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Changing Teacher Beliefs

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Purpose Statement

The purpose of the National Journal of Mathematics Education Leadership is to advance the mission and vision of the National Council of Supervisors of Mathematics by:

- Strengthening mathematics education leadership through the dissemination of knowledge related to research, issues, trends, programs, policy, and practice in mathematics education
- Fostering inquiry into key challenges of mathematics education leadership
- Raising awareness about key challenges of mathematics education leadership, in order to influence research, programs, policy, and practice
- Engaging the attention and support of other education stakeholders, and business and government, in order to broaden as well as strengthen mathematics education leadership

A Framework for Analyzing Differences Across Mathematics Curricula

Mary Ann Huntley
University of Delaware

Textbooks have a tremendous influence on *what* and *how* mathematics is taught. In a national study teachers reported that textbooks designated for a class influence their selection of content in nearly 5 out of 10 mathematics lessons, and that textbooks influence teachers' instructional strategies in roughly 7 out of 10 lessons (Weiss, Pasley, Smith, Banilower, & Heck, 2003). Given the importance of textbooks in mathematics classrooms, it stands to reason that choosing a mathematics textbook is an important task. But at the same time, this task can be both overwhelming and time consuming (Reys & Reys, 2006). Marketing materials provided by textbook publishers can be more confusing than helpful. Indeed, it seems that all textbook publishers claim their products are research-based and will produce student success.

Teacher leaders, and others who have responsibility for choosing textbooks, often resort to making decisions by ticking off topics in tables of contents that align with their state/district standards. Another popular method for selecting textbooks is the "flip test," which involves quick browsing of several textbooks for ease of readability, appealing design and color illustrations, and ready-made teaching aids and test questions, seizing on these attributes as proxies for quality.

Another impediment to selecting textbooks is that despite the plethora of rhetoric about mathematics textbooks generated by the Math Wars, mathematics programs tend to get lumped into one of two categories — reform or traditional. These categories are too broad and do not take into account the variation that exists across textbooks. There are major differences across traditional textbooks, just as there are major differences across reform textbooks. It is important to understand these differences, as they may differentially impact instructional practice and ultimately student learning.

In this article I offer a framework for looking beyond lists of topics and surface features of mathematics textbooks. For those with responsibility for choosing textbooks, I offer the framework as a tool for better understanding and appreciating the sometimes nuanced differences across mathematics curricular programs.

Method Used to Develop the Framework

In this section I outline the process by which I developed the framework. I explain my choice of textbooks that I use to illustrate the framework, my sources of data, and the specific aspects of textbooks that constitute the backbone of the framework.

Choice of Textbooks

I present my framework in the context of two comprehensive middle-grades mathematics curricula funded by the National Science Foundation (NSF): *Connected Mathematics* (Lappan, Fey, Friel, Fitzgerald, & Phillips, 2002) and *Math Thematics* (Billstein & Williamson, 1999-2005). I could have chosen any two curricular programs — two traditional programs, two reform programs, or one of each — but I decided to compare two NSF-funded curricula for several reasons. First, despite the growing literature base about how reform mathematics materials differ from more traditional materials, how to implement reform materials, and the effects of reform

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materials on students and teachers (e.g., Goldsmith, Mark, & Kantrov, 1998; Lloyd, 2002; Senk & Thompson, 2003; Smith & Star, 2007; Trafton, Reys, & Wasman, 2001), there has been surprisingly little discourse about how one set of NSF-funded materials differs from another. NSF funded the development of five comprehensive middle-grades programs, so surely there are important differences among them! The second reason I decided to anchor this discussion around these two particular curricula, *Connected Mathematics* and *Math Thematics*, is because these materials are likely to continue being used in schools in the future, as they are the two NSF-funded comprehensive middle-grades mathematics programs with greatest market penetration. And third, while being developed with common goals, these two curricular programs represent very different approaches to middle-grades mathematics.

Sources of Data

I reviewed written materials related to *Connected Mathematics* and *Math Thematics*, including student and teacher books. I read ancillary materials related to each program, including documents the authors provide for professional development providers (Denny & Williamson, 1999; Lappan, Fey, Fitzgerald, Friel, & Phllips, 2002). I talked extensively with the lead author of *Connected Mathematics* (Glenda Lappan) and the lead author of *Math Thematics* (Rick Billstein), as well as with several middle-grades teachers who have worked closely with the authors during field testing of the materials.

Textbook Features Examined

Authors of curricula are faced with many choices that affect how students experience a given set of instructional materials. For instance, authors must wrestle with issues such as the role of problem context, the amount of basic skills practice, and emphasis of cooperative groups versus whole-group discussion. Curriculum writers make decisions about these and other issues, which are often referred to as "curriculum variables." These decisions reflect the authors' explicit and implicit beliefs about mathematics, as well as their beliefs about the teaching and learning of mathematics.

The authors of *Connected Mathematics* and *Math Thematics* made decisions about their respective curricula within specified parameters that were outlined in the NSF program solicitation from which they received initial funding (NSF, 1989). According to this solicitation the curricula had to be aligned with the National Council of Teachers of Mathematics [NCTM] *Standards* (1989), and in comparison with most existing curricula they had to place greater emphasis on mathematical investigation, mathematics presented in real-world contexts, connections among content areas of mathematics and connections and between mathematics and other disciplines, and integration of technology with mathematics. These parameters set the general bounds for curriculum writing, but left considerable room for interpretation.

I developed the framework by examining differences across *Connected Mathematics* and *Math Thematics* in terms of two sets of curriculum variables. The first set relates to content, including structural organization, depth/breadth of content, presentation of content, worked-out examples, and definitions/rules. The second set of variables relates to instruction, including instructional model, use of class time, teacher's role, students' role, use of small group work, use of tools, assessment, and homework.

Framework

The framework that I developed to compare two mathematics curricula consists of three pieces. The first piece contains descriptive information about the curricula being compared. For each curriculum this includes the title, target grade range, authors, publisher and date of publication, list of ancillary materials provided by the publisher, and context (e.g., the funding source for the materials and extent to which the materials are aligned with the NCTM *Standards*). The second and third pieces of the framework contain comparative information regarding content variables and instructional variables, respectively. To illustrate use of the framework I discuss my analyses of *Connected Mathematics* and *Math Thematics*. In Figure 1a I provide descriptive information about each curriculum. My analyses around the two sets of curriculum variables (content and instruction) are shown in Figures 1b and 1c. In each figure the curriculum variables are in the center column, together with a description of the aspects of the variable that are common across the two curricula. The corresponding left- and right-hand columns indicate differences (if any) between *Connected Mathematics* and *Math Thematics*, respectively.

Content Variables (See Figure 1b)

In both *Connected Mathematics* and *Math Thematics*, at each grade level the mathematical content is partitioned into eight pieces. In *Connected Mathematics* these pieces are called "units" and in *Math Thematics* these pieces are called "modules." The "look and feel" of *Connected Mathematics* and *Math Thematics* books are considerably

Figure 1a. Comparison of *Connected Mathematics* and *Math Thematics* — curriculum information

CURRICULUM INFORMATION		
Connected Mathematics	**Title**	*Math Thematics*
6-8	**Grades**	6-8
Glenda Lappan, Jim Fey, Susan Friel, William Fitzgerald, & Betty Phillips	**Authors**	Rick Billstein & Jim Williamson
Prentice Hall (2002)	**Publisher (Year)**	McDougal Littell (1999-2005)
• Assessment Resources • Blackline Masters & Additional Practice • Transparencies • Lesson Planner • Computer Test Bank for Assessment & Practice (CD-ROM) • Spanish Resources • Implementation Guide • Teacher's Resource Kit (includes manipulatives, dot paper, & other resources) • Student Materials Kit	**Ancillary Materials**	• Professional Development Handbook • Teacher's Resource Books (module-by-module teaching strategies, classroom management tips, & blackline masters) • Spanish Resources • Notetaking Masters • Workbook (selected pages from Teacher's Edition) • Workbook (for additional skills development) • Transparencies • Tutor Place™ (laminated cards to help students with skills) • Multi-Language Visual Glossary • Test and Practice Generator (CD-ROM) • Personal Student Tutor (interactive, student tutorial software pack) • Student Manipulative Kit • Overhead Manipulatives Kit
Developed, with funding from the National Science Foundation, to align with the NCTM *Standards* (1989)	**Context**	Developed, with funding from the National Science Foundation, to align with the NCTM *Standards* (1989)

different. The *Connected Mathematics* materials include eight soft-cover books (units) for each grade 6-8, with each unit being organized around a big idea — a cluster of related concepts, skills, procedures, and ways of thinking. There is a focus on one content strand within each *Connected Mathematics* unit, with students studying one mathematical topic deeply before moving to another. In contrast, the *Math Thematics* materials contain one hardbound book for each grade 6-8. Each book contains eight modules, and each module has a theme that connects the mathematical content to the physical or social world. Each *Math Thematics* module includes a focus on multiple content strands, with content being presented in a spiral fashion, where students continually review previously-learned material.

With more traditional textbooks students are generally given rules and worked-out examples of how to apply the rules, and then they practice those rules. Traditional books contain collections of facts and skills to be memorized or mastered by students. By contrast, *Connected Mathematics* and *Math Thematics* are both problem-based curricula — mathematical content is presented as a sequence of problems or tasks. In both *Connected Mathematics* and *Math Thematics* the majority of the problems are set in real-world contexts, and the materials first present mathematical content in concrete examples before providing abstraction and formalization of the mathematical content. Each curriculum places emphasis on developing meaning of mathematical ideas before practice and skill using those ideas. A striking difference between the curricula is that the *Connected Mathematics* student books contain only a few worked-out examples that demonstrate solution methods, and contain only a few formal definitions/rules outside of the glossary.[1] This treatment is in sharp contrast with the *Math Thematics* materials, in which each module contains a reference section that includes a summary of key concepts and worked-out examples.[2]

Instructional Variables (See Figure 1c)

The sequence of activities in traditional mathematics classrooms has been characterized by Fey (1979) as follows:

Figure 1b. Comparison of *Connected Mathematics* and *Math Thematics* — content variables

	CONTENT VARIABLES	
Characteristics Unique to *Connected Mathematics*	**Common Characteristics**	**Characteristics Unique to *Math Thematics***
Eight units per grade, each in a soft-cover book	**Structural Organization** At each grade level the mathematical content is partitioned into eight pieces	Eight modules per grade in one hardcover book
Focus on one content strand within each unit; one mathematical topic is studied deeply before moving to another	**Depth/Breadth of Content** Emphasis on depth versus breadth of content coverage	Focus on multiple content strands within each module; content is presented in spiral fashion with continual review of previously-learned material
	Presentation of Content Mathematical content is presented as a sequence of problems/tasks motivated by real-world situations; Concrete examples are given before abstract formulation of ideas (students first learn "why" and then "how," i.e., understanding/meaning of concepts comes before practicing associated skills)	
Books contain few worked-out examples that demonstrate solution methods	**Worked-Out Examples**	Books contain numerous worked-out examples that demonstrate solution methods
Books contain few formal definitions/rules outside of the glossary	**Definitions/Rules**	Books contain many formal definitions/rules

Answers are given for the previous day's assignment, with the more difficult problems being worked by the teacher or students at the board. After a brief teacher-led presentation of new content and a few example problems being solved as a whole class, the remainder of the class time is devoted to students working on the homework while the teacher moves about the room answering questions. This sequence of activities, often referred to as the "transmission model of instruction," is based on the premise that students learn best by receiving information and practicing specific skills.

In contrast, reform mathematics curricula rest on the premise that students actively make sense of mathematical content. During class students are expected to investigate, discover, and make conjectures about mathematical ideas, reflecting the dynamic nature of what it means to "do mathematics." The teacher's role is that of a guide, or a facilitator, rather than a transmitter of knowledge. Students using reform mathematics materials are expected to engage in mathematical argumentation and produce mathematical evidence by talking or writing in ways that expose their reasoning to one another and to their teacher.

These characteristics of reform classrooms are consistent with the vision of mathematics teaching/learning as embodied in the *Connected Mathematics* and *Math Thematics* materials. Additionally, both sets of materials promote the "motivate[3] -explore-summary model of instruction." This model is characterized by the teacher first providing a "hook" to grab students' attention and relate the prior experiences of the students to the objectives of the lesson. In the explore phase students solve the problems presented in the curriculum materials, often working with other students in small groups. The summary phase provides closure by helping students bring mathematical ideas together in their own minds and make sense of what has just been explored. While the *Connected Mathematics* authors

[1] *Authors of Connected Mathematics encourage teachers to have students develop their own lists of definitions and examples because of their belief that students need to have descriptions of mathematical words that carry meaning at their level of verbal sophistication, which they can add to and refine as they gain new insight and encounter new examples.*

[2] *As with Connected Mathematics, each Math Thematics student book contains a glossary.*

Figure 1c. Comparison of *Connected Mathematics* and *Math Thematics* — instructional variables

INSTRUCTIONAL VARIABLES		
Characteristics Unique to *Connected Mathematics*	**Common Characteristics**	**Characteristics Unique to *Math Thematics***
The three phases occur each day; rare use of direct instruction	**Instructional Model** Three phrase instructional model: Motivate-Explore-Summary	The three phases occur over the course of several days; some use of direct instruction
Skill-based computational practice is reserved for homework	**Use of Class Time** Students work on problems from the curriculum	Some skill-based computational practice during classroom instructional time
Inquiry approach, with teachers providing only a little scaffolding	**Teacher's Role** Teachers are facilitators/guides	"Guided discovery" approach with teachers providing considerable scaffolding
	Students' Role Students actively make sense of the mathematical content by investigating, discovering, and making conjectures about mathematical ideas	
Students work in small groups 40-50% of the time	**Use of Small Group Work** Extensive use of small groups during class time	Students work in small groups 30-40% of the time
	Use of Tools Frequent use of manipulatives/technology	
	Assessment Multiple forms, both formal and informal, including student self and peer assessment	
	Homework Homework is assigned every night so students can practice content learned in class	

recommend that teachers incorporate these three phases into daily instruction, the *Math Thematics* authors recommend that teachers incorporate these three phases over the course of several days, with motivation provided one day, followed by several explorations over the course of the next several days, and then a summary.

The authors articulate other differences for how instruction with their respective curricula should play out. *Connected Mathematics* authors recommend only rare use of direct instruction, whereas *Math Thematics* authors recommend that teachers use some direct instruction of concepts and skills. *Connected Mathematics* authors believe that computational practice should be reserved for homework, whereas *Math Thematics* authors believe that some skill-based practice should occur during classroom instructional time. *Connected Mathematics* authors intend instruction to be less teacher-directed than *Math Thematics* authors, with *Math Thematics* authors using the phrase "guided discovery" to characterize instruction. *Connected Mathematics* authors recommend students work in small groups 40-50% of instructional time, and *Math Thematics* authors recommend small group work 30-40% of instructional time.

Consistent with the view of reform mathematics instruction outlined by NCTM (1989, 2000), *Connected Mathematics* and *Math Thematics* authors encourage teachers and students to make use of manipulatives and technology (as appropriate), and to use multiple forms of assessment (formal and informal, including student self and peer assessment). My talking with the lead authors of *Connected Mathematics* and *Math Thematics* revealed that they intend students to have homework every night in order to practice the content that was learned in class. Thus, for these curricular variables — tools, assessment, and homework — I found much in common, with little difference in philosophy across *Connected Mathematics* and *Math Thematics*.

[3] *In the Connected Mathematics materials, the motivate phrase is referred to as the "launch."*

Discussion

There is considerable public discourse and debate about different mathematics curricular approaches. What Reys stated several years ago bears repeating — it is time to move beyond the rhetoric and continuing controversy about various mathematics curricula and to "work together to improve children's mathematics education for the future" (Reys, 2001, p. 255). I believe one step in this direction is to discontinue the practice of lumping curricula into categories such as reform versus traditional, which disregards important differences between them. A second step (which is beyond the scope of this report) is to focus our energies on understanding how these differences differentially affect instructional practice and student learning. For example, what is the impact on students' learning when they are afforded concentrated time on one content strand before moving on to another (*Connected Mathematics*) versus a spiral approach with continual review of previously-learned material (*Math Thematics*)? The framework that I have developed and then have used to illustrate similarities and differences between *Connected Mathematics* and *Math Thematics* can be used by teacher leaders, and others who are responsible for choosing textbooks, to discern differences between any two curricula, reform or traditional. Figures 2a, 2b, and 2c contain a "stripped-down" version of the framework without reference to any specific curricula. Below I offer two specific uses of this framework.

- Textbook decision makers can complete the chart for textbooks being considered for adoption. Completing the chart, especially if done in a group setting, can result in productive discourse around curricular issues – discourse that moves beyond surface features of textbooks.

- Instantiations of the curriculum variables for the textbooks being considered can be examined to determine compatibility of the textbooks with teachers' beliefs about mathematics, and beliefs about teaching and learning mathematics. For instance, if teachers believe that students learn best by engaging in open-ended problems with minimal teacher guidance, then the following curriculum variables should be examined closely when considering a new textbook: presentation of content, worked-out examples, use of class time, teacher's role, and students' role.

Figure 2a. Framework to compare two mathematics curricula — curriculum information

CURRICULUM INFORMATION
Title *What is the title of the curriculum?*
Grades *What is the target grade range of the curriculum?*
Authors *Who are the authors of the curriculum?*
Publisher (Year) *What is the name of the publisher of the curriculum and in what year was it published?*
Ancillary Materials *What ancillary materials are provided by the publisher?*
Context *What was the funding source for the materials? To what extent do the materials align with the NCTM Standards?*

Figure 2b. Framework to compare two mathematics curricula — content variables

CONTENT VARIABLES		
Characteristics Unique to *Curriculum 1*	**Common Characteristics**	**Characteristics Unique to *Curriculum 2***
	Structural Organization *What are the physical features of the curriculum (e.g., number of units/modules per grade, softcover/hardcover)* **Depth/Breadth of Content** *Is depth or breadth of mathematical content emphasized and how does this play out (e.g., "layer-cake"/spiral/integrated approach)?* **Presentation of Content** *How is content presented (e.g., to what extent do students practice problems similar to worked-out examples vs. engage in a sequence of exploratory tasks; to what extent are problems set in real-world contexts)?* **Worked-Out Examples** *What is the extent of worked-out examples?* **Definitions/Rules** *What is the extent of definitions/rules? Where are they located (e.g., embedded in the text, glossary)?*	

Figure 2c. Framework to compare two mathematics curricula — instructional variables

INSTRUCTIONAL VARIABLES		
Characteristics Unique to *Curriculum 1*	**Common Characteristics**	**Characteristics Unique to *Curriculum 2***
	Instructional Model *What, if any, instructional model is explicitly articulated by the curriculum authors? What is the role of direct instruction?* **Use of Class Time** *What is a typical lesson like (e.g., to what extent do students explore content, watch the teacher demonstrate procedures, work on computational practice during class time)?* **Teacher's Role** *What is the role of the teacher during classroom instruction (e.g., what extent of scaffolding does the teacher provide)?* **Students' Role** *What is the role of students during classroom instruction?* **Use of Small Group Work** *To what extent do students work in groups?* **Use of Tools** *To what extent are students expected to use manipulatives and technology?* **Assessment** *What are major features of assessment (e.g., forms of assessment, formal/informal, self/peer)?* **Homework** *What is the frequency and role of homework (e.g., to practice newly-learned material, to review previously-learned material)?*	

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