

NATIONAL COUNCIL OF SUPERVISORS OF MATHEMATICS

NCSM

JOURNAL

of Mathematics Education Leadership

VOLUME 10, NUMBER 1

SPRING 2007



Table of Contents

| | |
|---|----|
| COMMENTS FROM THE EDITOR | 1 |
| Mark Driscoll, Education Development Center, Newton, MA | |
| TEACHERS NEED TO SELL MATHEMATICS TEACHING: | |
| <i>Reaching Out to Excellent High School Students</i> | 4 |
| Alice F. Artzt and Frances R. Curcio, with Naomi Weinman, Queens College, CUNY | |
| MAKING THE “CUT”: <i>One District’s Strategy for Algebra Placement</i> | 8 |
| Neal Grandgenett, Ph.D., University of Nebraska at Omaha Roberta Jackson, Ed.D, Westside Community Schools, Omaha, Nebraska | |
| THE PROFESSIONAL DEVELOPMENT OF LEADERS AND TEACHERS OF MATHEMATICS | 16 |
| Carole Greenes and Steve Rosenberg, Boston University; Kathleen Bodie, Arlington Public Schools; Donna Chevaire, Lawrence Public Schools; Charles Garabedian and Daniel Wulf, Watertown Public Schools; Ann Halteman and Kevin Wynn, Chelsea Public Schools; Eileen Herlihy, Waltham Public Schools | |
| THE INTERPERSONAL SIDE OF PROFESSIONAL DEVELOPMENT IN MATH | 25 |
| Myriam Steinback, TERC and Kimberly Sherman, University of Rhode Island | |
| WHAT?, WOW!, AND HMM...: | |
| <i>Video Clips that Promote Discussion of Student Math Thinking</i> | 32 |
| Katherine A. Linsenmeier, New Trier High School & Northwestern University Miriam Gamoran Sherin, Northwestern University | |
| THE PROBLEM-SOLVING CYCLE: | |
| <i>A Model of Mathematics Professional Development</i> | 42 |
| Jennifer Jacobs and Hilda Borko, University of Colorado at Boulder Karen Koellner, University of Colorado at Denver and Health Sciences Center Craig Schneider, Eric Eiteljorg, and Sarah A. Roberts, University of Colorado at Boulder | |
| AN AGENT OF CHANGE: <i>NSF Sponsored Mathematics Curriculum Development</i> | 58 |
| Barbara J. Reys & Robert E. Reys, University of Missouri | |
| STATE MATHEMATICS CURRICULUM STANDARDS AND REASONING | 65 |
| Ok-Kyeong Kim, Western Michigan University and Lisa Kasmer, Portage Public Schools | |
| INTEGRATING NRC PRINCIPLES AND THE NCTM PROCESS STANDARDS TO FORM A CLASS LEARNING PATH MODEL THAT INDIVIDUALIZES WITHIN WHOLE-CLASS ACTIVITIES | 72 |
| Karen C. Fuson, Northwestern University and Aki Murata, Stanford University | |

The Problem-Solving Cycle: *A Model of Mathematics Professional Development*

Jennifer Jacobs and Hilda Borko, University of Colorado at Boulder
Karen Koellner, University of Colorado at Denver and Health Sciences Center
Craig Schneider, Eric Eiteljorg, and Sarah A. Roberts, University of Colorado at Boulder

ABSTRACT:

There is a growing consensus that mathematics teachers need to significantly expand their content and pedagogical content knowledge in order to make instructional improvements and provide increased opportunities for student learning. Long-term, sustainable professional development programs can play an important role in this regard. Our research team has spent the past several years developing a program called the Problem-Solving Cycle (PSC). This professional development model is grounded in a situative perspective on learning and draws upon theoretical and empirical evidence regarding the importance of professional learning communities and the value of using artifacts of practice to situate teachers' learning in their classroom experience. The model takes into account the complexity of classroom teaching, the wide array of knowledge teachers need to promote the mathematical thinking of their students, and the long-term commitment required to develop such knowledge. In this article, we present the conceptual framework for the PSC, details of its enactment, and initial findings regarding its impact on teachers' knowledge.

The Problem-Solving Cycle (PSC) model of mathematics professional development is an iterative, long-term approach to supporting teachers' learning. One iteration of the PSC consists of three interconnected workshops in which teachers share a common mathematical and pedagogical experience, organized around a rich mathematical task. This common experience provides a structure within which the teachers can

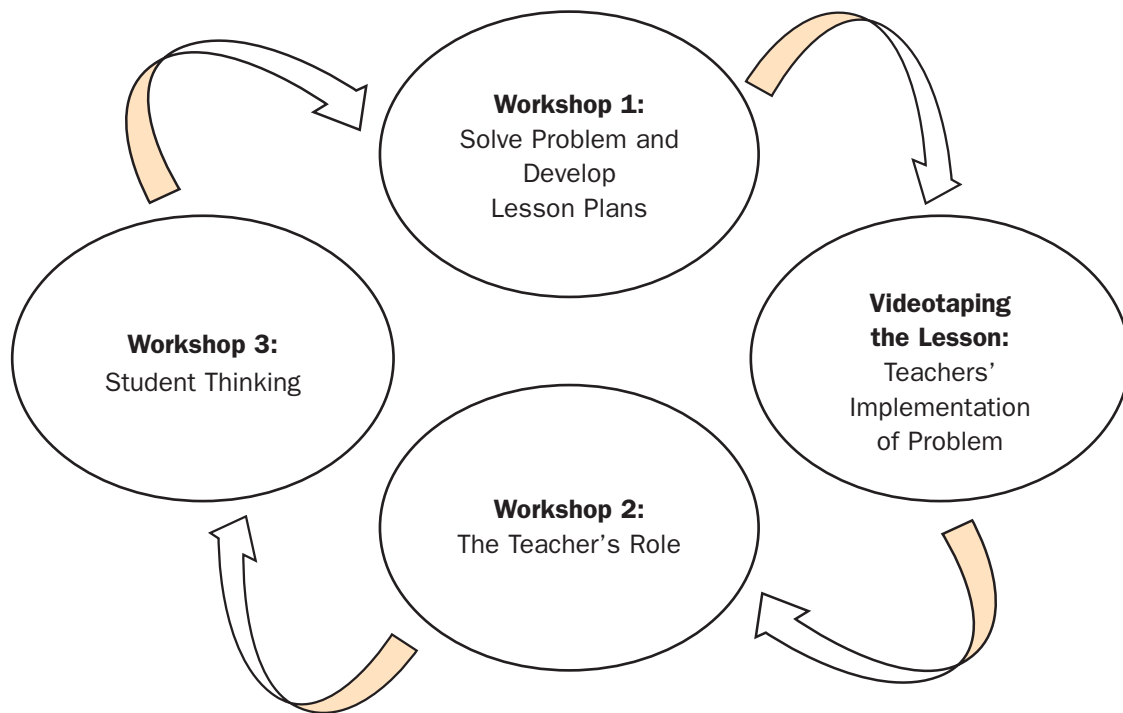
build a supportive community that encourages reflection on mathematical understandings, student thinking, and instructional practices.

During the first workshop of the PSC, teachers collaboratively solve a rich mathematical task and develop plans for teaching it to their own students. Workshops two and three focus on teachers' experiences implementing the task in their classrooms (see Figure 1). The teachers consider more about the mathematical concepts and skills entailed in the task, their instructional strategies, and their students' mathematical thinking. In all three workshops, there is an emphasis on using artifacts of practice to situate teachers' learning opportunities in the context of their work.

One iteration of the PSC roughly corresponds to an academic semester, so that teachers can participate in 2 iterations (6 workshops) per school year. Each iteration focuses on a unique mathematical task and highlights different aspects of teachers' instructional practices and students' mathematical thinking. Successive iterations of the PSC build on one another and capitalize on teachers' expanding knowledge, interests, and sense of community. The PSC model is designed to be implemented by a knowledgeable facilitator, who carefully plans and conducts each workshop and continually monitors the participating teachers' needs and interests. The facilitator might be a teacher leader, mathematics coach, department chair, professional development specialist, or other teacher educator.

The PSC model is flexible with respect to the domain of mathematics that is selected as well as the specific learning goals and instructional strategies that are addressed. In our

FIGURE 1: *The Problem-Solving Cycle model of professional development*



Note: The arrow from Workshop 3 to Workshop 1 represents movement from one iteration of the PSC to the next.

work, we have focused on algebra because of the growing concern regarding students' inadequate understanding and preparation in this domain of K-12 mathematics (U.S. Department of Education and National Center for Educational Studies, 1998). Algebra operates as a "gate-keeper" to higher mathematics and future educational and employment opportunities (Ladson-Billings, 1998; NRC, 1998). Students' difficulties in learning formal algebra are well documented (Kieran, 1992; Nathan & Koedinger, 2000), and our schools' approaches to algebra instruction are lacking. For example, first-year algebra courses have been characterized as "an unmitigated disaster for most students" (NRC, 1998, p. 1).

The enhancement of teachers' professional knowledge about algebra and the teaching of algebra is considered to

be a central component in the effort to support students' algebraic reasoning (Blanton & Kaput, 2005; Lacampagne, Blair, & Kaput, 1995; NCTM, 2000). The PSC model was developed and implemented as part of the Supporting the Transition from Arithmetic to Algebraic Reasoning (STAAR) project¹, which aimed to help teachers enhance their professional knowledge for the teaching of algebra and improve their instructional practices. We focused on middle school because it is becoming more common for school districts to require that algebra be taught during the middle school years, yet many middle school teachers have limited experience in teaching algebra. Furthermore, their experiences as algebra students typically emphasized learning procedures and manipulating symbols rather than reasoning about algebraic ideas (Ball, Lubienski, & Mewborn, 2001).

¹ The professional development program is one component of the STAAR Project, supported by NSF Proposal No. 0115609 through the Interagency Educational Research Initiative (IERI). The views shared in this article are ours, and do not necessarily represent those of IERI.

CONCEPTUAL FRAMEWORK FOR THE PROBLEM-SOLVING CYCLE

In this section, we present the conceptual and empirical grounding for the goals and processes of the Problem-Solving Cycle. We first explore the professional knowledge that mathematics teachers need and then discuss critical elements in designing professional development from a situative perspective.

PSC Goals: Enhancing Teachers' Professional Knowledge

Researchers and policymakers have come to agree that objectives for teacher learning should include becoming more proficient in the content they teach, gaining a better understanding of student thinking and learning, and improving their skills in content-based instructional practices (Secretary's Summit on Mathematics, 2003). These learning objectives provide the foundation for the PSC model of mathematics professional development.

In his seminal work in this area, Shulman (1986) identified subject-matter content knowledge and pedagogical content knowledge as two central domains of teachers' knowledge. Both domains are unique to the profession of teaching and can be enhanced over time as teachers gain expertise in their fields and participate in programs designed to foster such knowledge development (Wilson, Shulman, & Richert, 1987). Ball and her colleagues have extended Shulman's work in the field of mathematics education. Specifically, they have identified and elucidated "knowledge of mathematics for teaching" — the mathematical knowledge that teachers must have in order to do the mathematical work of teaching effectively (e.g., Ball & Bass, 2000; Ball, Hill, & Bass, 2005; Ball, Thames, & Phelps, 2005; Hill & Ball, 2004). This conception of knowledge of mathematics for teaching is multifaceted and incorporates both content and pedagogical content knowledge.

Mathematics content knowledge. Ball and Bass (2000) describe the mathematics content knowledge needed for teaching as including "common" and "specialized" knowledge of mathematics. Common content knowledge can be defined as a basic understanding of mathematical skills, procedures, and concepts acquired by any well-educated adult. Specialized knowledge involves a deeper, more nuanced understanding of mathematical skills, procedures, and concepts. Specialized knowledge enables teachers to evaluate the multiple, and novel, mathematical representa-

tions and solution strategies that students bring to the classroom; to analyze (rather than just recognize) errors; to give mathematical explanations; to use developmentally appropriate mathematical representations; and to be explicit about their mathematical language and practices (Ball & Bass, 2003). It is what Ma (1999) characterizes as "profound understanding of fundamental mathematics" (p. 120).

Pedagogical content knowledge. Mathematics teachers need a sophisticated understanding of instructional practices and student thinking related to specific mathematical content. Ball and her colleagues consider these two types of understanding as distinct components of pedagogical content knowledge: knowledge of content and teaching, and knowledge of content and students (Ball, Thames, & Phelps, 2005). Knowledge of content and teaching includes, for example, the ability to recognize instructional affordances and constraints of different representations, and to sequence content to facilitate student learning. Teachers draw upon this knowledge when they plan for the use of pedagogical strategies and instructional materials in a lesson, when they modify a task or introduce a new representation during instruction, and when they consider how to improve their instructional practices the next time they implement a lesson with related mathematical content. Knowledge of content and students includes the ability to predict how students will approach specific mathematical tasks, and to anticipate student errors. Teachers draw upon this knowledge when they create lesson plans that take into account the thinking that a task is likely to evoke in their students, when they interpret incomplete student ideas during a lesson, and when they consider how to respond to the various correct or incorrect pathways that students explore.

Although these domains of knowledge of mathematics for teaching can be separated for purposes of analysis, they are inextricably intertwined in teachers' instructional practices. Teachers routinely make decisions that draw upon all aspects of their knowledge as they engage in the numerous and complex activities of classroom instruction — activities such as selecting, modifying, and using mathematical tasks; selecting mathematical representations that are appropriate for a specific learning goal and group of students; understanding and building upon student conceptions; and establishing and maintaining a discourse community that enhances students' mathematical understanding and their capacity to reason mathematically.

Whereas knowledge of mathematics for teaching includes all strands of school mathematics, our research and professional development as part of the STAAR project focused specifically on algebra; hence we use the term “knowledge of algebra for teaching” (KAT)². Drawing upon the framework developed by Ball and colleagues, we conceptualize enhancing knowledge of algebra for teaching as enhancing both specialized content knowledge related to *algebraic reasoning* and pedagogical content knowledge related to *algebra instruction*.

Designing Professional Development from a Situative Perspective: Community and Artifacts as Tools for Teacher Learning

Situative perspectives on cognition and learning provide the conceptual framework that guided the design of the PSC. In the field of professional development, a situative perspective supports the value of creating opportunities for teachers to work together on improving their practice, and of locating these learning opportunities in the everyday practice of teaching (Ball & Cohen, 1999; Putnam & Borko, 1997; Wilson & Berne, 1999).

Professional learning communities. Situative theorists draw our attention to the social nature of learning and the central role that communities of practice can play in enhancing teachers’ professional knowledge and improving their practice (Greeno, 2003; Lave & Wenger, 1991; Little, 2002; Putnam & Borko, 2000). To create an environment in which teachers collectively explore ways of improving their teaching and support one another as they work to transform their practice, successful professional development programs must establish trust, develop communication norms that enable challenging yet supportive discussions about teaching and learning, and maintain a balance between respecting individual community members and critically analyzing issues in their teaching (Frykholm, 1998; Seago, 2004). Research also indicates that the development of teacher communities is difficult and time-consuming work. Although conversations in professional development settings are easily fostered, discussions that support critical examination of teaching are relatively rare (Grossman, Wineburg, & Woolworth, 2001; Stein, Smith, & Silver, 1999).

Artifacts of practice. Another central tenet of situative perspectives is that the contexts and activities in which people learn become a fundamental part of what they learn (Greeno, Collins, & Resnick, 1996). This tenet suggests that teachers’ own classrooms are powerful contexts for their learning (Ball & Cohen, 1999; Putnam & Borko, 2000). It does not imply, however, that professional development activities should occur only in K-12 classrooms. An alternative is to use artifacts of classroom practice—such as instructional plans and assignments, videotapes of lessons, and student work produced during a lesson—to bring teachers’ classrooms into the professional development setting (Kazemi & Franke, 2004; Little, Gearhart, Curry, & Kafka, 2003; Nikula, Goldsmith, Blasi, & Seago, 2006; Sherin & Han, 2004). Such records of practice make the work of teaching a central focus of professional learning experiences and anchor conversations in specific classroom events.

Video records of classroom practice are becoming increasingly popular as a tool for teacher professional development. Short video clips can be selected to address particular professional development goals. They can be viewed repeatedly and from different perspectives, enabling teachers to closely examine one another’s instructional strategies and student learning, and to discuss ideas for improvement. Although any video of classroom instruction can situate professional development in a setting that is likely to prove meaningful for teachers, there are conceptual and empirical arguments for using video from participants’ own classrooms. Video from teachers’ own classrooms situates their exploration of teaching and learning in a more familiar, and potentially more motivating, environment than does video from unknown teachers’ classrooms (LeFevre, 2004). In one comparative study, teachers who watched video from their own classroom, in a computer-based professional development environment, found the experience to be more stimulating than did teachers who watched video from someone else’s classroom, and they believed that the professional development program had greater potential for promoting instructional change (Seidel et al., 2005). The “video club” mathematics professional development program by Sherin and colleagues (Sherin, in press; Sherin & Han, 2004; Sherin & van Es, 2002) and the Video Case Studies in Scientific Sense Making Project by Rosebery and colleagues

² The term KAT is also used by the *Knowing Mathematics for Teaching Algebra* project at Michigan State University (Ferrini-Mundy et al., 2005). These two projects are unrelated, although our work draws upon their conceptualization of knowledge of algebra for teaching.

(Rosebery & Puttick, 1998; Rosebery & Warren, 1998) informed our thinking about how to create an effective professional development program that incorporates video from participating teachers' own classrooms.

Establishing community around video. . Establishing and maintaining a strong community is particularly important when teachers are asked not only to discuss teaching and learning but also to share video clips from their own classrooms with colleagues. Because classroom video clearly exposes actual teaching practices, sharing video is likely to seem more threatening to teachers than sharing other artifacts such as student work and lesson plans. To be willing to take such a risk, teachers must feel confident that showing their videos will provide valuable learning opportunities for themselves and their colleagues, and that the atmosphere in the professional development setting will be one of productive discourse.

In an appropriate professional development setting, analyzing video from teachers' own classrooms can help to foster a tightly knit and supportive learning community. As teachers share video records of their teaching with colleagues, they have the opportunity to create an atmosphere of openness and bonding that is rare in professional learning environments (Sherin, 2004). Creating and maintaining a productive learning community around video is an integral component of our professional development model (Borko, Jacobs, Eiteljorg, & Pittman, in press).³

THE PROBLEM-SOLVING CYCLE

In this section, we describe the three workshops that make up one iteration of the PSC, discuss decisions central to planning each workshop, and identify some of the variations enacted by the STAAR team. In another paper, we provide vignette descriptions of each workshop from one iteration of the PSC, illustrating the opportunities teachers had for learning about mathematics content, pedagogy and student thinking (Koellner et al., in press). In addition, our website (<http://www.colorado.edu/education/staar/>) includes a Facilitator's Guide to Planning and Conducting the Problem-Solving Cycle. The guide is intended to help professional development facilitators learn about the Problem-Solving Cycle and prepare to implement it.

WORKSHOP 1: Doing for Planning

The major objective of Workshop 1 is to support the development of teachers' mathematics content knowledge. Most of the workshop time is devoted to teachers collaboratively working on the selected mathematical task and debriefing their solution strategies. Additionally, teachers spend a significant portion of Workshop 1 developing unique lesson plans that will meet the needs of their students. Specifically, they identify learning goals, predict student solution strategies, and structure their lessons with specific pedagogical moves. Teachers then implement their lessons prior to Workshop 2. Thus, another aim of the workshop is to enhance teachers' *pedagogical content knowledge* through discussions about designing a lesson plan and considering different ways of teaching the selected task. We call the framework for this workshop "Doing for Planning" to highlight the dual focus on teachers' problem solving and instructional planning.

Selecting the task. As described above, the PSC is built around a rich mathematical task. Teachers work through the task, design a lesson incorporating the task, teach that lesson to their students, and discuss their classroom experiences in two subsequent workshops. For the PSC to be successful, facilitators must select a task that can foster a productive learning environment for the teachers over the course of three workshops. In our development and implementation of the PSC model, we have found that appropriate tasks meet the following criteria: (1) address multiple mathematical concepts and skills, (2) are accessible to learners with different levels of mathematical knowledge, (3) have multiple entry and exit points, (4) have an imaginable context, (5) provide a foundation for productive mathematical communication, and (6) are both challenging for teachers and appropriate for students.

Given our focus on algebraic reasoning, for each iteration of the PSC conducted by the STAAR team, the facilitators sought problems that contained mathematical ideas central to the middle school algebra curriculum. Facilitators selected problems that focused specifically on the algebraic concepts of patterns and functions; enabled teachers and students to utilize different representations of functions such as graphs, tables, and equations; and had connections

³ A discussion of the STAAR professional development program's approach to building professional community is beyond the scope of this article. For more information about the program, including our approach to building community, please see Authors 2005; and Authors, in press a.

to other areas of the mathematics curriculum such as number and operations, and geometry.

Conducting the workshop. The “Doing for Planning” framework guides the structure of Workshop 1. Teachers first read the selected problem and share ideas about the mathematical concepts and skills that are likely to be embedded in the solution strategies. They then work on the problem in small groups. During this time, the facilitator encourages the teachers to think about how they would create a lesson for their students incorporating the problem. At various points in the workshop, the teachers come together as a whole group to share their solution strategies and their ideas for using the problem in their teaching. As teachers create lesson plans tailored to their own students, they talk with colleagues and the facilitator about such issues as their mathematical goals for students, prior knowledge students will need for the lesson, and how they will adapt tasks to make them more accessible for their students. By the end of the workshop, teachers have explored the mathematical opportunities presented by the task, considered how their students might attempt to solve it, and developed a lesson plan for using the task in their classrooms..

Implementing and Videotaping the Lesson

Between Workshops 1 and 2, each participant teaches the problem in one of his or her mathematics classes, and the lesson is videotaped. In the STAAR program, we used two cameras to film each lesson. One camera followed the teacher throughout the lesson, and a second camera captured one group of students as they worked during small group activities. One of the most important components in Workshops 2 and 3 is the analysis of teachers’ pedagogical moves and students’ mathematical reasoning using video clips of the PSC lessons. Therefore, after the videotaping occurs, the facilitator selects short clips to serve as anchors for discussions about teaching and learning during Workshops 2 and 3.

WORKSHOP 2: Considering the Teacher’s Role

The central purpose of the second workshop is to foster teachers’ pedagogical content knowledge by guiding them to think deeply about the role they played in teaching the selected problem to their students. The majority of time in Workshop 2 is spent watching and discussing short video clips from one or more of the teachers’ lessons, and exploring aspects of the teacher’s role such as how they introduced the problem or orchestrated the classroom discourse. The workshop provides teachers the opportunity

to critically reflect on their own instructional practices, along with those of their colleagues, as they analyze video clips and participate in guided discussions. The rich task and accompanying video situate the workshop in particular mathematical content and classroom practices, and this interaction between content and pedagogy is highlighted throughout the workshop

Planning the workshop. In planning for Workshop 2, the facilitator identifies one or more aspects of the teacher’s role to explore. This decision depends on the particular needs and interests of the group of teachers as well as overall goals of the professional development program. Another key set of decisions for the facilitator involves selecting video clips to show and developing guiding questions for discussions during the workshop. We have found that video clips that work well in the PSC model have the following characteristics: (1) are relevant to the teachers, (2) are valuable, challenging, and accessible to the teachers, (3) cover a relatively short time period, and (4) provide an anchor for considering new instructional strategies. In addition, we have learned that it is important to prepare questions to help frame teachers’ viewing of and conversations about each video clip.

During our three iterations of the PSC, the STAAR facilitators focused on topics related to the teacher’s role such as introducing the task; posing questions to elicit, challenge, and extend students’ thinking; deciding when to provide explanations, ask leading questions, and let students follow their own line of reasoning; and wrapping up the lesson.

Conducting the workshop. Workshop 2 typically begins with teachers reflecting on and sharing their experiences teaching the problem. Subsequent activities are designed around the selected pedagogical topic and associated video clips. Teachers view the clips in both small group and whole group contexts, and the facilitator guides conversations about the instructional episodes they capture. Often, a video clip is viewed multiple times, as the conversation suggests another perspective to take or another interpretation to explore. Teachers are also given time to reflect critically and to consider ways of improving their instruction that they can take back to their classrooms.

WORKSHOP 3: Considering Student Thinking

The central objectives of Workshop 3 are to deepen teachers’ understanding of students’ thinking about the mathematics in the selected PSC task, and to extend their

ideas about how to foster and support students' mathematical reasoning. To situate teachers' explorations in their classroom practice this workshop relies heavily on clips from the videotaped lessons as well as additional artifacts that represent student thinking, such as students' written work and reflections. Throughout the workshop, teachers have opportunities to gain further insight into the complexities of both the mathematical concepts entailed in the problem and students' learning of those concepts.

Planning the workshop. A major task in planning Workshop 3 is selecting artifacts of practice that will provide opportunities for teachers to explore the various forms of mathematical reasoning their students applied to the problem and the different solution strategies they used. To select video clips, the facilitator considers the same characteristics as in planning for Workshop 2; however, rather than choosing clips to provide an anchor for examining instructional strategies, the facilitator selects clips to provide an anchor for considering student thinking. In a similar manner, facilitators select "rich" examples of student work such as individual student work on the task, posters created by groups of students, and written reflections. As in Workshop 2, the facilitator prepares guiding questions to help frame teachers' conversations about each video clip and example of student work, encouraging them to focus on the mathematical concepts and reasoning evident (or lacking) in the artifact.

The STAAR facilitators often chose video clips and student work that featured novel ways of solving the mathematical problem—in particular, solution strategies that none of the teachers noted during Workshop 1. We also addressed topics such as how students explained their solution strategies, and misconceptions or naïve conceptions.

Conducting the workshop. . In Workshop 3, teachers spend the majority of the time watching and discussing video clips and students' written work. Close analysis of the mathematical content in the clips and other artifacts often leads the teachers to rework the problem, and to engage in mathematically sophisticated conversations. For example, they may be prompted to discuss the affordances and constraints of various solution methods, the progression from naïve to more formal understandings of the content, and mathematical ideas embedded in the problem that they had not previously considered. Workshop 3 also includes time for teachers to reflect on what they have learned, in this workshop and over the course of one iteration

of the PSC. As they reflect, individually (in writing) and collaboratively (in small or whole group discussions), the teachers not only consider how they might improve their instructional practices based on knowledge gained thus far but also provide valuable input that the facilitator can use to shape successive iterations of the PSC.

RESEARCH METHODS

The STAAR professional development program began in 2003 and continued through spring 2005. During that time, we worked with a group of middle school mathematics teachers to develop and refine the PSC model. In fall 2003, we conducted three professional development workshops that focused on pedagogical practices associated with algebra. A central goal of these workshops was to develop norms for viewing and analyzing classroom video before conducting the first iteration of the PSC. We conducted the first PSC in spring 2004 and two more iterations during the 2004–2005 academic year. The three iterations used different mathematics problems and focused on different aspects of the teacher's role and students' mathematical reasoning. During the three iterations of the PSC, we utilized a design experiment approach (Cobb et al., 2003; Design-Based Research Collective, 2003) to study and refine the model

Participants

Eight teachers participated in the STAAR professional development workshops during the 2003–2004 academic year. All eight were middle school mathematics teachers, with classroom experience ranging from 1 to 27 years. They represented six different schools in three school districts within the state. In 2004–2005, seven teachers continued working with us and three additional teachers joined the project, as we further refined the PSC. Each new teacher was a colleague of one of the current participants.

Data Collection and Analysis

Throughout the professional development program we collected and analyzed a large amount of data on processes involved in developing and enacting the PSC model (see also Borko et al., in press and Koellner et al., in press). We also collected data on the teachers' experiences and learning outcomes over the course of the two years that they participated in the STAAR program. At the end of the second year we conducted both a written survey and individual face-to-face interviews asking the teachers to consider the impact of the professional development program on their learning of algebra, beliefs about learning and teaching algebra, and

instructional practices. In addition, we conducted a follow-up interview with each teacher during the school year following the conclusion of the professional development workshops, in order to assess their perception of the continuing impact of the professional development program.

To examine teachers' perceptions of the impact of the program, two coders analyzed three sets of self-report data: the written surveys completed during the final PD workshop, the post-program interviews conducted shortly after the final PD workshop, and the follow-up interviews conducted the next academic year. All of the interviews were transcribed. The coders independently marked all instances where the teachers wrote about or discussed the following categories:

- Impact on content knowledge,
- Impact on pedagogical content knowledge related to the teacher's role,
- Impact on pedagogical content knowledge related to student thinking, and
- Impact of watching video (including video of themselves and of their colleagues).

The coders then met to discuss and reconcile their coding decisions. In our analyses we report on the number of teachers who brought up these categories in at least one of the three data sources.

IMPACT OF THE STAAR PROFESSIONAL DEVELOPMENT PROGRAM

In this section we present initial results regarding the impact of the STAAR professional development program on the participating teachers' professional knowledge from their perspectives. In particular, we illustrate the perceived impact of the program on teachers' mathematics content knowledge and pedagogical content knowledge — specifically the teacher's role in promoting discourse and student thinking. We also explore the teachers' perspectives regarding a central component of the PSC model: watching video of themselves and their colleagues.

Impact on Content Knowledge

Analyses of the three self-report data sources suggest that the teachers believed their content knowledge was fostered

through participation in the professional development program. Specifically, we examined three categories of coded data related to content knowledge: a) learning mathematics content (generally), b) learning by working on the mathematics tasks that were part of the PD, and c) learning from using multiple approaches to solve the mathematics tasks. Looking across the three data sources for the eleven teachers⁴ who participated in the program, six teachers mentioned learning mathematics content, all eleven mentioned learning by working on the mathematics tasks, and ten mentioned learning from using multiple approaches (see Table 1).

Impact on Pedagogical Content Knowledge

We considered the impact of the professional development program on two aspects of teachers' pedagogical content knowledge that are emphasized heavily in the PSC model: the teacher's role and student thinking.

Knowledge about the teacher's role in promoting discourse.

Based on the participants' stated interests and instructional goals, the STAAR project focused on the teacher's role in improving classroom discourse. Therefore, in our analyses, we coded the three sources of self-report data for teachers' perceptions of the impact of the PD on their role in promoting discourse. All eleven teachers reported that the program helped to increase their knowledge about promoting classroom discourse, including learning about the importance of meaningful discussions and techniques for fostering discussions (see Table 2). Most of the teachers talked about the program as having an impact on specific aspects of their knowledge about classroom discourse. For example, ten teachers noted that they learned something about conducting groupwork, such as how important it is to provide time for groupwork or how to group their students more effectively. Nine teachers said that they learned how to foster better conversations in their mathematics classrooms, either within small groups or during whole class discussions. Eight teachers mentioned that they learned something about asking questions, including what types of questions are most effective and strategies for asking questions to elicit student thinking.

⁴ The data discussed in this section are from eleven teachers: seven participated in both years of the professional development program and 4 participated in one year.

Knowledge about student thinking.

Teachers' comments on all three self-report data sources suggest that participation in the PSC strongly impacted their knowledge related to student thinking. All eleven teachers commented that they gained a general awareness of students' mathematical thinking, including learning about how to listen to and promote their students' thinking (see Table 3). In addition, all of the teachers said that they learned about the importance of giving students more authority, for example by making their classrooms more student-centered or by decreasing their own role as the mathematical authority. Eight teachers said they became more knowledgeable about how to use or build on their students' mathematical thinking. Seven teachers reported learning about how to use mathematical tasks to promote student thinking, such as using rich problems that emphasize exploring processes rather than generating answers, or using fewer problems and exploring them for a longer period of time.

Impact of Watching Video

Because watching video is such a prominent feature of the PSC — and new to most teachers — we wanted to examine participants' perspectives on the value of this component of the professional development program. We coded and analyzed the teachers' self-report data to explore how they felt about watching video from their own lessons and from their colleagues' lessons.

Watching video of themselves. Ten teachers told us that being videotaped, although sometimes nerve-racking, was one of the most valuable aspects of the professional development. Many of these teachers pointed out that watching their own lessons on videotape enabled them to see what they were doing well and to identify areas for improvement. A number of teachers commented that by watching video they gained insight into what their students were thinking and what assistance they needed..

I was filled with anxiety when I thought someone was going to come in and videotape everything I was doing during classes with kids. But it turned out to be a powerful learning experience for me. (Pam, post interview)

Watching the video clips was great to see me in action and actually get to see what the students see. It allowed me to see the parts of my lessons that need improvement and what is good. (Laura, written reflection)

I think the most helpful [thing] was the videotaping, to watch myself on videotape, sometimes painfully so. Wanting to say, "Shut up, shut up. Why do you keep going on with that?" But it's so helpful to see how you come across to kids and how they are or are not responding ... and to think about what I might have changed in that lesson ... or how I could have connected with kids better. (Celia, final interview)

Watching video of other teachers. Eight teachers mentioned that they learned something by watching videos of their colleagues, such as new pedagogical strategies or how other students solve mathematical problems. Several teachers mentioned that it was informative as well as reassuring to watch their colleagues struggle with familiar issues.

We never get to see our colleagues doing what we're doing. We just assume they're doing the same things that we are, and that's not necessarily so. It's a great window into how other kids look and it's comforting when you see things that are the same. (Penny, final interview)

When I watched other teachers' videos, it wasn't critiquing, it was seeing what they do in their classroom and realizing [that] a lot of what's going on in their classroom is what's happening in mine. Or ... this person really does a great job at opening a lesson. Maybe I could try something they're doing. (Linda, final interview)

CONCLUSION

As this sample of findings from research on the STAAR professional development program illustrates, the Problem-Solving Cycle appears to be a promising model for enhancing teachers' content knowledge and pedagogical content knowledge. Our data suggest that as teachers engage in the PSC, they are prompted to think deeply about mathematics content and instruction as part of a collaborative and supportive learning community. In particular, teachers who participated in the STAAR program report a strong impact on specific areas of their professional knowledge that were targeted during the three iterations of the PSC: mathematics content (including the importance of working on tasks and generating multiple solution strategies), methods for improving classroom discourse (including how to conduct groupwork, foster conversations about mathematics and mathematical thinking, and ask effective questions), and ways of fostering and exploring student thinking (including giving students

authority, building on students' thinking, and using tasks that promote student thinking).

The PSC model provides a structure for the participating teachers to work together as professionals, to establish trust and develop communication skills that enable constructive yet respectful discussions about teaching and learning, and to share and expand their knowledge base. Drawing on a situative framework, the model emphasizes the use of classroom artifacts within a supportive professional community. Any professional development effort that foregrounds the analysis of video from teachers own classrooms is entering into relatively uncharted, and murky, territory. However, our experience suggests that when the necessary structure is in place, the impact on teachers can be extremely powerful and fundamentally positive.

We are particularly encouraged by the fact that teachers at four of the six schools represented in the STAAR project are spearheading new professional development efforts within their schools that contain some or all of the PSC elements. Teachers at several schools have decided to observe and videotape one another and then meet to discuss these videotapes. At one school, the mathematics instructors plan to all work on and then teach a selected problem, and get together to share their experiences. When asked about their reasons for initiating these professional development activities, the teachers explained that they felt empowered by their experiences in the STAAR program and wanted to share what they had learned with their colleagues. The

following remarks, from three of the teachers' final written reflections, are illustrative of these ideas:

I proposed this sort of "community" to my principal and next year we will meet once a week as grade-level math departments. The problem is that teachers have a mentality of "shut the door and let me teach." I hope my school's math people can get the same sense of community as we have here. (Peter, written reflection)

I have learned to become a leader in my professional community. I have been able to share my classroom with other teachers so they can take ideas about teaching and learning from me. (Nancy, written reflection)

I want to get my entire department involved with this process. As we put students in groups to work together, we as teachers need to do the same. We need to be doing math together. (Laura, written reflection)

Although our implementation of the PSC has been restricted to middle school teachers and focused on algebra content, the model is intentionally designed to be flexibly implemented and responsive to the needs of facilitators, teachers, and school district personnel. We anticipate that it can be adapted for use with teachers at elementary and high school levels and with different strands of the school mathematics curriculum. While our research and development work on the PSC model will continue, we encourage others in the mathematics education community to adapt, extend, and refine this approach and further explore its effectiveness.

References

- Ball, D. L., & Cohen, D.K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the Learning Profession* (pp. 3-31). San Francisco: Jossey-Bass.
- Ball, D.L., & Bass, H. (2003). Toward a practice-based theory of mathematical knowledge for teaching. Paper presented at the Proceedings of the 2002 Annual Meeting of the Canadian Mathematics Education Study Group, Edmonton, Alberta.
- Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on the teaching and learning of mathematics* (pp. 83-104). Westport, CT: Ablex.
- Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator*, 14(22), 43-46.
- Ball, D. L., Lubienski, S., & Mewborn, D. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of Research on Teaching* (4th ed.). New York: Macmillan.
- Ball, D. L., Thames, M. H., & Phelps, G. (2005, April). *Articulating domains of mathematical knowledge for teaching*. Paper presented at the annual meeting of the American Educational Research Association, Montreal, QC, Canada.
- Blanton, M. L., & Kaput, J. J. (2005). Characterizing a classroom practice that promotes algebraic reasoning. *Journal for Research in Mathematics Education*, 36(5), 412-446.
- Borko, H., Frykholm, J., Pittman, M., Eiteljorg, E., Nelson, M., Jacobs, J., Clark, K. K., & Schneider, C. (2005). Preparing teachers to foster algebraic thinking. *Zentralblatt für Didaktik der Mathematik: International Reviews on Mathematical Education*, 37(1), 43-52.
- Borko, H., Jacobs, J., Eiteljorg, E., & Pittman, M.E. (in press). Video as a tool for fostering productive discourse in mathematics professional development. *Teaching and Teacher Education*.
- Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher*, 32(1), 9-13.
- Design-Based Research Collective. (2003). Design-based research: An emerging paradigm for educational inquiry. *Educational Researcher*, 32(1), 5-8.
- Ferrini-Mundy, J., Floden, R., McCrory, R., Burrill, G., & Sandow, D. (2005). Knowledge for teaching school algebra: Challenges in developing an analytic framework. Paper presented at the American Education Research Association. Montreal, Quebec, Canada.
- Frykholm, J. A. (1998). Beyond supervision: Learning to teach mathematics in community. *Teaching and Teacher Education*, 14(3), 305-22.
- Greeno, J.G. (2003). Situative research relevant to standards for school mathematics. In J. Kilpatrick, W.G. Martin, & D. Schifter (Eds.), *A Research Companion to Principles and Standards for School Mathematics* (pp. 304-32). Reston, VA: National Council of Teachers of Mathematics.

- Greeno, J.G., Collins, A.M., & Resnick, L.B. (1996). Cognition and learning. In D. Berliner & R. Calfee (Eds.), *Handbook of Educational Psychology* (pp. 15-46). New York: Macmillan.
- Grossman, P.L., Wineburg, S., & Woolworth, S. (2001). Toward a theory of teacher community. *Teachers College Record*, 103(6), 942-1012.
- Hill, H. C., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's Mathematics Professional Development Institutes. *Journal of Research in Mathematics Education*, 35, 330-351.
- Kazemi, E., & Franke, M.L. (2004). Teacher learning in mathematics: Using student work to promote collective inquiry. *Journal of Mathematics Teacher Education*, 7, 203-35.
- Kieran, C. (1992). The learning and teaching of school algebra. In D. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 390-419). New York: Macmillan.
- Koellner, K., Jacobs, J., Borko, H., Schneider, C., Pittman, M., Eiteljorg, E., Bunning, K., & Frykholm, J. (in press). The Problem-Solving Cycle: A model to support the development of teachers' professional knowledge. *Mathematical Thinking and Learning*.
- Lacampagne, C.B., Blair, W.D., & Kaput, J.J. (Eds.). (1995). *The algebra initiative colloquium*. Washington, D.C.: U.S. Department of Education.
- Ladson-Billings, G. (1998). It doesn't add up: African American students' mathematics achievement. Paper presented at the conference on Challenges in the Mathematics Education of African American Children: Proceedings of the Benjamin Banneker Association Leadership Conference, Reston, VA.
- Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. New York: Cambridge University Press.
- LeFevre, D. M. (2004). Designing for teacher learning: Video-based curriculum design. In J. Brophy (Ed.), *Using video in teacher education: Advances in research on teaching* (Vol. 10, pp. 235-258). London, UK: Elsevier.
- Little, J.W., Gearhart, M., Curry, M., & Kafka, J. (2003). Looking at student work for teacher learning, teacher community, and school reform. *Phi Delta Kappan*, 85(3), 185-92.
- Little, J.W. (2002). Locating learning in teachers' communities of practice: Opening up problems of analysis in records of everyday work. *Teaching and Teacher Education*, 18(8), 917-46.
- Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, N.J.: Lawrence Erlbaum Associates.
- Nathan, M. J., & Koedinger, K. R. (2000). An investigation of teachers' beliefs of students' algebra development. *Cognition and Instruction*, 18(2), 209-237.
- National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- National Research Council. [NRC] (1998). *The nature and role of algebra in the K-14 curriculum: Proceedings of a national symposium*. Washington, DC: National Academy Press.

- Nikula, J., Goldsmith, L.T., Blasi, Z.V., & Seago, N. (2006). A framework for the strategic use of classroom artifacts in mathematics professional development. *NCSM Journal of Mathematics Education Leadership*, 9(1), 57-64.
- Putnam, R., & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning? *Educational Researcher*, 29(1), 4-15.
- Putnam, R. T., & Borko, H. (1997). Teacher learning: Implications of new views of cognition. In B. J. Biddle, T. L. Good, & I. F. Goodson (Eds.), *International handbook of teachers and teaching* (Vol. 2, pp. 1223-1296). Dordrecht, Netherlands: Kluwer.
- Rosebery, A., & Puttick, G. (1998). Teacher professional development as situated sense-making: A case study in science education. *Science Education* 82, 649-677.
- Rosebery, A., & Warren, B. (Eds.). (1998). *Boats, balloons and classroom video: Science teaching as inquiry*. Portsmouth, NH: Heinemann.
- Seago, N. (2004). Using video as an object of inquiry for mathematics teaching and learning. In J. Brophy (Ed.), *Advances in research on teaching, Volume 10: Using video in teacher education* (pp. 259-286). Orlando, FL: Elsevier.
- Secretary's Summit on Mathematics. (2003). *Teacher knowledge action plan: Mathematics* (Teacher Knowledge Working Group Report).
- Seidel, T., Prenzel, M., Rimmel, R., Schwindt, K., Kobarg, M., Meyer, L., Dalehefte, I.M., & Herweg, C. (2005). Do videos really matter: The experimental study LUV on the use of videos in teachers' professional development. Paper presented at the 11th Conference of the European Association for Research on Learning and Instruction, Nicosia, Cyprus.
- Sherin, M. G. (in press). The development of teachers' professional vision in video clubs. In R. Goldman, R. Pea, & S. Derry (Eds.), *Video research in the learning sciences*. Hillsdale, NJ: Erlbaum.
- Sherin, M. G. (2004). Video volunteers. *ENC Focus Review*, 11(3), 4-6.
- Sherin, M.G., & Han, S.Y. (2004). Teacher learning in the context of a video club. *Teaching and Teacher Education*, 20, 163-83.
- Sherin, M. G., & van Es, E. A. (2002). Learning to notice as a focus for professional development. *Classroom Leadership*, 5(9), 1, 6.
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4-14.
- Stein, M.K., Smith, M.S., & Silver, E.A. (1999). The development of professional developers: Learning to assist teachers in new settings in new ways. *Harvard Educational Review*, 69(3), 237-69.
- U.S. Department of Education and National Center for Educational Studies. (1998). *Pursuing excellence: A study of U.S. eighth-grade mathematics and science achievement in international context*. Washington, DC: U.S. Government Printing Office.
- Wilson, S.M., & Berne, J. (1999). Teacher learning and the acquisition of professional knowledge: An examination of research on contemporary professional development. In A. Iran-Nejad & P.D. Pearson (Eds.), *Review of Research in Education* (Vol. 24) (pp. 173-209). Washington, D.C.: American Educational Research Association.
- Wilson, S. M., Shulman, L. S., & Richert, A. E. (1987). 150 different ways of knowing: Representations of knowledge in teaching. In J. Calderhead (Ed.), *Exploring teachers' thinking* (pp. 104-124). London: Cassell.

TABLE 1. *Perceived impact of the professional development program on teachers' content knowledge*

| | Number of Teachers | Representative Quotes |
|--|--------------------|---|
| Learning mathematics content (generally) | 6 | <p>One of the things I was really weak at was trying to develop equations from patterns. I just could not do that for the life of me before [the STAAR program].... I actually forced myself to use those strategies... and it's really beginning to open my eyes. <i>(Nancy, final interview)</i></p> <p>I used to think in algorithm mode. Now I try to see or picture patterns. <i>(Deborah, written reflection)</i></p> |
| Learning from tasks | 11 | <p>[I learned] just how much insight you can get from working problems with other adults... When you do it on your own, you've got a much narrower view on it to start with. Whereas if you solve it with other adults before teaching, it broadens your view. And then the kids broaden it even more. <i>(Kristen, final interview)</i></p> <p>Before the STAAR workshops, I have to admit honestly that I did not try every single new rich problem, or non-rich problem, myself all the time. I'd look at the parameters of the problem, but not necessarily sit down and work them. You know what the STAAR project taught me? Feel their pain. Look at the problem, and work it either yourself or with someone else. <i>(Pam, post interview)</i></p> |
| Learning from multiple approaches | 10 | <p>I have learned that there's more ways than I could have imagined to solve problems. Without this program I would not have realized all the ways to solve a problem and the importance of looking at student work and thinking. <i>(Linda, written reflection)</i></p> <p>This group has enhanced my algebraic knowledge by listening to others' ideas to the same problem. Learning that multiple solutions do exist and [that it's important] to study them purposefully with kids. <i>(Penny, written reflection)</i></p> |

TABLE 2. *Perceived impact of the professional development program on teachers' knowledge about their role in promoting discourse*

| | Number of Teachers | Representative Quotes |
|---------------------------------------|--------------------|--|
| Teacher's role in discourse (general) | 11 | <p>I no longer think that math class is about me. It's about them and their learning. And it's about my facilitating... I can say, 'OK. Let's look at this and talk about it.' (Celia, final interview)</p> <p>I think learning to struggle is as important as anything else in math. [STAAR] helped me to know that because you put me through it! Now when kids say to me in class, 'Well I can't do it. Give me a hint,' I say, 'Maybe you better go talk to your group.' I step back and I step back for a good long time until we bring the large group back together again. (Pam, final interview)</p> |
| Conducting groupwork | 10 | <p>Working with other teachers on the math problems was really beneficial. And that led me to understand why it's so important for students to work in groups in the classroom. (Linda, final interview)</p> <p>I used to just kind of let the kids pair up and I didn't have much thinking behind it. Now I structure it and have a purpose between who's with whom. (Peter, post interview)</p> |
| Fostering conversations | 9 | <p>I realized the importance of talking about our thinking, and giving kids the opportunity to share their ideas. (Ken, written reflection)</p> <p>I now try to say to them, 'Please share that with the rest of your group.... Explain that to everyone.'.... I want them to pursue that and ask those questions of each other. (Celia, final interview)</p> |
| Asking questions | 8 | <p>I feel like I went from a lot of lecture ... a very broad kind of questioning [style] to asking deeper level — or higher level — thinking questions, provoking more of their thinking, rather than just "Is this right?" kinds of questions. (Ken, final interview)</p> <p>I try to get them to explain more about what they did. And so my questions ask for more than the answer. 'How did you get there?' 'Why did you do it that way?' 'Does anyone else want to tell us how they did it?' Those kinds of things. I think it's all about digging in deeper. (Kimber, post interview)</p> |

TABLE 3. *Perceived impact of the professional development program on teachers' knowledge about student thinking*

| | Number of Teachers | Representative Quotes |
|---|--------------------|--|
| Awareness of student thinking (general) | 11 | <p>I thought so much about looking at kids' work and trying to figure out what they were thinking with the STAAR program. <i>(Kimber, post interview)</i></p> <p>Watching other teachers allow their students to think and discover and digest a problem makes me realize that is a change I must make. <i>(Kristen, written reflection)</i></p> |
| Giving students authority | 11 | <p>I learned how to not just tell students how to do things, but have them participate... and share their information. Instead of me just standing up there and blabbing the hour and a half. <i>(Linda, final interview)</i></p> <p>I don't want to keep pushing them to get my answer and to follow my path. I want them to find their own path. <i>(Kristen, final interview)</i></p> |
| Building on student thinking | 8 | <p>Previously, I wouldn't allow my students to continue their thought process. I would stop them and have them go my way. Versus now, when I'm not quite understanding what they're doing, I will continue to ask questions. <i>(Laura, post interview)</i></p> <p>Before STAAR I would have said immediately, 'Oh yeah, that's right. Move on.' Now we explore it deeper than that. And they know, too, that I'm going to say to them, 'How can you prove it?' <i>(Celia, post interview)</i></p> |
| Using tasks to promote student thinking | 7 | <p>Student thinking takes time. [This knowledge has] helped me determine what part of the curriculum is more important, so I can do away with 'less important' problems. <i>(Nancy, written reflection)</i></p> <p>STAAR showed me that there are problems out there that have so many things to offer kids. So many things that they can talk about and experience and try to strategize. <i>(Pam, final interview)</i></p> |