

	Transforming Teaching:
	Examples that Support Taking Teachers from Dissonance to Depth Juli K. Dixon, Ph.D. University of Central Florida

	Intent...
	Cause dissonance (or at least provide examples that could be used to create dissonance in others). Share authentic examples used to initiate "transforming" conversations. Focus on topics in number such as: <ul style="list-style-type: none">- comparing and computing with fraction and- multiplying and dividing.

	Perspective...
	<p>When asked to compare $\frac{4}{5}$ and $\frac{2}{3}$ a student said,</p> <p style="text-align: center;">“I know that $\frac{4}{5}$ is greater than $\frac{2}{3}$.”</p> <p style="text-align: right;">How would you respond?</p>

	Perspective...
	<p>The student said,</p> <p style="text-align: center;">“I made both fractions using manipulatives. I knew that $\frac{4}{5}$ was bigger because $\frac{4}{5}$ has 4 pieces and $\frac{2}{3}$ only has 2 pieces and since 4 is greater than 2 then $\frac{4}{5}$ is greater than $\frac{2}{3}$.”</p>

	Perspective...
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	If students understand strategies for comparing fractions with depth, this problem should be appropriate for a student in grade 3.

	Premise...
	<p>If we provide authentic examples that cause disequilibrium...</p> <p>Teachers will be more likely to adjust their practices to teach for depth.</p> <p>The goal, then, is to create cognitive dissonance.</p> <p>(Festinger, 1957; Zaslavsky, 2005)</p>

	Tell which fraction is greater
	<ol style="list-style-type: none">1. $\frac{3}{7}$ and $\frac{5}{8}$2. $\frac{4}{7}$ and $\frac{4}{9}$3. $\frac{3}{8}$ and $\frac{5}{8}$4. $\frac{6}{7}$ and $\frac{8}{9}$

Think about this...

Alex and Jessica are racing their bicycles.
Alex is $\frac{3}{7}$ of the way to the finish line and
Jessica is $\frac{2}{3}$ of the way to the finish line.
Which racer is closer to the finish line? How
do you know?

Think about this...

Marc and Larry each bought the same type of
energy bar. Marc has $\frac{1}{8}$ of his energy bar left,
Larry has $\frac{1}{10}$ of his energy bar left. Who has
more energy bar left? How do you know?

Think about this...

Riley and Paige each bought a small pizza.
Riley ate $\frac{5}{6}$ of her pizza, and Paige ate $\frac{7}{8}$ of her pizza. Who ate more pizza? How do you know?

NOW tell which fraction is greater

1. $\frac{3}{7}$ and $\frac{5}{8}$
2. $\frac{4}{7}$ and $\frac{4}{9}$
3. $\frac{3}{8}$ and $\frac{5}{8}$
4. $\frac{6}{7}$ and $\frac{8}{9}$

	A new perspective...
	Would you ask a student to compare $\frac{22}{23}$ and $\frac{26}{27}$?

	A new perspective... on fluency
	What does it mean to compare fractions fluently?

A new perspective... on fluency

What does it mean to compare fractions fluently?

What does it mean to multiply fluently?

A new perspective... on fluency

Multiply this:

2	9	6
x		2

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	<p><u>Grade 4: <i>Develop understanding of and fluency with multi-digit multiplication</i>, and develop understanding of dividing to find quotients involving multi-digit dividends.</u></p> <p>Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations.</p>

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	Multiply 15×23.

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	<p><u>Grade 3:</u> Understand properties of multiplication... Apply properties of operations as strategies to multiply and divide.</p> <p><u>Grade 3:</u> Multiply and divide within 100 Fluently multiply within 100, using strategies such as the relationship between multiplication and division or properties of operations.</p>

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	Consider 6×7

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	Consider 6×7 How can applying properties of operations as strategies to multiply these factors help students look for and make sense of structure? (standard for mathematical practice 7) What strategies can we use?

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	What does it mean to multiply using strategies?
	Consider 6×7 Think: 6×6

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	<p>Consider 6×7</p> <p>Think: 6×6 add 6</p> <p>Think: 7×7 subtract 7</p> <p>Think: 6×5 add 12</p>

	<p>What does it mean to multiply using strategies?</p>
	<p>Consider 6×7</p> <p>Think: 6×6 add 6</p> <p>Think: 7×7 subtract 7</p> <p>Think: 6×5 add 12 $6 \times (5 + 2) = (6 \times 5) + (6 \times 2)$</p>

	<p>What does it mean to multiply using strategies?</p>
	<p>Consider 6×7</p> <p>How might a student who makes sense of the distributive property while exploring basic facts extend this thinking to solving 15×23.</p> <p style="text-align: center;">$6 \times (5 + 2) = (6 \times 5) + (6 \times 2)$</p>

	What does it mean to multiply using strategies?
	<p>Consider 15×23</p> <p>$15 \times 23 = 15 \times (20 + 3)$</p> <p>$15 \times 23 = (10 + 5) \times 23$</p> <p>$15 \times 23 = (10 + 5) \times (20 + 3)$</p>

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	What does it mean to multiply using strategies?
	<p>Multiply 3×298.</p>

	How should problems be presented?									
	<p>Multiply this:</p> <table border="1" data-bbox="743 491 889 680"> <tr> <td>2</td> <td>9</td> <td>6</td> </tr> <tr> <td>x</td> <td></td> <td>2</td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </table> <p>and this:</p> <p style="text-align: center;">3 x 298.</p>	2	9	6	x		2			
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x		2								

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	<p><u>Grade 5:</u> Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations.</p>

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How does this relate to how we divide multi-digit numbers?

Consider the case of long division.

So what does this mean???

$$\begin{array}{r} 134 \\ 4 \overline{) 536} \\ \underline{-4} \\ 13 \\ \underline{-12} \\ 16 \\ \underline{-16} \\ 0 \end{array}$$

	How does what we know about dividing whole numbers extend to dividing with fractions?

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	<p><u>Grade 5:</u> Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.</p> <p>Interpret division of a whole number and a unit fraction, and compute such quotients.</p>

	Solve this...
	$3 \div \frac{1}{7}$

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	<p>Perspective...</p>
	<p>When asked to justify the solution to $3 \div 1/7$</p> <p>A student said this...</p> <p>“Just change the division sign to multiplication and flip the fraction after the sign. $3 \div 1/7$ becomes $3 \times 7/1$. So I find $3/1 \times 7/1$ which is $21/1$ or 21.”</p> <p>Is this an acceptable justification?</p>

	Perspective...
	<p>When asked to justify the solution to $3 \div 1/7$</p> <p>Another student said this...</p> <p>"I know there are 7 groups of $1/7$ in one whole. Since there are three wholes, I have 3×7 or 21 groups of $1/7$ in 3 wholes so $3 \div 1/7 = 21$."</p>

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	<p>When asked to justify the solution to $3 \div 1/7$</p> <p>Another student said this...</p> <p>"I know there are 7 groups of $1/7$ in one whole. Since there are three wholes, I have 3×7 or 21 groups of $1/7$ in 3 wholes so $3 \div 1/7 = 21$."</p> <p>How is this justification different?</p>

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	Why is it important for students to explain and justify mathematics in these ways? How is this related to the Common Core State Standards (CCSS)?

	Perspective...
	“One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student’s mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from” (CCSS, 2010, p. 4).

	Represent this...
	$3 \div \frac{1}{7}$

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	$3 \div \frac{1}{7}$ <p>Create a story context for this and use a visual fraction model to show the quotient.</p>

	Represent this...
	$3 \div \frac{1}{7}$ <p>Create a story context for this and use a visual fraction model to represent it.</p> <p>Try this with $\frac{4}{5} - \frac{1}{2}$</p>

	How is this connected to the Standards for Mathematical Practice?
	$3 \div 1/7$ <p>Create a story context for this and use a visual fraction model to represent it.</p> <p>Try this with $4/5 - 1/2$</p>

	Describing the Standards
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	<h2>Describing the Standards</h2>
	<p>"... a lack of understanding [of mathematical content] effectively prevents a student from engaging in the mathematical practices" (CCSS, 2010, p. 8).</p> <p>What types of experiences help teachers to build content knowledge to support the mathematical practices?</p>

	<h2>Making the journey...</h2>
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	<p>Focus on relevant content. Number should be a focus area in elementary grades (National Research Council, 2009).</p> <p>Provide examples that lull teachers into a sense of security so that they use common procedures they have used over time without thinking about connections to other concepts (Zaslavsky, 2005).</p> <p>Create cognitive dissonance by contrasting common practices with teaching for depth (Festinger, 1957).</p>

	Thank you
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	Advice to help parents support their children:
	<ul style="list-style-type: none">■ Teach procedures only after they are introduced in school. Instead, ask your child to explain his or her thinking to you.■ Drill addition/multiplication facts only after your child explores strategies.■ Help your child become more proficient with using mathematics at home.

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