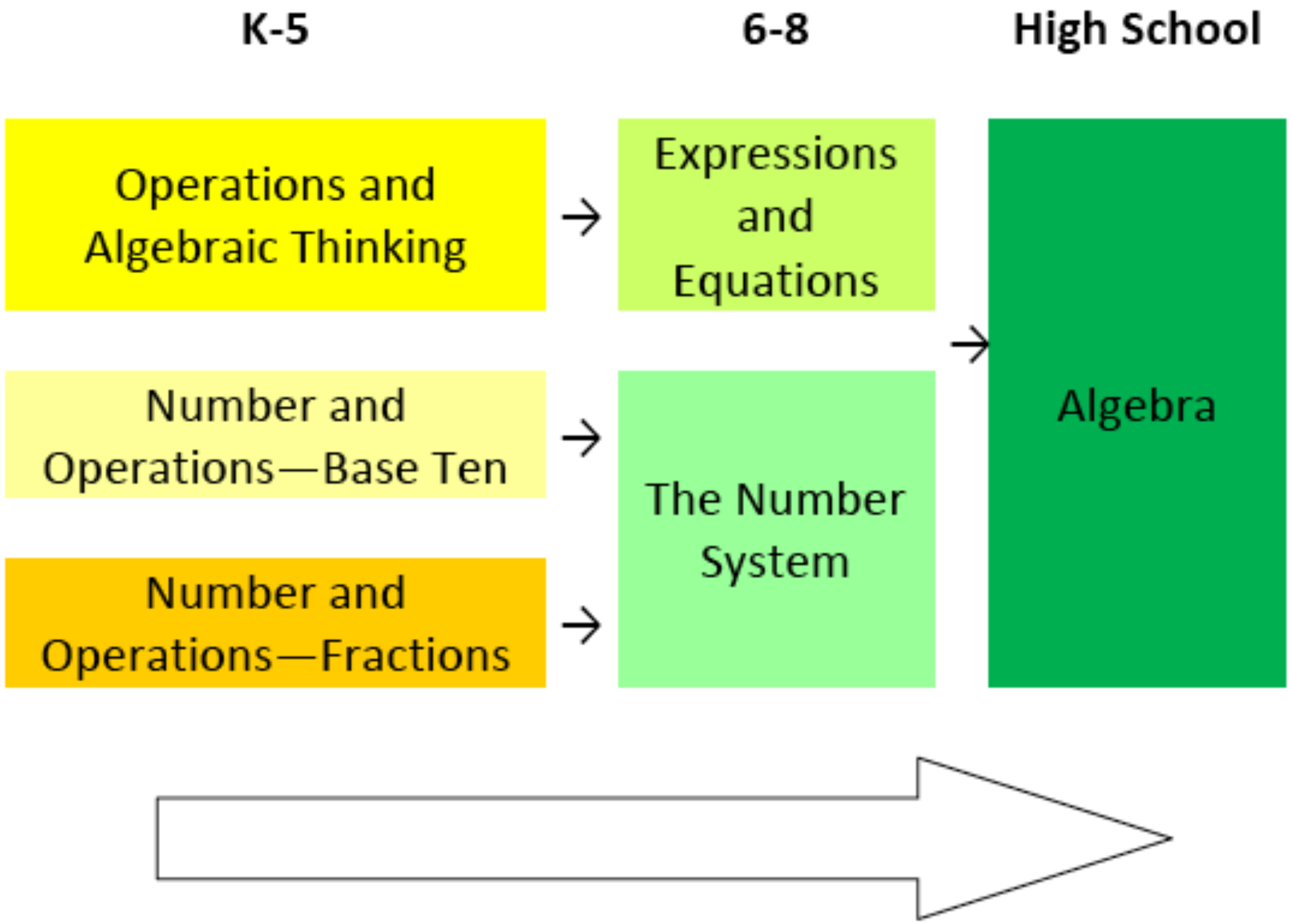
- Number and Operations
  - …
- Measurement and Geometry
  - …
- Algebra and Functions
  - …
- Statistics and Probability
  - …

U.S. standards organization

[12]

- Number and Operations
  - …
- Measurement and Geometry
  - …
- Algebra and Functions
  - …
- Statistics and Probability
  - …

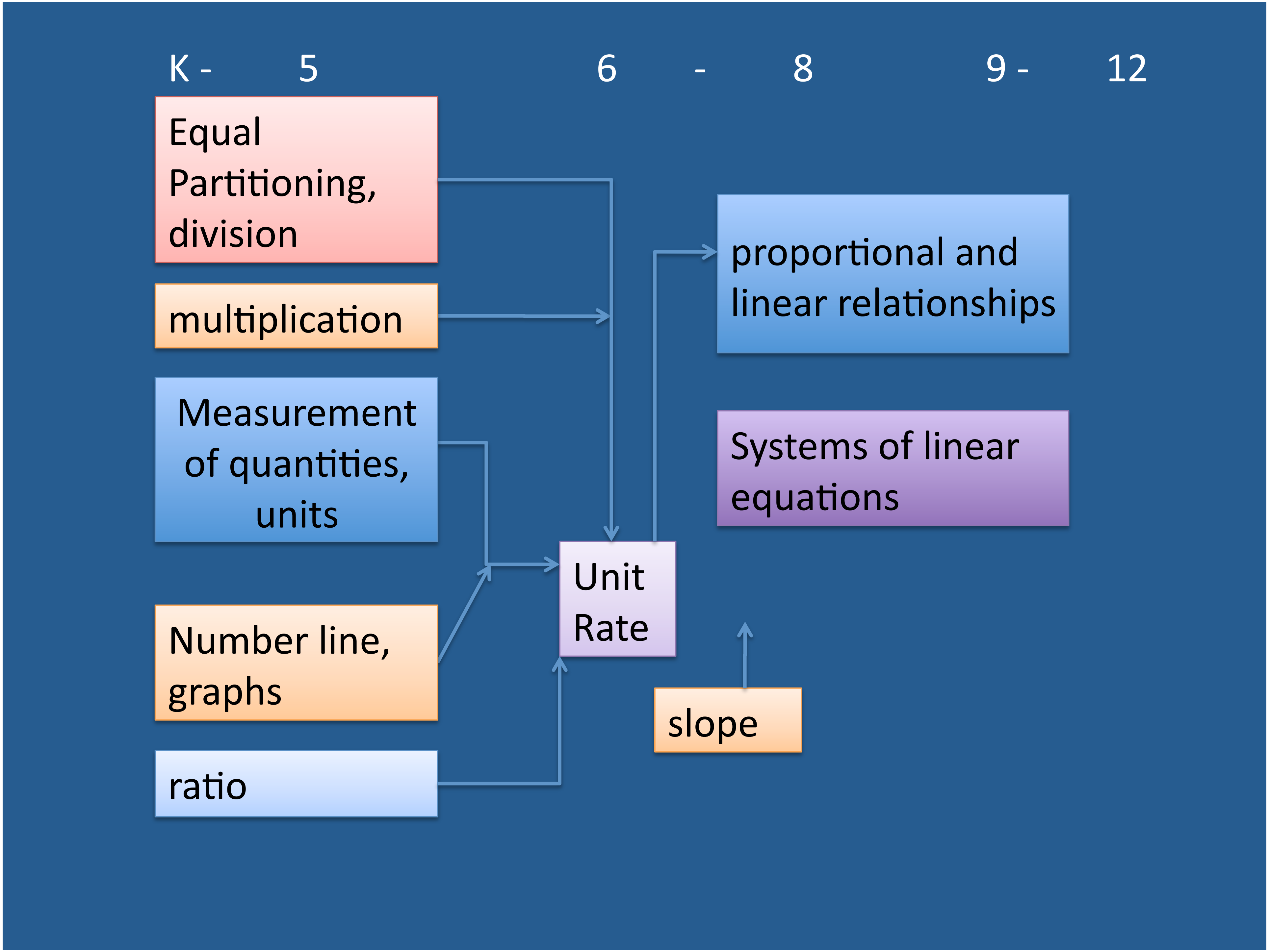
Focusing attention within Number and Operations



The most important ideas in the CCSS mathematics that need attention.

1. Properties of operations: their role in arithmetic and algebra
2. Mental math and [algebra vs. algorithms]
3. Units and unitizing
4. Operations and the problems they solve
5. Quantities-variables-functions-modeling
6. Number-Operations-Expressions-Equation
7. Modeling
8. Practices

Progression: quantities and measurement to variables and functions


K - 5
6 - 8
9 - 12
Equal Partitioning, division
multiplication
Measurement of quantities, units
Number line, graphs
ratio
Unit Rate
slope
proportional and linear relationships
Systems of linear equations

K-5: quantities and number line

- Compare quantities, especially length
- Compare by measuring: units
- Add and subtract with ruler
- Diagram of a ruler
- Diagram of a number line
- Arithmetic on the number line based on units
- Representing time, money and other quantities with number lines

Representing quantities with expressions

Mental math

72 -29 = ?

In your head.

Composing and decomposing

Partial products

Place value in base 10

Factor $X^2 + 4x + 4$ in your head

Fractions Progression

- Understanding the arithmetic of fractions draws upon four prior progressions that informed the CCSS:
  - equal partitioning,
  - unitizing,
  - number line,
  - and operations.

Partitioning

- The first two progressions, equal partitioning and unitizing, draw heavily from learning trajectory research.

Grade 3

1. The length from 0 to1 can be partitioned into 4 equal parts. The size of the part is ¼.
2. Unit fractions like ¼ are numbers on the number line

Unitizing

- Whatever can be counted can be added, and from there knowledge and expertise in whole number arithmetic can be applied to newly unitized objects.

Grade 4

1. $\frac{1}{4} + 1/4 + \frac{1}{4} = \frac{3}{4}$
2. Add fractions with like denominators
3. $3 \times \frac{1}{4} = \frac{3}{4}$
4. Multiply whole number times a fraction; $n(a/b) = (na)/b$

Grade 5

1. Add and subtract fractions with unlike denominators using multiplication by n/n to generate equivalent fractions and common denominators
2. 1/b = 1 divided by b; fractions can express division
3. Multiply and divide fractions

Units are things you count

- Objects
- Groups of objects
- 1
- 10
- 100
- ¼   unit fractions
- Numbers represented as expressions

Units add up

- 3 pennies + 5 pennies = 8 pennies
- 3 ones + 5 ones = 8 ones
- 3 tens + 5 tens = 8 tens
- 3 inches + 5 inches = 8 inches
- 3 ¼ inches + 5 ¼ inches = 8 ¼ inches
- ¾ + 5/4 = 8/4
- $3(x + 1) + 5(x+1) = 8(x+1)$

Unitizing links fractions to whole number arithmetic

- Students' expertise in whole number arithmetic is the most reliable expertise they have in mathematics
- It makes sense to students
- If we can connect difficult topics like fractions and algebraic expressions to whole number arithmetic, these difficult topics can have a solid foundation for students

Fraction Equivalence
Grade 3:

– Fractions of areas that are the same size, or fractions that are the same point (length from 0) are equivalent

– recognize simple cases: ½ = 2/4 ; 4/6 = 2/3

– Fraction equivalents of whole numbers 3 = 3/1, 4/4 =1

– Compare fractions with same numerator or denominator based on size in visual diagram

Fraction equivalence
Grade 4:

– Explain why a fraction a/b = na/nb using visual models; genrate equivalent fractions
– Compare fractions with unlike denominators by finding common denominators; explain on visual model based on size in visual diagram

Fraction equivalence
Grade 5:

– Use equivalent fractions to add and subtract fractions with unlike denominators

Fraction Item

4/5 is closer to 1 than 5/4. Show why this is true.

Operations and the problems they solve

- Tables 1 and 2 on pages 88 and 89

“Properties of Operations”

- Also called “rules of arithmetic” , “number properties”

From table 2 page 89

- $a \times b = ?$
- $a \times ? = p$, and $p \div a = ?$
- $? \times b = p$, and $p \div b = ?$

- *1.Play with these using whole numbers,*
- *2.make up a problem for each.*
- *3. substitute $(x - 1)$ for b*

Nine properties are the most important preparation for algebra

- Just nine: foundation for arithmetic
- Exact same properties work for whole numbers, fractions, negative numbers, rational numbers, letters, expressions.
- Same properties in 3rd grade and in calculus
- Not just learning them, but learning to use them

Using the properties

- To express yourself mathematically (formulate mathematical expressions that mean what you want them to mean)
- To change the form of an expression so it is easier to make sense of it
- To solve problems
- To justify and prove

Properties are like rules, but also like rights

- You are allowed to use them whenever you want, never wrong.
- You are allowed to use them in any order
- Use them with a mathematical purpose

Properties of addition

Associative property of addition	$(a + b) + c = a + (b + c)$ $(2 + 3) + 4 = 2 + (3 + 4)$
Commutative property of addition	$a + b = b + a$ $2 + 3 = 3 + 2$
Additive identity property of 0	$a + 0 = 0 + a = a$ $3 + 0 = 0 + 3 = 3$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$. $2 + (-2) = (-2) + 2 = 0$

Properties of multiplication

Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$ $(2 \times 3) \times 4 = 2 \times (3 \times 4)$
Commutative property of multiplication	$a \times b = b \times a$ $2 \times 3 = 3 \times 2$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$ $3 \times 1 = 1 \times 3 = 3$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$ $2 \times 1/2 = 1/2 \times 2 = 1$

Linking multiplication and addition: the ninth property

- *Distributive property of multiplication over addition*

$a \times (b + c) = (a \times b) + (a \times c)$

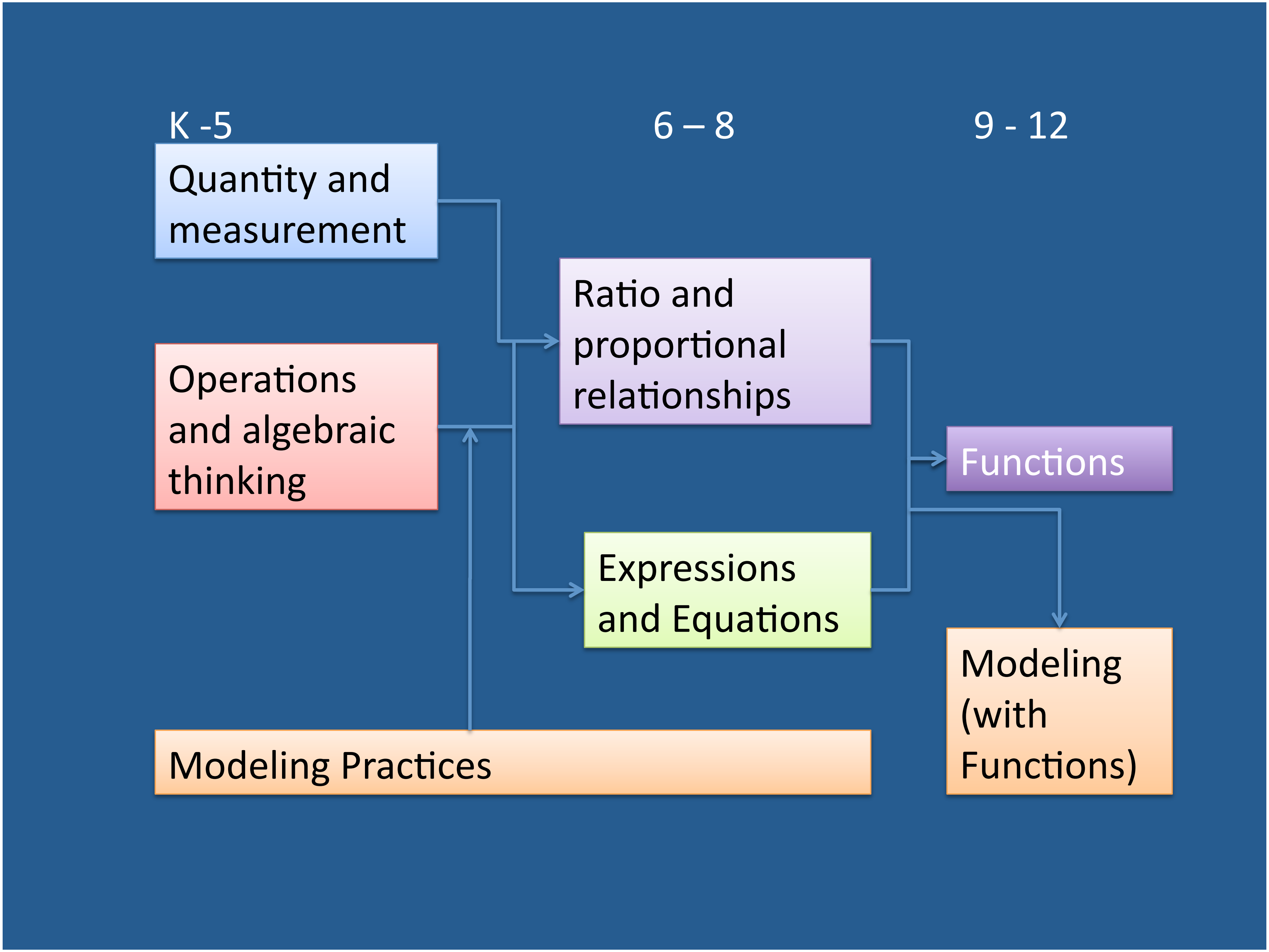
$a(b+c) = ab + ac$

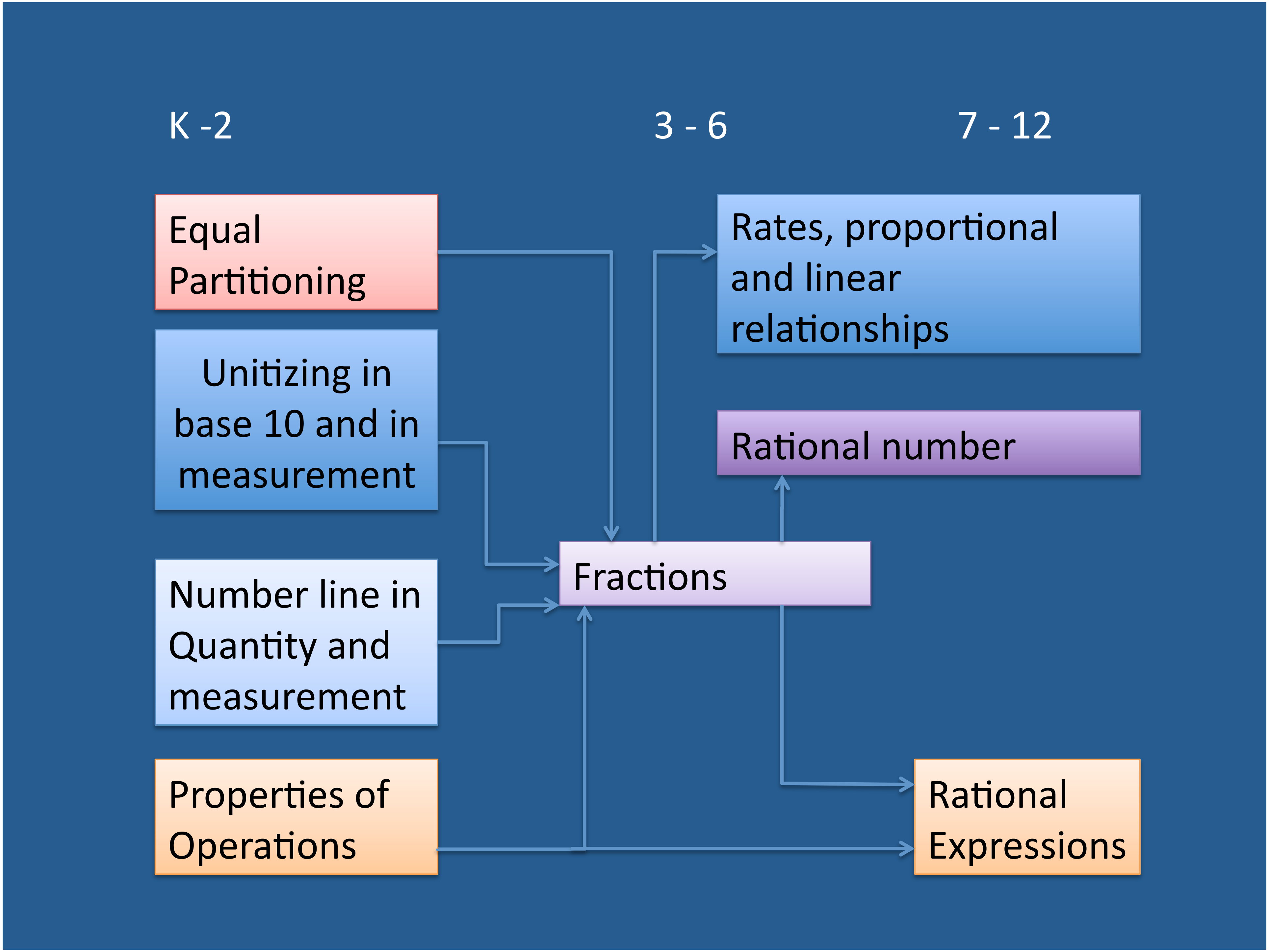
Find the properties in the multiplication table

- There are many patterns in the multiplication table, most of them are consequences of the properties of operations:
- Find patterns and explain how they come from the properties.
- Find the distributive property patterns

Grade level examples

- 3 packs of soap
- 4 dealing cards
- 5 sharing
- 6 money
- 7 lengths (fractions)
- 8 times larger (%)


K -5
6 – 8
9 - 12
Quantity and measurement
Ratio and proportional relationships
Operations and algebraic thinking
Functions
Expressions and Equations
Modeling (with Functions)
Modeling Practices


K -2
3 - 6
7 - 12
Equal Partitioning
Unitizing in base 10 and in measurement
Number line in Quantity and measurement
Properties of Operations
Fractions
Rates, proportional and linear relationships
Rational number
Rational Expressions

Functions and Solving Equations

1. Quantities-variables-functions-modeling
2. Number-Operations-Expressions-Equation

Take the number apart?

Tina, Emma, and Jen discuss this expression:

- 5 *1/3* x 6
- Tina: I know a way to multiply with a mixed number, like 5 *1/3* , that is different from the one we learned in class. I call my way “take the number apart.” I’ll show you.

Which of the three girls do you think is right?
Justify your answer mathematically.

First, I multiply the 5 by the 6 and get 30.

Then I multiply the 1/3 by the 6 and get 2. Finally, I add the 30 and the 2, which is 32.

- Tina: It works whenever I have to multiply a mixed number by a whole number.
- Emma: Sorry Tina, but that answer is wrong!
- Jen: No, Tina's answer is right for this one problem, but "take the number apart" doesn't work for other fraction problems.

What is an explanation?

Why you think it's true and why you think it makes sense.

Saying "distributive property" isn't enough, you have to show how the distributive property applies to the problem.

Example explanation

Why does 5 1/3 x 6 = (6x5) + (6x1/3) ?

Because

5 1/3 = 5 + 1/3

6(5 1/3) =

6(5 + 1/3) =

(6x5) + (6x1/3) because a(b + c) = ab + ac

Mental math

72 -29 = ?

In your head.

Composing and decomposing

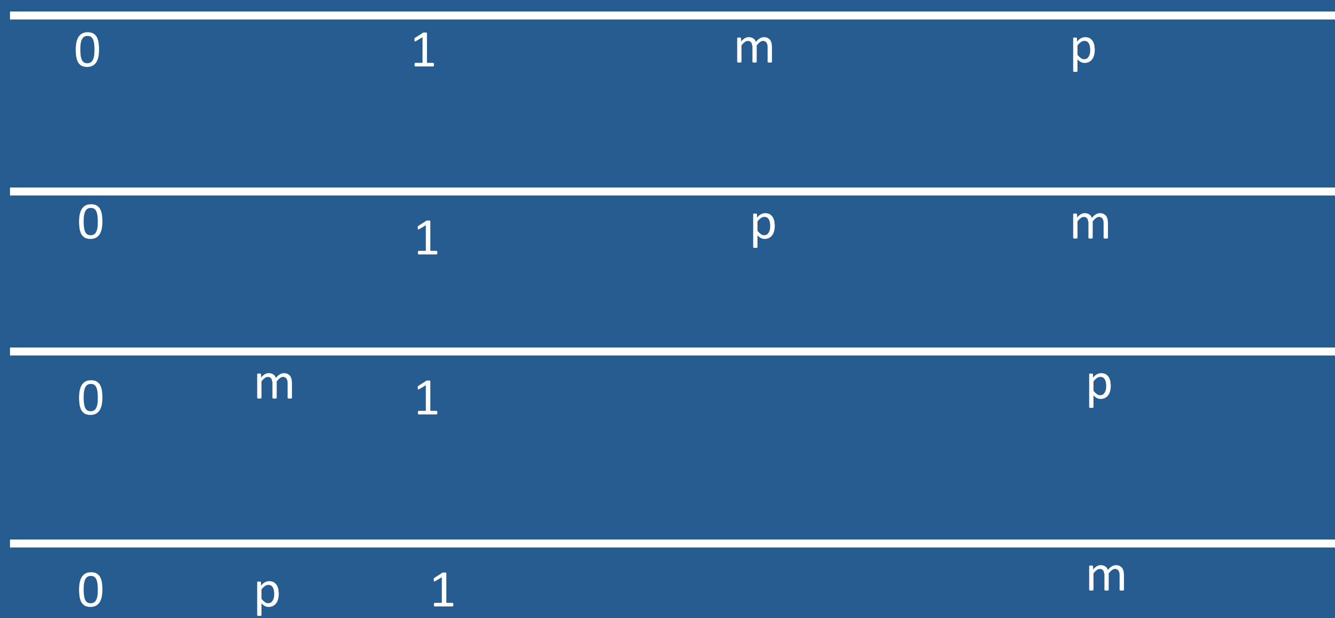
Partial products

Place value in base 10

Factor $X^2 + 4x + 4$ in your head

Locate the difference, p - m, on the number line:

For each of the following cases, locate the quotient *p/m* on the number line :



Misconceptions about misconceptions

- They weren't listening when they were told
- They have been getting these kinds of problems wrong from day 1
- They forgot
- The other side in the math wars did this to the students on purpose

More misconceptions about the cause of misconceptions

- In the old days, students didn't make these mistakes
- They were taught procedures
- They were taught rich problems
- Not enough practice

Maybe

- Teachers’ misconceptions perpetuated to another generation (where did the teachers get the misconceptions? How far back does this go?)
- Mile wide inch deep curriculum causes haste and waste
- Some concepts are hard to learn

Whatever the Cause

- When students reach your class they are not blank slates
- They are full of knowledge
- Their knowledge will be flawed and faulty, half baked and immature; but to them it is knowledge
- This prior knowledge is an asset and an interference to new learning

Second grade

- When you add or subtract, line the numbers up on the right, like this:
- 23
- +9

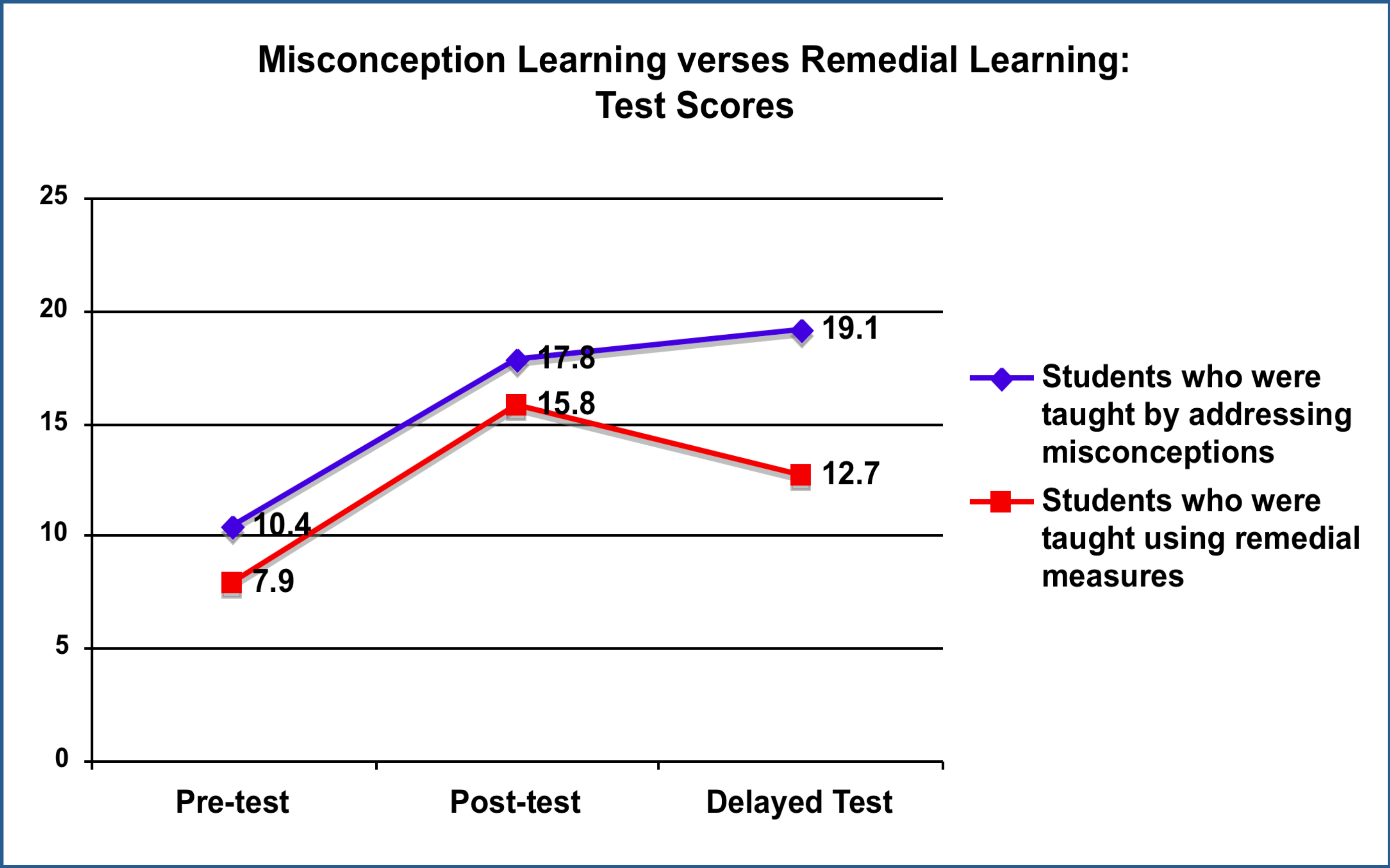
- Not like this
- 23
- +9

Third Grade

- 3.24 + 2.1 = ?
- If you “Line the numbers up on the right “ like you spent all last year learning, you get this:

- 3.2 4
- + 2.1
- You get the wrong answer doing what you learned last year. You don’t know why.
- Teach: line up decimal point.
- Continue developing place value concepts

Research on Retention of Learning: Shell Center: Swan et al



Lesson Units for Formative Assessment

- **Concept lessons**
  “Proficient students expect mathematics to make sense”
  – To reveal and develop students’ interpretations of significant mathematical ideas and how these connect to their other knowledge.

- **Problem solving lessons**
  “They take an active stance in solving mathematical problems”
  – To assess and develop students’ capacity to apply their Math flexibly to non-routine, unstructured problems, both from pure math and from the real world.

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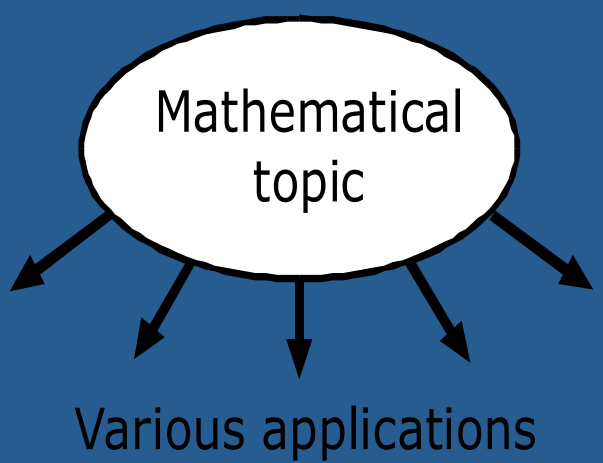
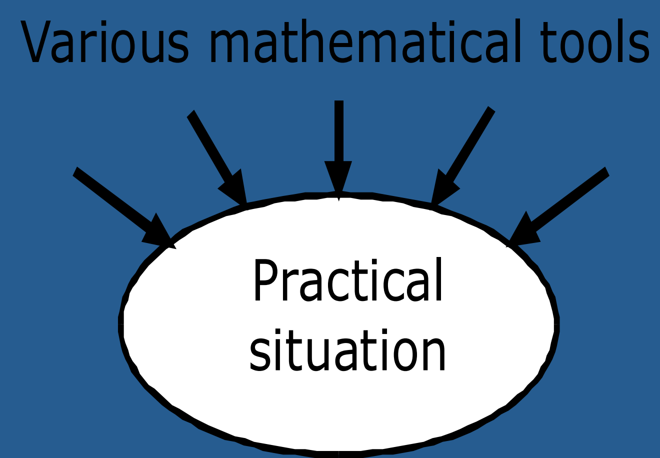
College and Career Readiness Standards for Mathematics

Mathematical Content Standards

- Number & Quantity
- Algebra
- Functions
- Modeling
- Statistics and Probability
- Geometry

“Concept focused” v “Problem focused”:





Optimization Problems: *Boomerangs*

Projector Resources

Boomerangs

Phil and Cath make and sell boomerangs for a school event.
The money they raise will go to charity.

They plan to make them in two sizes: small and large.

Phil will carve them from wood.
The small boomerang takes 2 hours to carve and the large one takes 3 hours to carve.
Phil has a total of 24 hours available for carving.

Cath will decorate them.
She only has time to decorate 10 boomerangs of either size.

The small boomerang will make \$8 for charity.
The large boomerang will make \$10 for charity.
They want to make as much money for charity as they can.

How many small and large boomerangs should they make?

How much money will they then make?

Evaluating *Sample Responses to Discuss*

- What do you like about the work?
- How has each student organized the work?
- What mistakes have been made?
- What isn't clear?
- What questions do you want to ask this student?
- In what ways might the work be improved?

Alex’s solution

Phil can only make 12 small or 8 large boomerangs in 24 hours

12 small makes $96

8 large makes $80

Cath only has time to make 10, so $96 is unipossible.

She could make 10 small boomerangs which will make $80.

So she eather makes 8 large or 10 small boomerangs

and makes $80.

Danny’s solution

No of small s	S x 8	No of large	l x 10	Profit
0	0	8	80	80
1	8	~~9~~ 7	70	78
2	16	6	60	76
3	24	5	50	74
4	32	5	50	82 ←
5	40	4	40	80
6	48	3	30	78

The most Profit is $82

Jeremiah's solution

Small boomerangs = x

Large boomerangs = y

Time to carve $2x + 3y = 24$ ①

Only 10 can be decorated $x + y = 10$ ②

$2x + 2y = 20$ ③

① − ③ $y = 4$ $x = 6$

So make 4 large boomerangs

6 small boomerangs.

Tanya's solution