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Start at Square One

Which Way Will You Effect Change in Our Profession?

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Table of Contents

Purpose Statement

The purpose of the National Journal of Mathematics Education Leadership is to advance the mission and vision of the National Council of Supervisors of Mathematics by:

- Strengthening mathematics education leadership through the dissemination of knowledge related to research, issues, trends, programs, policy, and practice in mathematics education

- Fostering inquiry into key challenges of mathematics education leadership

- Raising awareness about key challenges of mathematics education leadership, in order to influence research, programs, policy, and practice

- Engaging the attention and support of other education stakeholders, and business and government, in order to broaden as well as strengthen mathematics education leadership

Effective Use of Manipulatives Across the Elementary Grade Levels:
Moving Beyond Isolated Pockets of Excellence to School-Wide Implementation

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Imagine a school where first grade students solve problems and discuss their mathematical thinking with the support of manipulatives, but when students enter the second and third grades, they are no longer able to use manipulatives. When these same children enter fourth grade, the manipulatives become available again. What are the implications for the children in this scenario? Now imagine you are the fourth grade teacher. The children have not used manipulatives in the mathematics classroom for two years. What are the implications for your instruction in this scenario? Obviously, this situation suggests that uneven use of manipulatives is not in the best interest of children or teachers. Therefore, it is important to not only consider the effective use of manipulatives within individual classrooms, but also their appropriate use across elementary grade levels. This article discusses the research base on the use of manipulatives and strategies for leaders to help colleagues begin to use or strengthen their use of manipulatives so that effective school-wide implementation becomes a reality in more elementary schools.

What Are Manipulatives?

Manipulative materials are objects that appeal to several senses — sight and tactile, so they can be touched and moved about. Ideally these manipulative materials serve as physical models allowing mathematical ideas to be abstracted from use with them. Manipulatives have become prevalent in curriculum materials and in elementary classrooms. Commercial manipulatives abound, including, copyrighted Cuisenaire Rods® to generic base ten blocks, pattern blocks, and interlocking cubes. In addition to commercial manipulatives, the use of teacher made/gathered manipulatives, such as buttons, ten-frame tiles, mirrors, and straws add to a variety of materials that can be used to model mathematical concepts and facilitate active engagement in learning mathematics. Advances in technology have also resulted in many applets that have expanded the notion of "hands-on" manipulatives (Clements and McMillin 1996) to include "virtual manipulatives" (Hodge 2003). For example, see the Math Forum (http://mathforum.org/mathtools); National Library of Virtual Manipulatives (http://www.matti.usu.edu); and NCTM Illumination Activities (http://illuminations.nctm.org/ActivitySearch.aspx). Overall, elementary teachers have an overwhelming number of choices and decisions to make when it comes to not only selecting but also using manipulatives to improve the teaching and learning of mathematics. For example, teachers may decide to model or demonstrate a mathematical idea using a specific manipulative. Teachers may also provide manipulatives to students to use as tools to investigate mathematical problems they do not know how to solve.

What Does the Research Say About Using Manipulatives?

A steady line of research on manipulatives and their impact on mathematics teaching and learning has been reported for decades (Beougher 1967; Suydam and Higgins 1977; Sowell 1989; Uttal, Scudder, and DeLoache 1997). While research related to manipulatives in school environments is complex,

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the research findings are overwhelmingly positive in their support of teachers using manipulative materials in mathematics classes. Despite strong support from research and the existence of more manipulatives, many elementary teachers are reluctant if not resistant to using manipulatives as a regular part of their mathematics teaching. Unfortunately, this reality can lead to situations that are similar to the one described in the opening paragraph.

While research supports the use of manipulatives in helping children learn mathematics, research on the value and impact of manipulatives is complicated by many factors, such as which manipulatives were used, the length of time they were used, how they were used, and who used them (children/teacher). Nevertheless, a number of reasonable conclusions can be drawn from the existing research base that may help dispel some myths about manipulatives. Take the true/false quiz in Figure 1 to assess your own knowledge regarding the research/policy base on the use of manipulatives in elementary school classrooms.

Why Are Some Teachers Reluctant to Use Manipulatives?

For a number of reasons, teachers' use of manipulatives in elementary classrooms has grown significantly in the past twenty years. Yet, in some schools effective use of manipulatives has been in isolated pockets. Even when manipulatives are available and included in mathematics textbooks, some teachers make decisions to limit their use. This reluctance to use manipulatives may be a consequence of teachers' lack of familiarity with the available manipulatives. It may be influenced by the fact that their own experience as learners in K-12 mathematics classes did not include manipulatives. It may be based on their experiences with using manipulatives during instruction that led to challenging classroom management situations or frustrated students. Regardless of the influencing factors, many teachers show reluctance to using manipulatives to help children learn mathematics. This leads to uneven or ineffective use of manipulatives across the grade levels and it raises the question:

How Can Leaders Support Teachers?

How can you support teachers to effectively use manipulatives in every elementary grade level in your school or district? We asked experienced classroom teachers this question, and received many excellent suggestions.[1] As we examined their suggestions, we recognized that their ideas would be useful to other leaders. Multiple stages of action were suggested, with the first step to understand why teachers are resistant to use manipulatives and then to identify some specific actions that might help promote change. Throughout this process, it is essential to proceed with caution, being careful not to overwhelm or push too hard in bringing about change. The following suggestions may be useful for your school or district.

Figure 1

Write "True" or "False" for each statement.

_____ 1. Teachers' use of manipulatives decreases as the grade levels increase.
_____ 2. Good mathematics teaching always includes the use of manipulatives.
_____ 3. Manipulatives are more useful with less-experienced students than more-experienced students.
_____ 4. Students need not necessarily manipulate the materials to gain mathematical understanding.
_____ 5. Teachers sometimes overestimate the value of manipulatives because they know and understand the mathematical concept being represented.
_____ 6. Manipulatives may be used before or after a procedure is learned with generally equal success.
_____ 7. Teachers need to help students connect the mathematical concept(s) being explored with the manipulatives.
_____ 8. Students need to reflect on their actions with concrete materials to maximize their learning.
_____ 9. Almost any manipulative can be used to teach any mathematical concept.
_____ 10. Manipulatives are more useful in the elementary grades than in the upper grades.

See Figure 2 for answers.

Determine what is available. A teacher survey may be used to determine what manipulatives are available (Hatfield 1994; Scott 1983). It may lead to an inventory of manipulatives that are available by room, grade or building. Such information is helpful in determining the range of manipulatives that exist, and may reveal shortages or areas of need for additional materials. This information may also lead to a discussion about characteristics of manipulatives to be used in mathematics teaching. Discussions of physical as well as pedagogical criteria for manipulatives can be informative and generate healthy discussions about home made and commercially available manipulatives (Hynes 1986; Reys 1971). It may result in teachers reflecting

on and discussing their current mathematics curriculum, mathematical concepts, and more specifically student mathematical thinking.

Understand why teachers are resistant to using manipulatives. The challenge here is to learn why teachers are resistant to using manipulatives in a non-threatening and non-critical way. Allow teachers to present their thoughts, concerns, fears, and experiences. For example, teachers may describe objections related to prior unsuccessful classroom use of manipulatives, lack of access to manipulatives, lack of understanding the connections between the manipulatives and the mathematical concepts, or difficulty managing children and manipulatives. Identifying specific objections is the first step in finding solutions.

Address concerns. After assessing the concerns of teachers throughout the school, you may realize that several teachers have similar concerns while other individuals have unique concerns. In either case, teachers will need to work together to address concerns that have been raised.

Determine who has expertise about and experience with manipulatives. Teachers who have used specific manipulatives effectively to address mathematical goals can share how they use them. This sharing will allow other teachers to contribute additional ideas and suggestions, as well as provide a climate where teachers may ask questions about using the manipulatives. This setting may also lead to discussions about how these manipulatives actually facilitate mathematics learning, and the important role that teachers play in helping children make connections between manipulatives and mathematical concepts.

Start with a few lessons. Focus on a few teachers or a specific grade level. As a group, collaborate and plan a few lessons together. Identify the teachers' mathematical goals for each lesson and then help them select and use appropriate manipulatives to accomplish their goals. After each teacher in the group has taught the lessons, meet to discuss how to improve the lessons. Focusing on a few lessons each semester that target the use of manipulatives will create a collection that can be slowly expanded without overwhelming teachers. More importantly, the establishment of this regular process of teachers working together to develop, teach, and reflect on their mathematics lessons will improve mathematics teaching and learning.

Watch others. Providing structured opportunities for teachers to observe one another teach can facilitate more effective teaching. Observations and related discussions regarding manipulatives may help teachers with issues related to classroom management and student learning. Observations focused on one small group of students using manipulatives or focused on how a teacher helps students make connections between the manipulatives and the mathematics may increase the effectiveness of the observations by providing structure. If teachers are uncomfortable observing colleagues, observing and discussing videotapes (e.g., Cognitively Guided Instruction or Project Construct videos) in a group provides an alternative approach. This approach allows larger groups of teachers to observe classrooms and allows video segments to be replayed and analyzed more carefully.

Provide a rationale for using manipulatives. Much has been written about the value of manipulatives and their potential role in elementary classrooms. Discussing a few of the true/false questions in Figure 1 or professional articles (Kennedy 1986; Moyer, Bolyard, and Spikell 2002) can both inform and stimulate discussion. Such readings help establish an important intellectual foundation for using manipulatives as well as provide guidance on how to use manipulatives to facilitate mathematical learning. An examination of some research, either first hand reports (Uttal, Scudder, and DeLoache 1997) or research summaries (Driscoll 1981; Suydam 1996, Thompson and Lambdin 1994) can be very helpful. However, the most persuasive evidence will be gathered from actual student performance in your building. Therefore, showing examples of student work that documents improved understanding and performance in mathematics as a result of using manipulatives will be very powerful.

Tread carefully. While manipulatives can be powerful instructional tools for helping children learn mathematics, their use alone does not guarantee success. The challenges of using manipulatives effectively and related issues that teachers need to keep in mind have been voiced (Ball 1992; Baroody 1989) and need to be considered. Keep lines of communication open with teachers to provide support and discuss questions/challenges that are bound to arise. As you interact with and listen to teachers, ask how you can support them and encourage their efforts. Involve teachers and administrators in planning a course of action. In general, tread lightly and respect the bee hive!

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Conclusion

Developing teachers' knowledge and comfort in using manipulatives is an on-going challenge. The challenge exists for teachers who are resistant or hesitant to use manipulatives, as well as for lead teachers who have acquired expertise with manipulatives. As new models/manipulatives continue to become available, it takes time and energy to learn when and how to use them well to facilitate mathematical learning. These stages—from initial awareness to hesitation to instructional attempts to continuing refinement—are learning cycles that every teacher experiences. Even though this discussion has focused on supporting teachers who have shown reluctance to using manipulatives, we believe the identified strategies will support the professional growth of all teachers and thus improve mathematics teaching and learning at the school-wide level. Uneven or ineffective use of manipulatives at the school-wide level is not in the best interest of children or teachers. Ensuring that isolated pockets of success are expanded across the grade levels to achieve effective school-wide implementation requires a conscious, sustained effort facilitated by effective leaders.

Figure 2

True-False Answers

True 1. Teachers' use of manipulatives decreases as the grade levels increase.
Use of manipulatives is greatest among primary grade teachers, and manipulative use decreases as the grade levels increase (Hatfield 1994; Grouws and Smith 2000).

False 2. Good mathematics teaching always includes the use of manipulatives.
Teaching is a complex practice and good teaching of mathematics is "not reducible to recipes or prescriptions." (NCTM 1991, p. 22).

False 3. Manipulatives are more useful with less-experienced students than more-experienced students.
Manipulatives have been recommended as a means of improving performance for all levels of students, including gifted students (Peterson, Mercer, and O'Shea 1988).

True 4. Students need not necessarily manipulate the materials to gain mathematical understanding.
Teacher demonstration of manipulatives can be effective in facilitating mathematical learning (Suydam 1996).

True 5. Teachers sometimes overestimate the value of manipulatives because they know and understand the mathematical concept being represented.
Children do not have the same understanding as their teachers so it becomes very important that teachers help children make connections between the manipulative and the mathematical concept being developed (Suydam and Higgins 1977; Fuson, et. al. 1997).

False 6. Manipulatives may be used before or after a procedure is learned with generally equal success.
Models and manipulatives seem to be most effective in the developmental stages and prior to procedures or algorithms being learned (National Research Council 2001).

True 7. Teachers need to help students connect the mathematical concept(s) being explored with the manipulatives.
Manipulatives have many components and children may not always focus on the key variables. Teachers need to help children make connections between relevant variables and the mathematics (Beishuizen, Gravemeijer, and van Lieshout 1997; National Research Council 2001).

True 8. Students need to reflect on their actions with concrete materials to maximize their learning.
Children may use the manipulatives without making any connections to relevant mathematical concepts. Teachers need to ask questions and encourage students to reflect on their actions with the materials (Uttal, Scudder, and DeLoache 1997).

False 9. Almost any manipulative can be used to teach any mathematical concept.
One size does not fit all. Manipulatives need to be carefully selected to embody the mathematical concepts being developed (Dienes 1969).

False 10. Manipulatives are more useful in the elementary grades than in the upper grades.
Manipulatives have been shown effective in supporting mathematics learning and achievement with elementary, middle and high school students (Driscoll 1981; Sutton and Krueger 2002).

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