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The Professional Development of Leaders and Teachers of Mathematics

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In 2003 with funding from the National Science Foundation (NSF/EHR-0314692), *Focus on Mathematics* (FOM) began its 5-year project to study, design and implement solutions to the problem of the large number of secondary school students experiencing difficulty with the learning and mastery of key ideas of mathematics. FOM is a collaboration of five Boston area school districts, their middle and high school mathematics leaders, teachers and students, and four institutions of higher education with their faculty in mathematics and mathematics education.

One of the first FOM activities was the establishment of the Curriculum Review Committee (CRC) composed of the math coordinators from the school districts and two Boston University faculty, one in mathematics and one in mathematics education. The initial charge for the committee was to review the districts' course and program syllabi for grades 6 through 12 to ensure that their topics and goals were in concert with the Massachusetts Curriculum Framework for Mathematics) and the Massachusetts Comprehensive Assessment System (MCAS) tests (www.doe.mass.edu/mcas).

After reviewing the materials, it was clear to the CRC that all programs and courses were aligned with the framework and with the tests. What then could account for students' poor performance on MCAS items? Several hypotheses were offered by the committee. The CRC's exploration of the hypotheses resulted in what we believe to be a fruitful and new approach to the professional development of leaders and teachers of mathematics.

We share this story with you because it 1) describes the genesis of our professional development program, 2) offers insights into students' fragile grasp of big mathematical ideas and skills, and 3) suggests the need to reconsider existing models used to represent key mathematical concepts. Although we focused our work in algebra, other mathematical content areas or ideas could just as effectively serve as the centerpiece of the activities. First, we present our story. This is followed by the Steps of the PD Model.

OUR STORY

In January of 2004, the mathematics leaders from the five school districts involved in FOM and the mathematics and mathematics education faculty at Boston University met for the first time. At that meeting, we realized that in order to tackle the problem of low performing students, we needed greater understanding of students' difficulties with major mathematical concepts. To gain some insight into those difficulties, we decided to spend several meetings analyzing student performance on the most recent MCAS tests, and on algebra items, in particular. The choice of algebra seemed to be a good one since the vast majority of teachers of students in grades 8 through 12 teach algebra or teach courses that involve the application of algebraic concepts and skills. Because the concept of linearity is fundamental to the study of algebra and a major topic in introductory algebra, the committee narrowed its focus to analysis of the linearity items and grade 8 student performances on those items.

What knowledge of linearity is required by the state test?

As a committee we "unpacked" each linearity item. The unpacking involved identifying 1) the mathematical concepts and skills that students need to bring to bear to the solutions of the problems, the reasoning methods required to solve the problems, and the types of displays or formats to be interpreted, and 2) possible reasons for students' difficulties with the items. What was particularly interesting in the discussion of #2 was that we teacher veterans did not reach consensus on the reasons for student difficulties with the MCAS items.

Why did students have difficulty with items on the state tests?

To gain consensus, we decided that we needed more input from the students. We selected three of the released linearity MCAS items that had appeared on the previous year's test and developed a clinical interview around the selected items. District math coordinators conducted and taped their interviews of grade 8 students solving the problems and describing their thinking. From the middle range of achievers, we selected students who liked to talk!

The entire committee viewed and reviewed the interviews and compared the "real" difficulties with our speculations. Yes, we were on target about 60% of the time. Surprisingly, we were not correct the rest of the time. Student difficulties were often totally unsuspected. We decided that we needed more information about the difficulties that we observed and the ones that we didn't suspect.

How can we get more information about student difficulties?

There was no existing vehicle for probing the specific difficulties so we spent several months developing our own assessment tool, which we will refer to as the Mini-Assessment Tool or MAT. For each item we developed scoring directions that would take note of specific types of errors.

The MAT consists of seven items that can be administered in one class period (see Appendix A). To reflect the formats of the MCAS items, the MAT contains one essay item, three short-answer items, and three multiple-choice items. A brief description of the seven items follows.

The Essay Response Item: Given coordinates of a point and the equation of a line, students determine if the point is on the line and describe their decision-making process.

The Short Answer Items: 1) Given an equation of a line with a negative slope, students create a table of values (coordinates) of points on the line. 2) Given a graph of a line, students identify the slope of the line. 3) Given a distance-time graph, students identify the part (one of three) of the graph that represents the car moving slowest; the slope of another part of the graph; and the car's speed in that other part of the graph.

Multiple-Choice Items: 1) Given a table of (x, y) values representing points on a line, students identify one of four graphs that contains all points. 2) Given a table of values showing the number of cars sold each week, students identify one of four linear equations that represents the relationship between number of cars sold and number of weeks. 3) Given a linear equation that is not in slope-intercept form, students identify one of the five possibilities for the value of the y-intercept.

One week after students completed the MCAS tests, Grade 8 classroom teachers in the five districts administered the MAT to all of their students and scored the tests. District leaders compiled results by school and by district.

What new information did we gain from our mini-test?

Our committee met shortly after all tests were scored to analyze the performance of the more than 3000 grade 8 students who completed the MAT. Findings revealed that across districts, students have minimal understanding of two major topics: points on a line and slope. (As an example, Figure 1 on page 18 shows the Data Reporting Sheet for Problem 1.)

With regard to points on a line, many students didn't know or weren't sure that: 1) all points on a line have two coordinates, including the y-intercept, 2) coordinates of points on the graph of a line satisfy the equation for that line, and 3) coordinates of points on a line that are presented in tabular form satisfy the equation for the line, and when plotted, produce a graph of the line.

With regard to slope, students demonstrated difficulty determining if lines shown in the coordinate plane had positive or negative slopes. Particular difficulty was noted when lines with positive slopes were pictured in the third quadrant of the coordinate plane. Many students could

FIGURE 1

PROBLEM 1	TALLY	TOTAL
a) Number of students who responded: **Yes**		
No		
b) Number of students whose explanations are: 1) **Replace *x*** with 2 and ***y*** with –8. Check that the two expressions are equal or **Replace *x*** with 2. Compute 3***x*** – 14(3 (2) – 14). Check result with the ***y*** value of –8		
2) **Replacement Error:** Error in computation after replacement		
3) **Graph:** Identify 2 points on the line. Construct graph. Locate (2,–8) on graphed line.		
4) **Graph Error:** Error in graphing or computing.		
5) Number of students who gave incomplete or incorrect responses different from those above.		
6) Number of students who didn't respond.		

not compute slopes from tables of data, graphs of lines, or equations of lines. Almost all students had difficulty recognizing the relationship between slope and speed in a time versus distance graph.

What more can we learn from the students themselves?

To validate our suspicions about the nature of the errors on the MAT (we learned a valuable lesson after interviewing students on MCAS items and discovering that our hypotheses about their errors were not always on target), district leaders conducted taped interviews of grade 8 students solving two of the MAT problems; one to determine the slope of a line from its graph and the other to interpret slope in an application problem. The difficulties cited above were confirmed in the interviews.

What could be contributing to student difficulties?

After viewing the tapes, and re-examining performance results on the MAT, district leaders were convinced that student difficulties stemmed from the format of problems, the grade level of the students, or the language and terminology used in the problem statements. Each of these sources of difficulty was investigated, and in the order listed.

Problem Format: In the original item on the MAT in which the slope of a line had to be determined from its graph, one of the points on the line was labeled with its coordinates and the line intersected the y axis at y = 4. However, there were no grid lines shown, and although the axes had hash marks, the scales were not indicated. The committee believed that the lack of grid lines and the unmarked axes presented a new situation for the students, one for which they were unprepared. To check out this hypothesis, three forms of that problem were developed and administered to about 1000 grade 8 students. One of the forms was the original presentation of the problem, a second showed grid lines, and a third showed grid lines and scales on the axes. Scores were analyzed and no significant difference by format was found.

Appropriate Grade Level: To check out the hypothesis that the concept of slope is too abstract for grade 8 students and should be explored with expected mastery by older students, the MAT was administered to all grade8, 9 and 10 students in the five districts. Performance was not markedly different by grade level. Grade 9 and 10 students experienced the same difficulties.

It was during the time when we were checking the relationship of grade level to success with MAT items, that our committee was joined by two international visiting faculty, Dr. Kyung Yoon Chang from Seoul, Korea and Dr. David Ben Chaim from Haifa, Israel. These faculty were quite interested in translating our MAT into their respective languages and administering the test to students in their countries. Dr. Chang gave the algebra test to Grade 8 students and Dr. Ben Chaim administered the MAT to grade 9 students. Students in those countries performed much like our students; they had difficulty with the same items and made the same types of errors.

Familiarity with Language: To explore difficulties posed by language, the mathematical language used in the 2000 through 2005 MCAS tests was compared with terminology in the instructional programs used in the five districts. Results of the comparison showed that all vocabulary in MCAS appeared in the various curricula.

Do instructional programs make a difference?

For two of the MAT items, one that focuses on identifying slope of a line from its graph, and the other that requires identifying the relationship between slope and speed, performance data were disaggregated by instructional program. Five different grade 8 programs were examined. In four of the districts, all students had difficulty with the two items regardless of the type of instructional program. In the fifth district, there remains the question of whether the ability levels of the students or the type of program made the difference.

As each new hypothesis was tested and results offered no explanation for student difficulties, the frustration level of the committee increased. The suggestion was made to examine the instructional programs. Perhaps the cause of the difficulties could be found in the programs themselves.

How do instructional programs differ?

How do instructional materials used in grade 8 introduce, maintain, and enrich concepts and skills of linearity, particularly those with which students have difficulty? Special attention was given to the models employed. Although some programs introduced the concepts in real-life settings and others used mathematical contexts, the models and language were essentially the same. Major differences were observed in 1) the particular chapter or unit when the concepts are introduced; 2) the nature of the types of applications presented; and 3) the frequency with which the concepts are revisited and practiced. We noted that, in all instructional programs, fundamental concepts of linearity were introduced but they were not reviewed and practiced systematically in order to enhance understanding and recall.

Do mathematics programs in grades 5 through 7 prepare students for algebra?

Another suggestion was made that the middle school curricula be studied to determine if students are being well-prepared to study algebra in grades 8 or 9. We are now asking ourselves, 1) What constitutes good preparation for Algebra I? 2) Is there a well-articulated development of mathematical concepts and skills in the grades preceding Algebra I that lead to the formal study of algebra? 3) For those applications of linearity that appear in middle school programs (and perhaps upper elementary school levels, as well), are students and their teachers aware that these are applications of the concept? Are these problems

THE *FOCUS ON MATHEMATICS* PROFESSIONAL DEVELOPMENT MODEL

Step 1: Select a content area focus and grade level(s).
We selected algebra and then looked more closely at an aspect of algebra. You can choose any content area of interest. Be sure that the focus area is not too broad.

Step 2: Analyze student performance on state or other tests.
Unpack items. Identify concepts and skills assessed, as well as displays to be interpreted. Speculate about student difficulties.

Step 3: Conduct video-taped interviews of students solving problems on the tests.
View tapes together, analyze student performances, and compare observed errors with your speculations about student difficulties.

Step 4: Design a mini-assessment tool (MAT) to probe student difficulties.
The format of items should reflect types used in your state assessments. Construct a scoring form that will help test administrators/teachers identify particular errors.

Step 5: Administer your MAT to all students and score the tests.

Step 6: Analyze performance on the MAT.
Speculate about student difficulties.

Step 7: Interview students solving MAT items.
As a group, view and analyze the tapes. Confirm/negate speculations about student difficulties.

Step 8: Examine instructional programs.
In what ways do they attend to student difficulties? Are other instructional materials/activities needed? How do earlier grades treat these concepts, skills and displays? Is there a clear articulation across grades?

Step 9: Develop a plan for scaffolding instruction of key concepts across grades.

highlighted and developed in such a way as to help students gain understanding?

What have we learned as a committee?

Although our committee has met once or twice each month since 2004, we still don't have answers to all of our questions. However, as teacher educators, we've learned a great deal of mathematics by "unpacking" problems and thinking about the concepts, sub-concepts and skills required to solve problems. We've become smarter about how to interpret student performance in mathematics and particularly, performance on local and state tests. We've gained greater insight into what students know and are able to do mathematically by interviewing them, and as a team, listening to and analyzing their comments, their interview behaviors, and their written work. We've become better at speculating about the nature of student errors. And, we've been humbled by how much about the learning of mathematics we still don't know.

We not only intend to continue our work as a committee, but we also want to engage our middle and high school teachers in the same type of study that we have done. We believe that the series of steps that were generated by our curiosity and observations constitute a new model for the professional development of leaders and of teachers of mathematics.

A FINAL NOTE

We believe that our own professional development was enhanced by the variety of districts and mathematics educators that were involved. If you decide to use this model, we strongly recommend that you join with at least one other school district and include university faculty in mathematics and mathematics education on your committee. If you are interested in administering our algebra MAT and comparing results with our districts, please contact Carole Greenes at cgreenes@bu.edu.

APPENDIX A

Please show all work on these pages

1. (a) Is the point with coordinates (2, –8) on the graph of the line $y = 3x–14$?

(b) How did you decide?

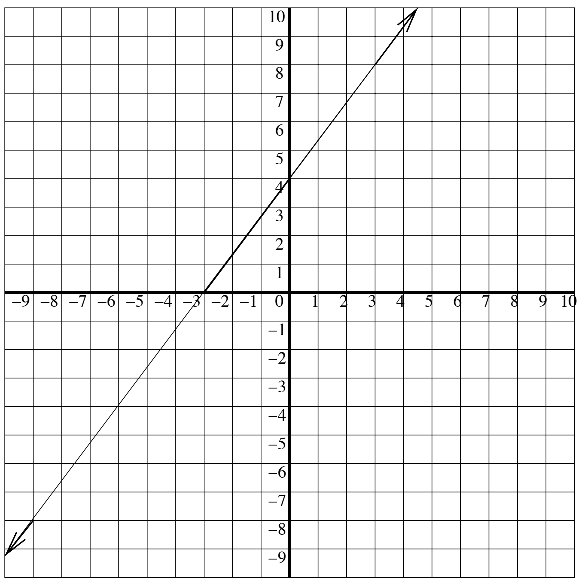
2. $y = 2x + 3$

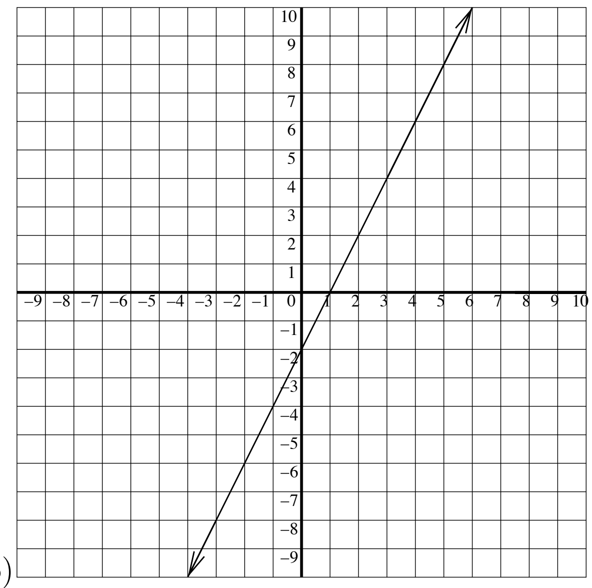
Create a table of values for the equation. Complete 5 rows of the table.

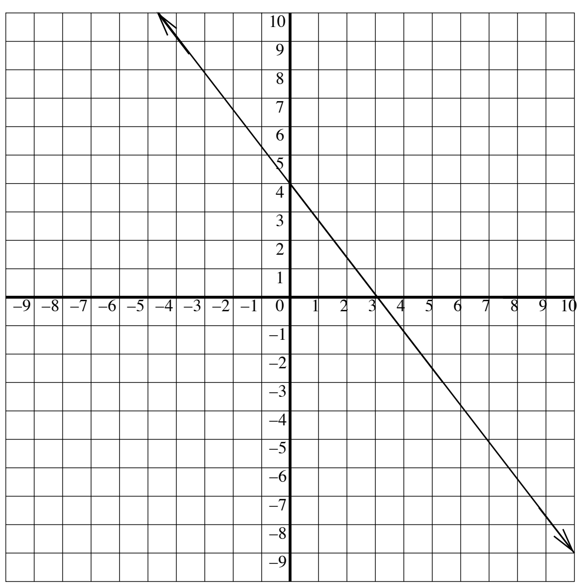
x	y
a.	
b.	
c.	
d.	
e.	

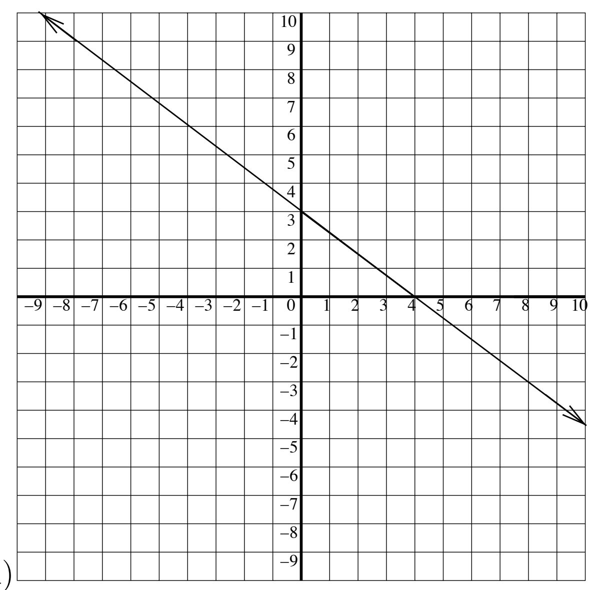
x	*y*
3	4
6	10
9	16
12	22

3. Which of the graphs below contains the points given in the table?

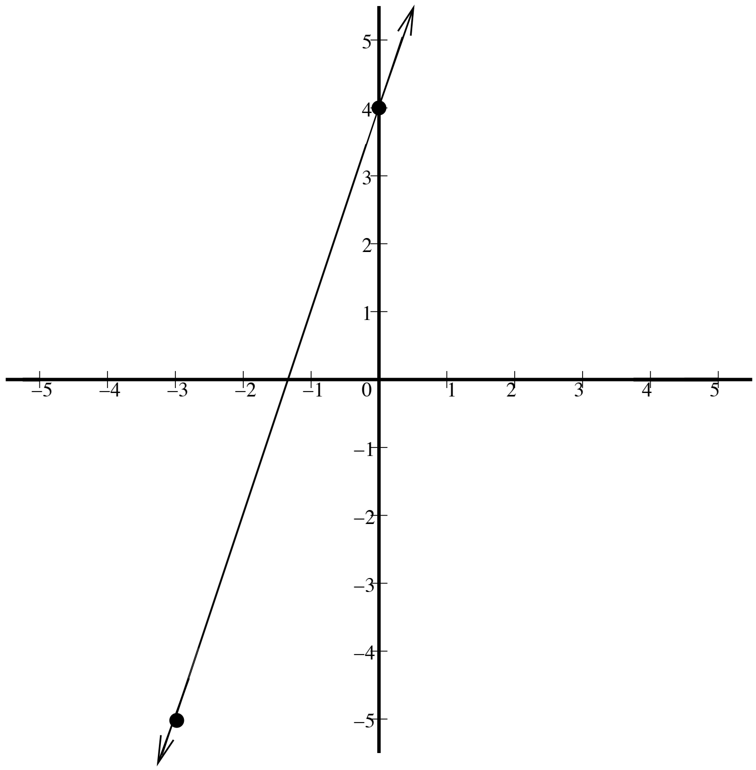
a) 

b) 

c) 

d) 

4. In the graph below, what is the slope of the line? Show your work.



5. The owner of a car dealership noticed a pattern in the weekly car sales, as shown in the table below.

Weekly Car Sales

Week *(w)*	**Number of Cars Sold** *(s)*
1	12
2	18
3	24
4	30

Which of the following equations represents the relationship between the number of cars sold (s) and the number of weeks *(w)*?

(a) $s = 6w$
(b) $s = 12w$
(c) $s = 6w + 6$
(d) $s = 6w + 12$

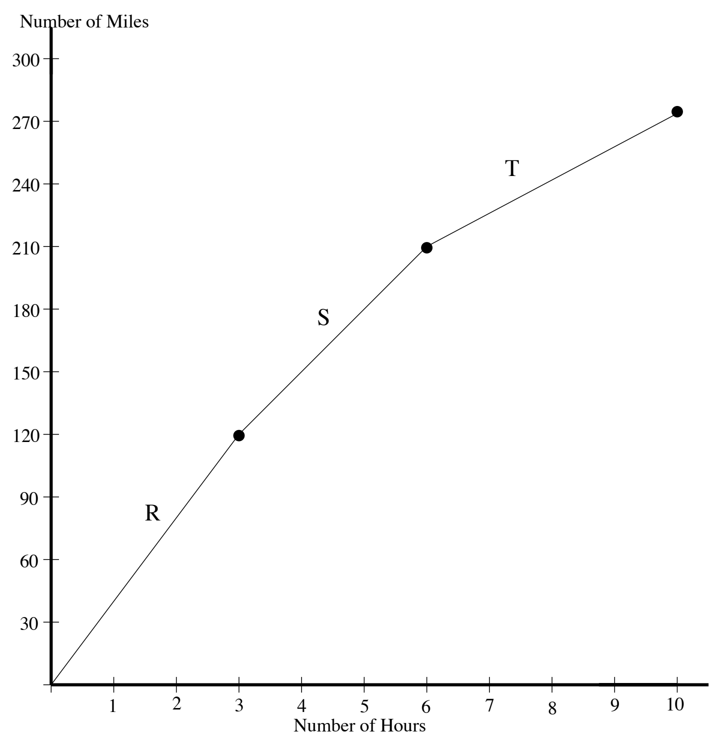
6. Given this linear equation

$$2x + 3y = 12,$$

which of the following is the y-intercept?

(a) 0
(b) 2
(c) 4
(d) 6
(e) 12

7. The graph below represents the distance that a car traveled after different numbers of hours



(a) Which part of the graph(R, S, or T) represents the hours when the car moved the slowest?

(b) What is the slope of part R of the graph?

(c) What is the speed of the car in part R?