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Changing Teacher Beliefs

PLCs in Guatemala

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Purpose Statement

The purpose of the National Journal of Mathematics Education Leadership is to advance the mission and vision of the National Council of Supervisors of Mathematics by:

• Strengthening mathematics education leadership through the dissemination of knowledge related to research, issues, trends, programs, policy, and practice in mathematics education

• Fostering inquiry into key challenges of mathematics education leadership

• Raising awareness about key challenges of mathematics education leadership, in order to influence research, programs, policy, and practice

• Engaging the attention and support of other education stakeholders, and business and government, in order to broaden as well as strengthen mathematics education leadership

Teacher Knowledge and Student Achievement: Revealing Patterns

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Universities researchers and teacher facilitators implemented a state-funded professional development project during the 2005-06 academic year to help county middle school teachers improve student achievement in mathematics. In this paper, we discuss lessons and results from this innovative model, whose iterative cycle includes teacher content knowledge, item analysis from a high-stakes test, pedagogical content knowledge, big mathematical ideas behind test items, and designing-implementing-reflecting on lessons to address critical problem areas in student learning and understanding.

Theoretical Framework

Too often professional development focuses narrowly on changing teaching behaviors (e.g., on helping teachers learn how to use a new technology or new teaching strategy) with no attention to the impact of such tools on what students know and can do. Although teachers need to learn to use new techniques and tools, most importantly they need to step back from their own learning and consider the implications for the students' learning and achievement.

An intervention was driven by a professional development model designed by the project team. The model focused on a "teaching on evidence" approach, using item analysis as a main component. We used an iterative, cyclical model (see Figure 1) which included assessment of teacher content knowledge, item analysis from a high-stakes test, identifying (ID) and assessing low-performing items (LPI), teacher-driven discourse with pedagogical content knowledge (PCK) consideration, big mathematical ideas behind test items, vertical alignment of mathematical concepts, and evidence-based lesson study cycle (designing, implementing, and reflecting on lessons) to address critical problem areas in student learning and understanding. The model also included peer-observation and analysis of designed lessons. Teacher participants kept journal logs and regularly submitted their reflections on assigned items during the yearlong professional development.

We used data from our state's mandated standardized test (TAKS: Texas Assessment of Knowledge and Skills; see Texas Education Agency website: www.tea.state.tx.us) as a source to create cognitive pedagogical conflict for teachers. The conflict occurs between teachers' assumption of what happened in the classroom (i.e., "I taught this topic/objective") and actual student learning (i.e., "Did

student learn what I taught?"). Usually, teachers have no ready explanations of why students would do poorly on a TAKS item based on a topic/objective that was taught in class. Our main professional development strategy was to identify error patterns in students' TAKS performance and use *item analysis* to engage teachers in conceptual discourse on how to overcome the disequilibrium and change their teaching practices in a way that would help student achievement. An example of a similar approach with elementary school teachers is discussed by Fisher and Kopenski (2007). During each professional development session, the workshop was launched and driven by the particular TAKS items and corresponding TEKS (Texas Essential Knowledge and Skills) competencies correlated with a content-specific objective — with most of the focus on the TAKS items on which student performance was the poorest.

The idea clearly is not to "teach to the test" and focus only on these particular items, but to unpack big mathematical conceptual ideas (e.g., reversibility, flexibility, and generalization) and effective teaching strategies that might help teachers to improve student achievement on a much larger collection of items, and to situate this understanding of big ideas in a greater set of curriculum objectives in the K-12 continuum.

Figure 1. Professional development model used in the study
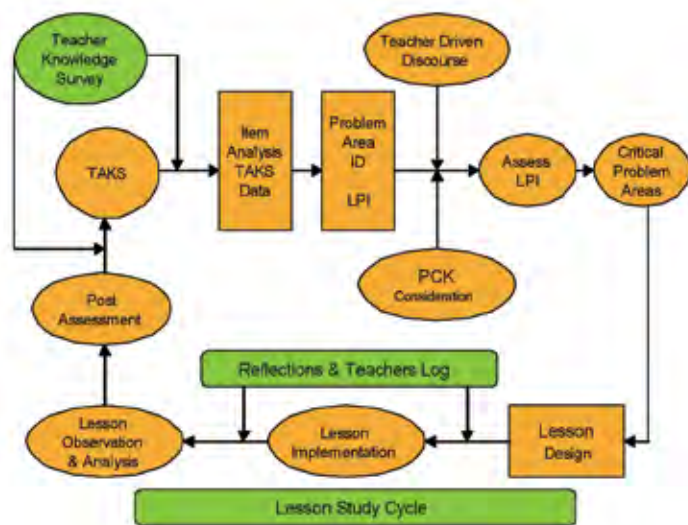
An example of one of those big-picture teaching strategies was multiple representations (Lesser and Tchoshanov 2006). The professional development intervention included 13 three-hour workshop sessions (and a follow-up session during the summer), which was broken down into two sessions for each of the six objectives of the

high-stakes middle school TAKS mathematics test: numbers, operations, and quantitative reasoning; patterns, relationships, and algebraic reasoning; geometry and spatial reasoning; measurement; probability and statistics; mathematical processes and tools. Sessions were driven mainly by analyzing low-performing TAKS items. Teachers worked on items individually, shared their work with group members, and then presented ideas from their group to the entire room, following the five-step *item analysis approach* designed by the project team:

(1) Solve the problem.

(2) What is/are the big mathematical idea(s) or core concept(s) of the problem? How does this connect to concepts from elementary school mathematics? How does this connect to concepts from high school mathematics?

(3) What TAKS objective and what TEKS knowledge and skill does this problem address?

(4) What do you think caused student low performance on this problem?

(5) What would you change/ modify in your teaching so students will be more successful solving this problem? What questions would you ask students during your teaching to prevent low performance with this problem? Make a list of two or three questions.

During professional development sessions, the project team used a *teaching on evidence* approach, which focused on the following main goals:

• Develop a culture of evidence by using TAKS data (item analysis approach) to improve teaching practices

• Develop an understanding of big mathematical ideas and vertical alignment of concepts across grade levels

• Develop mathematical habits of mind such as generalization (e.g., from a sequence of numbers to an algebraic rule) or reversibility and effective general strategies such as multiple representations and high-level questioning skills

• Investigate a topic not as a single item but as part of a set of connected ideas.

All six TAKS objectives were addressed before the TAKS test in an order informed by teachers' scope and sequence to maximize the opportunity for teachers to have

a professional development session on a particular TAKS objective before they taught it.

One part of the model that was not fully realized was lesson study. While each teacher received feedback on her teaching, participated in regular reflection writing, etc., a true full-scale lesson study cycle was not possible to conduct within the time and resource constraints of the grant.

Methodology

In order to assess the impact of the intervention on teacher knowledge and student achievement, the project team used a *mixed-methods design* with the following measures.

1. **Texas Assessment of Knowledge and Skills (TAKS).** TAKS scores were collected to assess teacher impact on student achievement, which will be discussed in the Results section.

2. **Teacher Observation Protocol.** An existing standardized protocol used in a local district was used for documenting observations of teacher lessons. The form allows the observer to provide narrative comments as well as to choose among three rating levels for each indicator within each category. The categories are class structure, methods, teacher-student interaction, and content. (While each teacher had the benefit of being observed and receiving peer feedback from this form, the data was not rich and detailed enough to yield meaningful results with respect to the workshop's focus areas and this protocol is being revised accordingly for future use.)

3. **Teacher Knowledge Survey.** At the beginning of the series of workshops, this survey was used to assess teacher content knowledge and consisted of 33 multiple choice problems addressing corresponding TAKS objectives and using three different levels of cognitive demand. Further discussion of the construction of the survey appears later.

4. **Teacher Reflections.** Between professional development sessions, teacher participants submitted written reflections using the framework of applying the aforementioned five-step item analysis organizer (which includes addressing what you would change in your teaching) to new TAKS problems the authors supplied.

The research sample consisted of 22 in-service teachers from high-need (based on percentage of students at the school not passing the mathematics portion of the high stakes TAKS test)

and low-SES schools (based on the percentage of students participating in free or reduced-price lunch programs). These schools' student bodies are about 80-90% Latino/Hispanic. One of the main variables and measures of the project was student achievement. Below, we provide statewide students' TAKS performance (Figure 2). The low pattern of achievement in the middle grades was a strong reason we targeted teachers of these grade levels.

Figure 2. Statewide Students' TAKS Performance

All Students, Percent Met Standard

	2003	2004	2005	2006
Elementary grades				
3rd Grade	74%	83%	83%	82%
4th Grade	70%	78%	81%	83%
5th Grade	65%	73%	79%	81%
Middle Grades				
6th Grade	60%	67%	72%	**79%**
7th Grade	51%	60%	64%	**70%**
8th Grade	51%	57%	61%	**67%**
High School Grades				
9th Grade	44%	50%	56%	**56%**
10th Grade	48%	52%	58%	60%
11th Grade	44%	67%	72%	77%

Results

Teacher Knowledge and Student Performance
A body of existing research claims that U.S. teachers lack essential knowledge for teaching mathematics and that teachers' intellectual resources affect student achievement (Coleman et al., 1966; Ball, 1991; Stigler & Hiebert, 1999; Ma, 1999; Hill et al., 2005). Our study supports this claim and shows that teacher knowledge and student achievement parallel each other (Figure 3). Teacher knowledge (as measured by the Teacher Knowledge Survey given at the beginning of the intervention) is denoted by the largest icon symbol in Figure 3.

Through the analysis of the TAKS data, the project team discovered how things are very similar from campus to campus within the feeder pattern. We believe that it makes sense since campuses in the same feeder pattern are supposed to work together and align their instruction. In Figure 3, we compare one of these campuses to the district as well as to the state. The same pattern occurs everywhere, which would seem to say that the teaching everywhere across the state is about the same. It seems that teachers across the state are about equally effective in conveying the same material to students, no matter what

curriculum is used. Or, regardless of scheduling differences (e.g., scope and sequence of curriculum topics), teachers across the state teach relatively in the same manner.

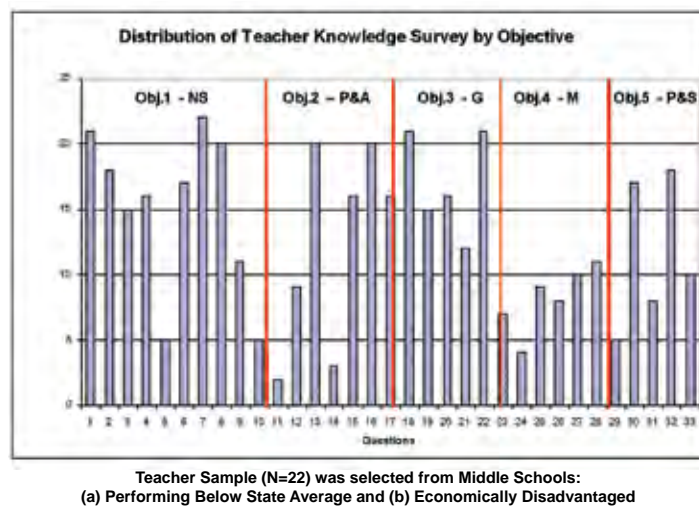Figure 3. Student achievement (measured by TAKS) and teacher knowledge (measured by Teacher Knowledge Survey) for each objective
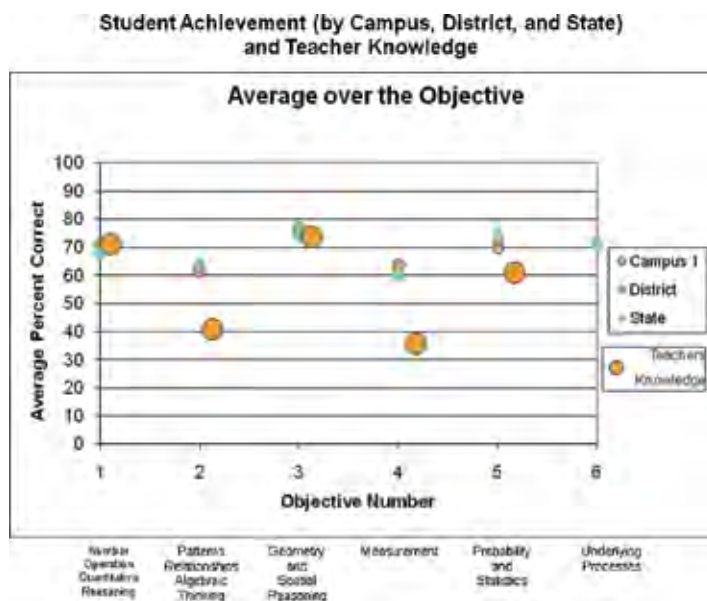
Another important observation is that student performance pattern by objectives mirrors teacher performance on the Teacher Knowledge Survey. In a sense, it means that if teachers have difficulty in mastering a particular objective then it impacts student achievement in the same objective. Figure 3 shows, for example, that low teacher knowledge on objectives #2 (patterns, relationships, and algebraic thinking) and #4 (measurement) is correlated with low student achievement on the same objectives compare to other objectives (e.g., objectives #1, 3 and 5). The project team did not collect data for the process objective #6 (underlying processes), focusing mainly on content specific objectives (#1 through #5). It does not mean that objective #6 is not important for the study, though, and we will talk about one of the process standards — problem solving — in more detail later.

Distribution of teacher performance by items on the Teacher Knowledge Survey is presented on Figure 4. The data shows the same pattern: teachers have a lack of knowledge on objective #2 (P&A=Patterns & Algebra) and objective #4 (M=Measurement) compare to objectives #1 (NS=Number Sense), #3 (G=Geometry), and #5 (P&S=Probability & Statistics).

Figure 4. Teacher performance on the Teacher Knowledge Survey items by objective

Teacher Sample (N=22) was selected from Middle Schools: (a) Performing Below State Average and (b) Economically Disadvantaged

The lowest-performing item (out of 33 items in the Teacher Knowledge Survey), item #11 from the Patterns and Algebra objective, is shown below (Figure 5):

Figure 5. Graph used in the lowest-performing item from the Teacher Knowledge Survey
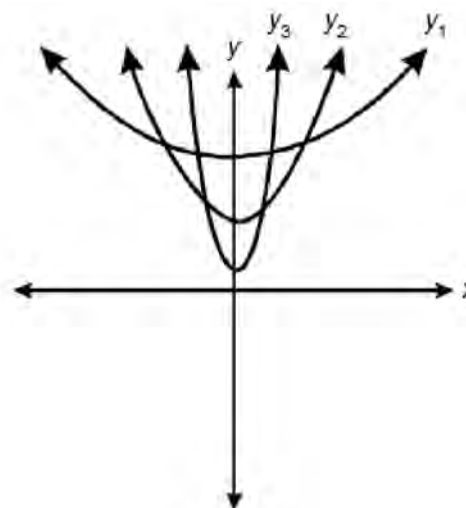
(11) In the figure [Figure 5] above, the function y_3 is translated 4 units left and 7 units down. Which of the following equations best describes the new function?

A. $y = ax^2 + 11x + 28$
B. $y = ax^2 + 4x + 7$
C. $y = ax^2 + 8ax + c$
D. $y = x^2 + 28x + 11$

Only two out of 22 teachers were able to solve the item correctly. In contrast, item #7 (shown below) from the Number Sense objective was successfully solved by all teacher participants.

(7) What is the rule for fraction division?

A. $\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{ac}{bd}$

B. $\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{ab}{cd}$

C. $\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{cd}{ab}$

D. $\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{ad}{bc}$

As you see, there is a difference not only in content objectives but also in cognitive demand level between items #11 and #7. Whereas item #7 is addressing correctly identifying a mathematical procedure (fraction division rule) recorded with algebraic notation, item #11 focuses on applying a non-routine mathematical procedure with understanding. The problem can be viewed as a Level 2 cognitive demand (which will be explained shortly) in which a student has to choose the more appropriate form of a quadratic function to use — namely, the vertex form. After substituting the -4 and -7 numbers in the standard form of the quadratic function $y = a(x - h)^2 + k$ and expanding the equation, it becomes consistent with the choice C. The construct of cognitive demand provided guidance for developing the teacher knowledge survey.

The Roles of Task and Cognitive Demand in the Teacher Knowledge Survey

One indicator of teachers' conceptual understanding of mathematics is an ability to engage students in meaningful discourse in the classroom through selecting instructional and assessment tasks that embody learning goals (Shepard et al., 2005). Why are tasks important? Students learn from the kind of work they do during class, and the tasks they are asked to complete determines the kind of work they do (Doyle, 1988). Mathematical tasks are critical to students' learning and understanding because "tasks convey messages about what mathematics is and what doing mathematics entails" (NCTM, 1991, p. 24). "The tasks make all the difference" (Hiebert et al., 1997, p. 17). Tasks provide the context in which students think about mathematics and different tasks place different *cognitive demands* on students' learning (Doyle, 1988; Henningsen & Stein, 1997; Porter, 2004).

Cognitive demands can be defined as the kind and level of thinking required of students in order to successfully engage with and solve the task (Stein et al., 2000, p. 11). Such thinking processes range from memorization to the use of procedures and algorithms (with or without attention to concepts, understanding, or meaning), to complex thinking and reasoning strategies that would be typical of "doing mathematics" (e.g., conjecturing, justifying, or interpreting) (Henningsen & Stein, 1997, p. 529).

Given the importance of tasks, the next issue is: "What do teachers need to know to select or make up appropriate individual tasks and coherent sequences of tasks? The simple answer is that they need to have a good grasp of the important mathematical ideas and they need to be familiar with their students' thinking" (Hiebert et al., 1997, p. 34). Similarly, Grossman, Schoenfeld, & Lee (2005) posed a critical question: "What do teachers need to know about the subject they teach?" (p. 201), and provided a fairly straightforward answer: "Teachers should possess deep knowledge of the subject they teach" (*ibid*, p. 201).

In constructing the teacher knowledge survey, we generated items for each of the following levels of cognitive demand. Note that our Level 1 includes Stein's first two levels (memorization; procedures without connections), our Level 2 is like Stein's third level (procedures with connections), and our Level 3 is close to Stein's fourth level (doing mathematics, which includes conjecturing, etc.). Further discussion addressing the reliability and validity of our instrument appears at the end of the article.

Level 1: Facts and Procedures
- Memorize Facts, Definitions, Formulas, Properties, and Rules;
- Perform Computations;
- Make Observations;
- Measure;
- Solve Routine Problems

Level 2: Concepts and Connections
- Justify and explain solutions to problems;
- Use and select multiple representations to model mathematical ideas;
- Transfer knowledge;
- Connect concepts to solve non-routine problems;
- Communicate "Big Ideas";
- Explain findings and results from analysis data

Level 3: Models and Generalizations
- Generalize;
- Make and test conjectures;
- Prove statements;
- Design mathematical models

To illustrate the difference between levels, we include an example using a fraction division problem under objective #1 "Number Sense."

Level 1.   What is a rule for fraction division?

Solve the following fraction division problem

$$1\frac{3}{4} \div \frac{1}{2}$$

Level 2.   Solve the same problem in more than one way, for example, draw a model or illustrate the problem with manipulatives.

Make up a story for the fraction division problem

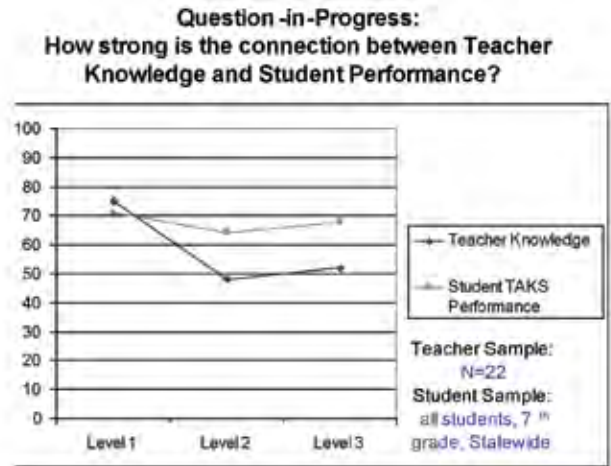Level 3.   Is the following $\frac{a}{b} \div \frac{c}{d} = \frac{ab}{cd}$ ever true?

Figure 6 consists of the same Teacher Knowledge Survey results (displayed in Figure 4) distributed by cognitive demand levels. Each bar represents a particular item.

Figure 6. Teacher Knowledge Survey results by cognitive demand levels

Considering the selection criteria for teacher participants in the project, we expected that the percentage of correct responses would be highest for Level 1. However, the finding that Level 3 scores were higher than Level 2 scores was unexpected.

Following the teacher knowledge survey, we performed the same activity with TAKS items, classifying them by cognitive demand level. We triangulated the process by having three mathematics educators independently conduct the classifications, with 90% agreement. We distributed 7th grade student statewide TAKS performance results by cognitive demand levels and compared it with teachers' data. The project team was surprised to observe a similar pattern between teacher and student performance (Figure 7). A Cohen's *d* effect-size calculation between the teachers' level 1 performance and the teachers' mean performance on levels 2 and 3 resulted in the large effect size of 2.3.

Figure 7. Teacher and student performance by cognitive demand levels

We wonder if the data in Figure 7 could add insight into the pattern we saw in Figure 2. In other words, the similarity on student TAKS achievement across the state might reflect similarities on cognitive demand level of tasks and assignments used in mathematics classrooms all over the state.

Does the Number of Steps in the Problem Matter?
It is well known that students do not perform well in solving word problems. Solving story or word problems is a challenging task at every level of schooling, including middle grades. Research shows that students' poor performance in word problem solving could be a result

Figure 8. Example of TAKS item sorted by cognitive demand and 'stepness' level

Here is an Example

28 For storage Mrs. Lin uses cylindrical containers like the one shown below.

If Mrs. Lin uses 2 of these containers, which is closest to the total volume of both containers?

F* 13 cubic feet (42%)
G 6 cubic feet (28%)
H 8 cubic feet (18%)
J 16 cubic feet (11%)

This item was graded by our team as a Level 1 item having two steps.
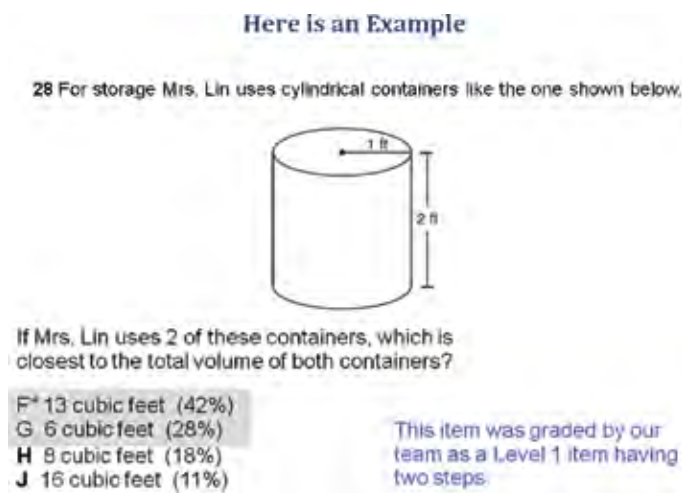
of their misunderstanding of the problem (Cummins et al., 1988). Among factors influencing misunderstandings are: difficulties in perception of mathematical language (Kane, 1967), insufficient subject matter knowledge (Mayer, 1992), problem posing (Butts, 1980), language deficiencies (Mestre, 1988), ineffective text processing (Nathan, Knitsch, & Young, 1992), and lack of effective reading strategies in problem solving (Shuard & Rothery, 1988). We examine a very specific component of this broad issue: does number of steps in solving problems (including word problems) impact student performance on standardized testing?

In order to answer this question, we conducted an analysis of TAKS items using the following criterion - number of steps required to solve the item. The project team called this criterion 'stepness.' We performed a similar triangulation process (as we did with sorting problems by cognitive demand level) for sorting the TAKS items by criterion of 'stepness.' Experts reached higher percentage of agreement in this sorting task — 97%. So, along with a cognitive demand level a particular item was assigned a 'stepness' level. We used the following scale for the 'stepness' criterion: 1, if an item requires one-step solution; 2, if an item requires two-steps for solution; 3, if the solution requires 3 or more steps. Figure 8 illustrates one of the items (item #28 from the April 2006 7th grade math TAKS test released to the Texas Education Agency website) by both cognitive demand and 'stepness' level. This item was sorted by experts as Level 1 item (e.g., performing procedure) having two steps: step one - to find/ estimate a volume of one cylindrical container, and step two – to multiply it by 2. We included the percentage of students' responses in parenthesis for each given choice. For

instance, the right choice F (marked by *) was picked by 42% of students and the choice G was selected by 28% of students. Imagine that this problem was phrased as a one-step problem (asking to estimate a volume of one container), then it is not unreasonable to think that $42\% + 28\% = 70\%$ of the students would answer this problem correctly. So introducing the second step would seem to reduce the student success rate by $(.70-.42)/.70 = 40\%$!

After the project team sorted TAKS items by the 'stepness' criterion, the team looked for a connection between student TAKS performance and corresponding 'stepness' of the item. Figure 9 shows a negative correlation ($r = -0.34$) between student performance (black curve) and 'stepness' (scaled to 100% by dividing each mean rating for a problem's number of steps by the overall maximum number of steps) of the item (red curve). The black curve reflects the apparent tendency of the TAKS test-makers to put the least difficult items at the beginning (when students need that initial boost of confidence?) and end (when students are getting tired?) of the test. Inspired by a reviewer's suggestion, we wondered if the

Figure 9. Connection between student performance (*black*) on item and its 'stepness' (*red*)

**Question -in-Progress:
How strong is the connection between Student Performance and " Stepness"?**

r=-.34, n=48, p<.01
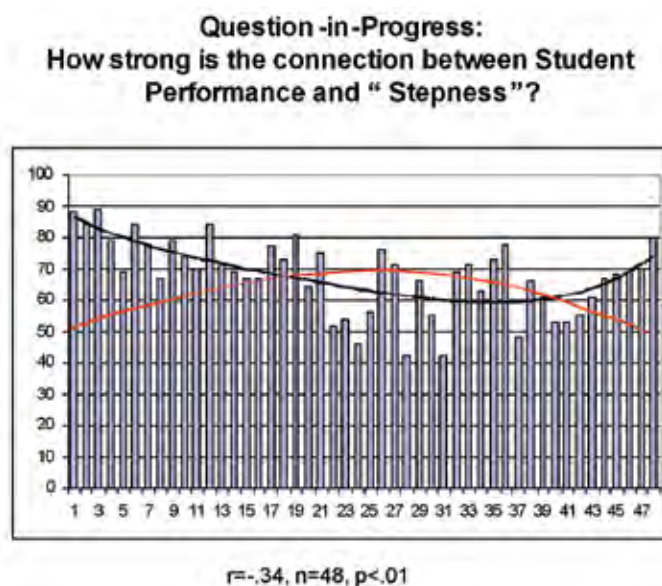
stepness connection to student performance was mirrored by stepness connection to teacher performance, in the same spirit as the pattern we saw in Figure 3. The first two authors independently coded each of the 33 problems on the Teacher Knowledge Survey (TKS) for number of steps involved. The authors agreed on the exact number of steps for the majority of the problems and the correlation between the two sets of ratings was significant ($p < .05$).

Figure 10. Connection between teacher performance on item and its 'stepness' (diamond icons with extensions denote multiple points at that spot)
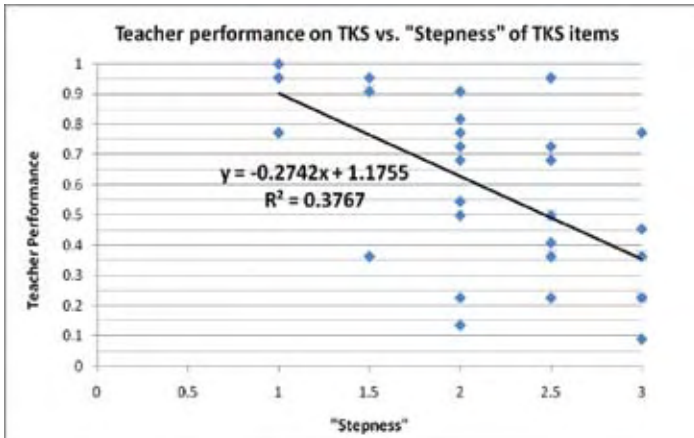
The authors then took the mean stepness rating and found its correlation with the proportion of teachers who correctly solved the TKS problems (see Figure 10). This correlation was found to be $r = -0.61$, $n = 33$, $df = 31$, 1-tailed p-value $= 7.3 \times 10^{-5} < 0.0001$. The R^2 value tells us that more than one-third of the variation in TKS scores can be explained by problem stepness. This result is an even stronger correlation than the one found for students.

Teacher Knowledge and Reflective Thinking

These quantitative findings are also supported by reviewing the teachers' reflection papers that used the five-question format mentioned near the beginning of the article. In particular, we see that teachers who were "high-high" (i.e., they scored higher than average on TKS and their students had higher than average improvement on passing rates, and this latter phrase will be discussed in the next section) approached their reflections qualitatively differently than teachers who were "low-low." To be more precise, we are comparing teachers from the upper right quadrant of the scatterplot in Figure 12 to those in the lower left quadrant.

As an example, let us look at a problem on the Teacher Reflection Paper assigned for January 9, 2006: "Which of the following is NOT true about similar figures?"

- A. Similar figures always have the same shape.
- B. Similar figures always have the same size.
- C. Similar figures always have corresponding angles that are equal.
- D. Similar figures always have corresponding sides that are proportional.

This question was problem #9 on the Spring 2004 7th grade mathematics TAKS test and the student response pattern was: A (8%), B (63%; correct answer), C (10%), D (18%). It was most revealing when teachers went through the five-step organizer (see p. 40) and addressed what they thought caused student low performance on this problem and what they would change (including questions to ask students) in their teaching. One of the high-high teachers' answers had a focus on mathematical concepts and terms, saying students had trouble with "Vocabulary of proportional, corresponding, etc. "and would need "practice on properties and scale factor." Typical answers from the low-low teachers, however, had more of a focus on test-taking or surface features, saying that "Students do not read the 'NOT' and therefore miss what the question is asking of them" and therefore would "stress reading the question."

Does Content-Focused Professional Development Matter?

We started the paper with the statement that too often professional development focuses narrowly on changing teaching behaviors with not enough attention paid to the impact of such tools on what students know and are able to do. The true focus of this project was on the effect of professional development on student achievement. In other words: does content-focused professional development make a difference?

We collected teacher participants' TAKS scores for two consecutive years: 2005 and 2006. By comparing the passing rate of teacher A's 6th graders in 2005 to the passing rate of teacher A's 6th graders in 2006, we were being consistent with the performance target specified in the grant application, which focuses on passing rates not for individual cohorts of students, but on a teacher or district level. Figure 11 below shows the change in TAKS passing rates for every participating teacher

Figure 11. TAKS passing rate gains for participating teachers from 2005 to 2006
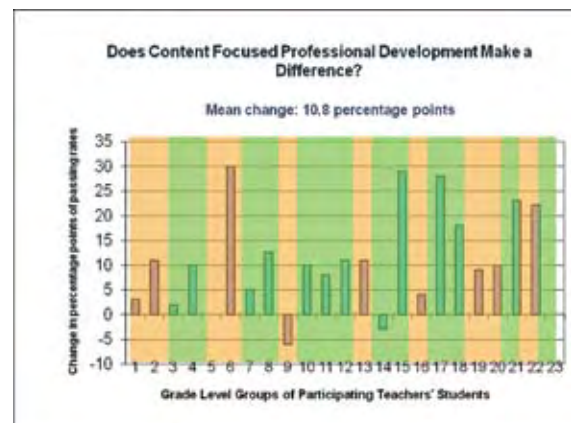
by grade level assignment. In total, we have the results for 14 different teachers, each of whom taught either one, two, or three grades' worth of students. For instance, columns 5 and 6 are the data for the same middle school teacher who was teaching two different grade levels, one of which had no change in passing rates.

The mean gain in TAKS passing rates of participating teachers from 2005 test administration to 2006 was 10.8 percentage points, compared to 4.4 for the state's mean gain for middle grades students. It should be noted that 10.8 actually underestimates our teachers' improvements because two instances of negative change (#9 and #14) both involve teachers whose previous groups of students (including gifted and talented classes) had passing rates of 90% and 100%, respectively, which created a ceiling effect that limited further improvement. By removing these two groups from the dataset, the mean change actually would have been 12.2 percentage points. There may be much hope in these numbers: content-focused professional development based on the proposed model (Fig. 1) can make a difference in student achievement. Although it seems likely that the professional development may have also increased teacher knowledge, this is something we can only conjecture. The TKS was given only as a pretest because the direct goal of the grant (improving teachers' students' achievement) did not justify administering the time-consuming TKS survey again.

Last, but not least: could the level of teacher knowledge affect student achievement? We conducted a regression analysis using teacher participants' knowledge survey scores and correlated it with the teacher's students' TAKS gain over the period of the study. A promising finding here is that the level of teacher knowledge is highly related to improvement in students' passing rate (Figure 12), with $r = 0.486$ ($n = 22$, $p < .01$).

Further Discussion

The overall conclusion of this paper: there is a connection between teacher knowledge and student achievement in general, and there are revealing patterns in the connection with regard to specific mathematical domains, processes and levels of cognitive demand in particular. The Teacher Knowledge Survey (TKS) showed the lowest performance on the "patterns, relationships, and algebraic reasoning" and "measurement" objectives, which are precisely the lowest performing two out of the six TAKS objectives for students! Within each objective, items on the 33-problem Teacher Knowledge Survey were also sorted by levels of

Figure 12. Positive relationship between teacher knowledge and student performance
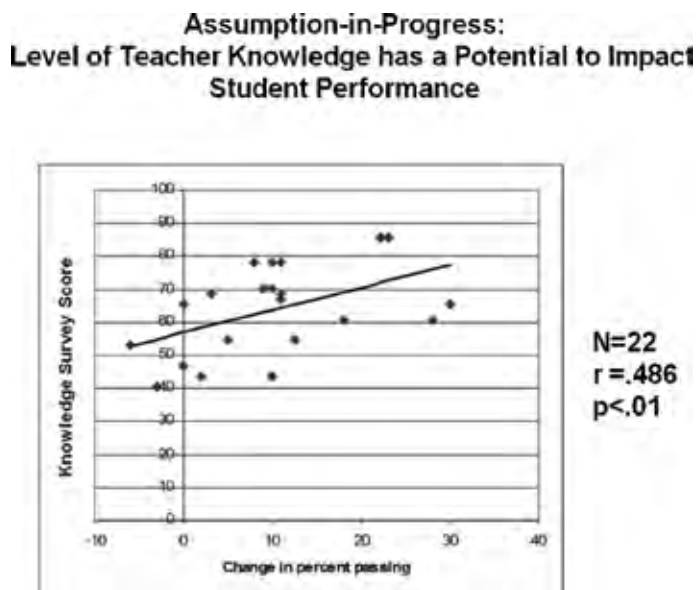
cognitive demand. Not surprisingly, teachers did the best on problems involving the lowest level of cognitive demand. Surprisingly, teachers did slightly better on problems at the highest level of cognitive demand than on problems at the middle level. The same pattern was observed in student performance on the state standardized test.

The TKS is itself an important accomplishment of this study, as its correlation with student performance is a measure of predictive validity and the instrument also had a respectably high level of reliability (Cronbach alpha = 0.76), especially considering the instrument's varying level of difficulty of problems. This is especially significant in light of the report of the National Mathematics Advisory Panel (United States Department of Education 2008, p. 37): "Evidence about the relationship of elementary and middle school teachers' mathematical knowledge to students' mathematical achievement remains uneven and has been surprisingly difficult to produce. One important reason has been the lack of valid and reliable measures. The literature has been dominated by the use of proxies for such knowledge, such as certification status and mathematics course work completed."

There appears to be much promise for this content-focused professional development model for identifying performance patterns and impacting some teacher variables on student achievement, and it will be interesting to explore how it might scale up or transfer to additional contexts.

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