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Making the "Cut":
One District's Strategy for Algebra Placement

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ABSTRACT:
Some of the most discussed issues in mathematics education today involve Algebra and its instruction. These issues include the optimal timeline for when students first take a formal algebra course, the related selection process for getting into that first course and what algebra instruction should generally look like throughout the curriculum. Algebra is being recognized as a key "gate-keeper" course for high school and college success and has even been called an emerging "civil rights issue" by some researchers and authors. When to place students into an algebra class and how to ensure that a student is ready for Algebra are both critical curriculum decisions for a district. In many districts, algebra placement is a process that may be undergoing considerable revision along with how algebra is integrated across the curriculum. This article describes one district's approach for evaluating and revising their placement strategy for admitting students into their first middle school algebra course.

"Not every child has an equal talent or an equal ability or equal motivation, but all children have the equal right to develop their talent, their ability and their motivation."

~ John Fitzgerald Kennedy, 1963

John Kennedy's famous civil rights quote that "all children have the equal right to develop their talent, their ability, and their motivation" was made in a speech to the American people in a radio address on the morning of June 11, 1963. That was the morning that President Kennedy sent in the Alabama National Guard to open up the University of Alabama to two well-qualified black students. Access to a college education, for all qualified students was of course one of the most important civil rights issues of that day. In many ways, that civil rights issue is still with us in mathematics education and is often represented within the discussions of when students take Algebra and how they study it throughout their K12 coursework.

In mathematics education, the timeline for when students take Algebra, the related selection process, and what algebra instruction should look like throughout the K12 curriculum are some of the most discussed issues in the profession today. For example, algebra instruction and placement have been strongly represented in the last several National Council of Teachers of Mathematics and National Council of Supervisors of Mathematics annual conferences, with numerous sessions and presentations dedicated to algebra instruction. Another example of this professional dialogue is the new 2006 document by the National Council of Teachers of Mathematics, called "Curriculum Focal Points" which details topics of particularly important focus for pre-kindergarten to grade 8 mathematics instruction. This document has algebra well identified as a focus area, with consistent references to "number operations and algebra" as focal points from first grade through fifth grade, and an emphasis on "algebra" itself as one of the key focal points in grades 6-8. Algebra is obviously continuing to become an ever more important topic in K12 mathematics instruction.

The importance of algebra is also increasing as computer technology impacts the ways in which we have to teach mathematics (Heid, 2005; Hegedus & Kaput, 2004). Instructional tools such as graphing calculators, computerized algebra programs and homework helping websites are allowing schools and teachers to more effectively provide the instructional depth to algebra that it deserves in its growing importance in the K12 mathematics curriculum (Heid & Edwards, 2001). In fact, professional associations such as the Association of Mathematics Teacher Educators are commonly mentioning algebra as an instructional area particularly compatible with new technologies of instruction (Association of Mathematics Teacher Educators, 2006).

In a direct reference to the civil right passions of the 1960's, algebra has even been called an emerging "civil rights issue" for the next decade (Checkley, 2006; Moses, 2000; Moses, 1994). From a research perspective, an early understanding of algebra has been shown to be a key (and perhaps THE key) predictor for success in high school mathematics coursework and even entry into college (Burris, Heubert, Levin, 2004). A study by Horn and Nunez (2000) illustrates the importance for students in taking the advanced mathematics coursework that follows an early algebra placement. In their study, students of parents who never attended college more than doubled their chances for enrolling in a four-year college when taking coursework past Algebra 2. A well-prepared student that gets into an "early algebra sequence" may well have a distinct academic advantage over a student who does not get into that sequence. In addition, a poorly prepared student who fails at an early algebra course, may well be doomed to struggling in mathematics or even discarding mathematics as something that they are only minimally interested in learning (Schoenfeld, 2002).

Thus, how a school district selects students to enter a formal algebra course and when that selection process occurs is becoming critically significant within a district's mathematics program. With an awareness of just how important such an algebra selection process can be for students, the Westside Community Schools and the University of Nebraska at Omaha carefully examined Westside's algebra selection process by reviewing past placement data, holding a series of collaborative discussions, and then modifying the selection process to try to be as fair as possible to students within the context of limited district resources. This article describes an evidence-based investigation of Westside's algebra placement process and the related changes that the district made in its placement procedures as a result of this inquiry.

The Historical Context at Westside

First, it is important to get a sense of the Westside Community Schools. The district is an urban school district of approximately 6,000 students, 1,400 of whom are not residents of the district, but rather attend through Nebraska's school choice program. Eighty-six percent (86%) are white. Approximately 20% of the students qualify for free or reduced price lunch. The district has a K-12 curriculum with ten elementary schools (grades K-6), one middle school (grades 7-8), and one high school (grades 9-12). The district has always prided itself on having a strong and vibrant mathematics program, which has been recognized within the context of several awards, including students qualifying for the National Math Counts competition for five consecutive years, several students achieving perfect scores on the American Mathematics Competition and a high number of student qualifiers in the state's annual mathematics competitions.

During 2001, the Westside Community Schools adopted a new mathematics curriculum at the elementary level in order to better challenge their elementary students in mathematical problem solving as well as other higher level mathematics skills. The curriculum blends basic skills development with conceptual understanding activities in a mix that has been shown to be a positive component of effective mathematics instruction in several districts across the country (Cavanagh, 2006). The Westside program was carefully planned and adopted with considerable input from teachers, parents and even students (Grandgenett, Jackson, Willits, 2004). The elementary program revisions also included the adoption of *Everyday Mathematics* instructional materials, which appeared to align well with district desires to better challenge students. Elementary teachers also went through an extensive professional development program to help prepare them for a more challenging elementary curriculum. This professional development process also systematically included the early integration of algebra's big ideas, such as variables, patterns and functions, and proportions and proportional reasoning as recommended by authors such as Greenes (2004).

Teachers and students have embraced this revised elementary curriculum. Along with better preparing students for mathematical problem solving, reasoning, and mathematical connections, the curriculum also carefully covers introductory algebra topics which are well integrated into all grade levels at the elementary level. For example, in the Everyday Mathematics curriculum, algebra-related topics appear in each elementary grade and are indexed within the instructional materials (Everyday Learning Corporation, 2002).

Like most school districts today that have worked hard to develop an effective elementary mathematics program, placement into a formal algebra or pre-algebra course (leading to Algebra) at the middle school level has now surfaced at Westside as an important focus area for further revisions within the K-12 mathematics program. The district's strong elementary preparation in algebra readiness has only increased a need to offer strong middle school coursework options for students. Thus, the early integration of algebra concepts at the elementary level has essentially encouraged a more systematic approach to algebra at the middle school. This need for a careful transition for algebra instruction is consistent with research that suggests that successful instructional efforts for algebra should be well paced and systematic across the curriculum (Noddings, 2000; Steen, 1992).

In the National Research Council's 2005 report "How Students Learn," a total of 179 out of the 600 pages are dedicated to the learning of mathematics. Within this extensive discussion, Fuson, Kalchman, and Bransford (pgs. 217-256) reinforce that there are three important principles for teachers to follow in helping provide a foundation for the learning of mathematics, and particularly algebra. These principles include: 1) teachers must engage student prior understandings; 2) teachers must help students build a deep foundation of factual knowledge, give students a conceptual framework, and help them to organize knowledge; and 3) teachers need to help students take a metacognitive approach in taking control of their own learning within challenging coursework.

Challenging coursework has always been a strong component of Westside's mathematics program and student selection for such coursework has always been an important district concern. Historically, in the Westside district, two assessments were used to identify students who were perceived as "ready" for a challenging Pre-algebra course in the middle school after an aggressive elementary school curriculum. Students who received a score above the established cut scores were placed in Pre-algebra and others were placed in the "regular" 7th grade mathematics curriculum. This practice had a long history but no real documentation of the validity of the assessments or the predictive capability of the established cut scores. One of the primary assessments was even a "district-made" test that was initially constructed nearly 20 years ago by a group of middle school teachers and revised periodically over the years based upon the further input of later teachers.

The tests and the cut scores used for algebra placement had essentially not changed for more than a decade, but in recent years the proportion of students qualifying for Pre-algebra had steadily increased. The following table shows the percentage of students that took the placement tests each year and the percent qualifying within the district during the four years before changes were made in the selection process.

PRE-ALGEBRA TESTING

Year	6th Grade Enrollment	Number Taking Test	Percent Taking Test	Number Qualifying	Percent Qualifying
2001-2002	405	250	61.7%	137	33.8%
2002-2003	422	384	91.0%	248	58.8%
2003-2004	468	420	89.7%	283	60.5%
2004-2005	452	390	86.3%	258	57.1%

Although the tests and qualifying scores hadn't changed generally between 2001 and 2005 other things had. Historically, letters were sent to parents of students identified by sixth grade teachers as potential candidates for Pre-algebra. These parents were invited to have their child take the screening tests at the middle school on a Saturday morning or designated weekday evening, a practice that was eventually found to penalize students whose parents were not aware of, or initially interested in, providing this opportunity for their children. Procedures were then changed in the spring of 2002. Middle school teachers and counselors continued to administer the tests, but the tests were given during the school day at each elementary school and all students were encouraged to take the tests. As mentioned previously, the elementary curriculum had also changed during this period. The new curriculum placed greater emphasis on problem solving, reasoning, mathematical connections and had students apply their mathematical understanding to a greater extent than the previous curriculum. The curriculum also systematically introduced the "big ideas" of algebra at the lower grade levels. Standardized test scores in mathematics went up after the adoption of the new curriculum and teachers believed that the new curriculum also may have positively impacted students' performance on the Pre-algebra screening test.

As the numbers of students placed in Pre-algebra increased, middle school teachers recognized that the students arriving in these classes were representing a wider range of backgrounds and also observed that some students within this increased pool of students appeared to be struggling more than in the past. Two additional concerns led administrators to the conclusion that the placement tests and cut scores needed to be carefully examined. First, the validity of the tests themselves was in question. One test was a basic teacher-developed computational mathematics test, which had been refined over time, but without any formal reliability and validity testing. The other test was the Orleans Hanna, a commercially published assessment of algebra readiness (Harcourt Brace and Company, 1998). However, this more established test was not being used in connection with student grades as the test publisher prescribed. Secondly, there was no documentation of the formal procedures used to set passing scores on either of the assessments. There essentially was no evidence that the tests, or the established cut scores, were effective predictors of student success in Pre-algebra. Thus, the district felt it was time to carefully examine and better formalize the algebra placement process.

Looking at the Situation Statistically

To look at the algebra placement situation statistically and to better examine the algebra placement process, Westside partnered with the University of Nebraska at Omaha, to review the existing data related to the district's seventh grade mathematics placement process and compare the statistical power of the historical cutoff procedure with an alternate procedure thought to be more consistent with the new mathematics program. These two contrasting selection procedures included 1) the current use of the district constructed mathematics survey test (called the Westside Survey Test) and the commercially prepared *Orleans Hanna Test*, and 2) a potential alternate procedure using student grades and the *Orleans Hanna Test*. The alternate procedure using grades in combination with *Orleans-Hanna* scores, was also an assessment strategy recommended by the publisher of the *Orleans-Hanna Test*. In this context, grades were changed to a numerical score (again following *Orleans-Hanna*), using a scale of 0-12 for each grade assigned from F (assigned 0 points) to A+ (assigned 12 points). A total of 373 past student records were available to help investigate the relative statistical power of these two procedures.

As a first step in the statistical investigation, correlations were conducted to examine the overall relationships of various fifth grade and sixth grade mathematics variables (e.g., scores on mathematics assessments administered in fifth or sixth grade) with seventh grade mathematics achievement as represented by grades (see table below). The district also had a practical desire to have the qualifying procedure include a written test to aid in parent discussions. Another desire by the district was to somewhat emphasize the 6th grade scores since these scores would be more closely associated in time to the seventh grade year.

SAMPLE CORRELATIONS (6th GRADE)	r
Total 6th Grade Score	0.62
Mathematics Grade	0.60
Reading Grade	0.56
* Grades and Orleans Hanna Test Combined	**0.55**
Social Studies Grade	0.53
* Survey Test and Orleans Hanna Test Combined	**0.43**
Survey Mathematics Test	0.42
Orleans Hanna Raw Score	0.40
Science Grade	0.37
SAMPLE CORRELATIONS (5th GRADE)	**r**
Gr 5 SAT9 Total (Complete) Battery	0.45
Gr 5 SAT9 Total Math	0.42
Gr 5 SAT9 Math Proc	0.40

In examining the correlations, it appeared that the potential alternate selection procedure of combining semester "report card grades" with the *Orleans Hanna Test* was a viable alternative to the earlier procedure.

Multiple regression procedures were then used to compare the relative strengths of the two data models: the new model (Grades + *Orleans Hanna*) with the old model (Survey Test + *Orleans Hanna*) in their predictive relationships to student grades in seventh grade mathematics. The new model of combining grades and the *Orleans Hanna* scores was found to be statistically stronger when considering its effectiveness for achievement predictions within the available sample of 373 past student records. The new model accounted for 38% of the variance in scores, approximately double that of the old model, which accounted for only 19% of the variance. Actually, these findings are quite consistent with research that suggests that combinations of coursework grades and testing can be useful in predicting future mathematics performance (Burris, Heubert, Levin, 2004; Fenton, 2002).

Again using the historical data, the relative effectiveness of the two cutoff score strategies were then examined by considering how many "true predictions" and "false positives" the different cutoff score procedures represented while looking at the historical distribution of the 373 scores. For purposes of this comparison process, the following operational definitions were used:

> ***True Prediction:*** This term referred to the situation where a student made the cutoff score and then was successful in seventh grade mathematics.
>
> ***False Positive:*** This term referred to the situation where a student made the cutoff score, but was then unsuccessful in seventh grade math.
>
> ***Successful in seventh grade Math:*** A student was considered to be successful in seventh grade math if they received a grade of at least a "B" in their seventh grade math course.

As mentioned earlier, the current cutoff score procedure used a combination of tests that included the *Orleans Hanna Test* and a district created mathematics survey test. This traditional cutoff score process included the following criteria identified in district communications to parents:

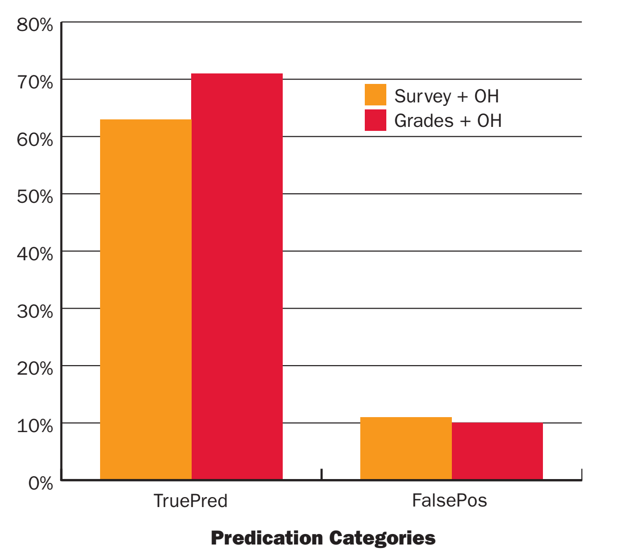
> "Students who are recommended for enrollment in the Pre-algebra course demonstrate the knowledge to be successful in Pre-algebra by meeting one of two criteria: 1) a score of 60% or higher on the Orleans-Hanna Algebra Prognosis Test and a score of 70% or higher on the Westside Mathematics Survey Test or 2) a combined average score on the two tests of 67% or higher."

This traditional cutoff score procedure predicted 63% of the sample's mathematics achievement (true prediction). About 11% of the sample were false positives (student made cutoff score but then struggled). It was also found by examining the 373 records that the two options within the criteria for qualifying (meeting the cut score on both tests or the mean of the two) statistically overlapped and were not both needed. All students either met both criteria or neither.

The recommended new student selection model used the *Orleans Hanna Test* and student grades. This selection process included a procedure recommended by the test publisher for combining student grades in four subjects (Math, Science, Social Studies, Reading/Writing). This approach uses the scale of 0-12 for each grade assigned from F (0 points) to A+ (12 points), and when combining all four grades, this point summation then accounts for a total grade value ranging from 0 to 48. This grade value is then combined with the *Orleans Hanna Test* scale of 0-50, to give an overall combined score ranging from 0 to 98. When examining the historical data, the new cutoff score procedure was found to be potentially superior based on this past data and a cutoff score of 64 was considered to be statistically optimum. Using this cutoff score, the prediction of student success (true prediction) was generally maximized and the false positives were relatively minimized (student makes cutoff score but is unsuccessful). This cutoff score predicted 71% of the population successfully, with 10% false positives.

Based on this analysis, the new cutoff score process was expected to statistically increase the true prediction of student success by roughly 8% while also potentially decreasing the false positives (student makes cutoff score but then

SURVEY TEST + ORLEANS HANNAH PREDICTIONS VS. GRADES + ORLEANS HANNAH PREDICTIONS



struggles) by roughly 1%. These two approaches are compared side by side on the graph.

Using the historical sample of 373 students to "predict" how many students would be expected to make the new cutoff score, it was determined that the new cutoff score process would most likely have about 67% percent of the district's students expected to quality for the initial middle school algebra course.

In essence, by using the new assessment procedure (combining student grades and the Orleans Test) it was concluded that there would be a more effective assessment process than the current procedure (using the Westside Survey Test and Orleans Hanna). The analysis of the historical data suggested that the new procedure would be more accurate, have slightly less of a chance of admitting students who would then struggle and would admit a few more students into the program. This new procedure would also make use of a test with greater demonstrated reliability and validity than a district constructed test.

The New System in Action

As expected, the new selection procedure resulted in nearly 67% of the students qualifying for Pre-algebra and has made the selection process easier to administer. Adding students' grades to the selection process using the numerical assignments as recommended by the *Orleans Hanna Test* is continuing to be monitored. Including grades and assigning the overall grade score to have an equal weight to the test itself, resulted in 35 students qualifying for Pre-algebra that would not have on the basis of the test score alone and disqualified 9 students that would have qualified on the basis of the test alone. The performances of these students are now being carefully observed.

As one might expect, we are finding that more advanced middle school mathematics coursework has significant implications for the mathematics curriculum throughout the secondary years. Increasing the number of students taking Algebra as eighth graders has the direct effect of increasing the number of students in advanced level mathematics in high school. The student who takes Pre-algebra as a seventh grader typically goes through a secondary course sequence that concludes with Calculus as a senior. Currently approximately 25% of the district's seniors take Calculus, roughly the same percentage that took Pre-algebra as seventh graders. Beginning with the new selection process for Pre-algebra in the 7th grade (and then Algebra in the 8th grade) the number of Calculus students at the high school level will potentially double.

As the district continues to review and adjust its mathematics placement process, some particularly talented students may well eventually become potential candidates for Calculus III as seniors. Historically the district has paid tuition for such students to enroll in Calculus III at a local University, but this will not be of interest for large numbers of students since Calculus III is required for only a few university majors. AP Statistics is being added to the high school course offerings to provide another option, but almost certainly, as more students are placed into early advanced coursework, the demand for higher-level mathematics courses in high school will grow.

Teacher perceptions continue to be mixed with the initial implementation of the selection process. Some teachers are skeptical that a larger percentage of students are able to handle Algebra and would still prefer a cut score resulting in fewer students being placed into the Pre-algebra sequence. Fewer identified students would indeed mean fewer students placed in Pre-algebra who do not perform well. However, it would also increase the number of students in seventh grade "General Mathematics" who might have been more appropriately placed in Pre-algebra.

The larger number of Pre-algebra students has also resulted in a scheduling challenge at the Middle School. Rather than six sections of seventh grade Pre-algebra, as was the case prior to the new selection process there are currently 11 sections. This change brings staffing and staff development implications. Teachers who have previously taught only seventh grade mathematics must be prepared to teach more challenging courses.

Although the greater numbers of accelerated students have required significant changes in middle school scheduling and staffing, the change has been particularly positive for scheduling in one important respect. Having a traditionally small number of accelerated students resulted in that group of students also taking other core curriculum courses such as English, Science and Social Studies together. This traditional procedure had the unfortunate effect of tracking throughout the system. With a larger number of students, it has been possible to schedule those students in a way that they can be better integrated throughout the system, minimizing the tracking across the middle school curriculum.

Next Steps: Where Do We Go from Here?

The changes related to algebra placement have been significant, but they are only just beginning. We will continue following the effectiveness and practicality of this new selection process. As greater numbers of students are placed and complete the courses, the statistical analyses we will conduct should be able to provide a more complete picture of how the new placement process is working. Curriculum review and staff planning is ongoing. High school staff and administrators have been involved throughout the change process and are fully aware of the implications. As more accelerated students advance through the system, significant changes will need to occur at the high school level. The high school will likely need to add Calculus III and certainly more sections of advanced mathematics classes will be needed. Who will teach these advanced level classes? That discussion is currently underway. Teachers who have taught Algebra and Geometry in the past will undoubtedly be asked to also teach these higher-level mathematic courses.

Finally, it is important that we continue the philosophical debate. There are those district educators who believe that only a very select group of students should be accelerated or take more advanced mathematics coursework. While at the other extreme, some educators believe that all seventh grade students should take Pre-algebra and that there should be no placement tests at all. We see such debate within the district as healthy and an important key to providing the best and most appropriate mathematics program for all students. Although we are still evolving toward a truly equitable and effective algebra placement strategy, we believe that we have made an important step forward with this revised and more inclusive placement process. As suggested by the John Kennedy, we also believe that "all children have an equal right to develop their talent, their ability and their motivation." Hopefully, the students in the Westside Public Schools are a step closer to realizing this important right with our mathematics curriculum.

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