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An Activity-Based Approach to Technology Integration in the Mathematics Classroom

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Tim was so learned, that he could name a horse in nine languages. So ignorant, that he bought a cow to ride on.

Benjamin Franklin, 1914, p.54

Becoming a mathematics teacher today can be a challenging endeavor, requiring teachers to learn difficult content and specialized pedagogies as well as becoming fluent with new technological tools and techniques. Pre-service teachers at the secondary level are faced with programs of study that often begin with Calculus and include abstract topics such as non-Euclidean geometry, discrete mathematics, and modern algebra (NCTM, 2000; U.S. Dept. of Education, 2005). With increased federal mandates for mathematics instruction, today's pre-service elementary teachers are also faced with significant mathematics content involving topics such as number sense, geometry, and probability (NCTM, 2006). At the same time, both pre-service and practicing teachers of mathematics at all levels are encouraged to consider relatively sophisticated strategies for instruction such as problem-based learning, student-centered teaching, and scaffolding (Davis, Maher, Noddings, 1990; Fuson, Kalchman, & Bransford, 2005).

Technologies such as graphing calculators, symbolic processing programs, mathematical simulations and cross-discipline instructional tools such as robotics kits are becoming ever more available to teachers as an aid to instruction (Heid, 2005). For many teachers, these are new tools to consider in their instruction, but it is important for these tools to be used thoughtfully and strategically to strengthen instruction (NCTM, 2003, 2006). As Mr. Franklin admonished (albeit indirectly), how do we help teachers make good technology choices as they plan and enact instruction? How do we ensure they do not “buy a cow to ride on,” but rather, consider mathematics content and pedagogy along with their choice of technology to make decisions that are instructionally sound and promote student learning?

One answer may lie in the ways we help teachers to consider how mathematics content, pedagogy and technology might be combined to plan effective instruction in today's quickly evolving mathematics classroom, keeping in mind that the field of mathematics is changing rapidly as mathematics educational technologies evolve, and these technological changes often have implications for mathematics content and pedagogy (Heid, 2005; Peterson, 1988; Sinclair & Crespo, 2006). For example, computational technologies have helped mathematicians explore the use of fractal geometry to model and examine real-life phenomena such as lightening strikes, plant growth, cloud formation, coastline erosion, and blood circulation yet the integration of fractals into mathematics textbooks and coursework is comparatively new and often requires the use of technology and appropriate pedagogy (Falconer, 2003). Technology use is similarly changing the mathematics of statistics, graphing, plane geometry, matrices, and probability, to name just a few and teachers of mathematics are encouraged to use a wide range of educational technologies to help their students to learn about such topics (Heid, 2005; Rosen, 1999). Given the proliferation of new mathematics content, new instructional strategies, and new mathematics-based educational technologies, how can we help these teachers make optimal choices when so much is changing so quickly?

Technological Pedagogical Content Knowledge

Technological Pedagogical Content Knowledge or TPACK (Koehler, & Mishra, 2008) is a framework that describes the interconnected and interdependent content, pedagogy, and technology knowledge that teachers must have to make good instructional choices when planning and enacting a mathematics lesson. TPACK is built upon Shulman's (1986, 1987) notions of pedagogical content knowledge, which is the knowledge necessary to teach particular curriculum content, including both disciplinary and general pedagogical knowledge. TPACK designates knowledge of educational technologies—especially how to use these rapidly proliferating tools and resources instructionally in varying educational contexts—as pedagogical content knowledge that requires deliberate examination and development. Teachers who have well-developed TPACK demonstrate this knowledge by incorporating a strategic mix of mathematical content, appropriate pedagogies, and well-chosen technologies within their lessons (Grandgenett, 2008).

In some ways, mathematics educators have a bit of a head start on TPACK development, since the profession's integration of instructional technology to date, when compared with other disciplines, has been relatively strong. For example, the use of graphing calculators in high school mathematics classes has been suggested by several authors to be one of the most successful integration of technologies into teaching and learning (Fuson, Kalchman, & Bransford, 2005; Kaser, Bourexis, Loucks-Horsley, & Raisen, 1999; Reece, Dick, Dildine, Smith, Storaasli, Travers, Wotal, & Zygas, 2005). Technology-based applications like *Geometer's Sketchpad* and *Excel*, and Web-based resources like the *National Library of Virtual Manipulatives* are relatively common and well embraced in today's mathematics classroom (Heid, 2005). However, current integrations of digital technologies, such as graphing calculators or Excel, only scratch the surface of the educational opportunities that these tools and resources make possible in mathematics instruction. Innovative software programs such as *Inspire Data* or the newly enhanced *Mathematica*, or new technologies such as robotics and global positioning systems (GPS) are providing exciting opportunities for the learning of mathematics.

On one hand, when considering the possibilities for effective technology use in the mathematics classroom, there appear to endless possibilities. On the other hand, if one considers the learning activities that a teacher of mathematics might *typically* plan, a more limited list of possibilities would probably be generated. We suggest that providing teachers with a comprehensive list of possible mathematics learning activities along with some specific educational technologies that might be considered useful tools to support that activity might help better integrate educational technologies into mathematics instruction. We believe that such a resource could contribute significantly to the TPACK development of teachers and strengthen their mathematics teaching practice overall.

Supporting the Integration of Technology into Mathematics Instruction

We are attempting to generate and categorize a comprehensive taxonomy of mathematics learning activities and useful technologies to support each activity so a teacher planning a lesson for a particular mathematical topic or concept can review the taxonomy, select several learning activities to combine in a lesson, unit, or project plan, and consider any technology tools that might be useful to incorporate into their instruction. The taxonomy was generated through a careful review of the technology-based activities published during the past five years in the three teaching-related journals published by the National Council of Teachers of Mathematics: *The Mathematics Teacher*, *Mathematics Teaching in the Middle School*, and *Teaching Children Mathematics*. In all, more than 180 journal issues were examined. We have identified 31 distinct mathematics learning activity types and acknowledge that some may need to be edited or combined, some may need to be added, and some may need to be removed. Our taxonomy is presented as a beginning point for others to consider and the list will no doubt grow and evolve along with advancements in the discipline. We have set up a wiki at http://activitytypes.wmwikis.net/ to help to facilitate this ongoing process of identifying mathematics learning activity types and we invite the readers of this article to contribute to this effort.

By creating and sharing this taxonomy of mathematics learning activity types, and by including any associated technologies, we hope we can support the development of "TPACK in action" for teachers of mathematics so they are better able to select instructional strategies and technology tools to help students meet particular curriculum content standards. The mathematics learning activity types are intended to represent possibilities for instruction, conceptualized primarily in terms of student actions, and focused on what students might actually be *doing* during a mathematics lesson. For example, a teacher planning to address the concept of algebraic slope might use the taxonomy to consider using a "interpreting a phenomenon mathematically" mathematics learning activity that has students driving an electronic car up different sloped ramps and then using an interactive graphing program to represent the changing equation of the slope of the ramp.

Mathematics Learning Activity Types

The purpose of presenting a learning activity types taxonomy for mathematics is to introduce a full range of possible learning activities for teachers to consider when building lessons that effectively integrate technology, pedagogy, and content. In doing so, we attempt to scaffold teachers' thinking about how to best structure learning activities, how to best support those activities with educational technologies, and how to creatively engage their students in learning mathematics. The mathematics learning activity types are designed to be catalysts for thoughtful and creative instruction by teachers.

We have conceptualized seven genres of activity types for mathematics that are derived from NCTM's process standards. These activity types are expressed using active words to represent the pursuit of a dynamic and student-centered learning environment: *Consider, Practice, Interpret, Produce, Apply, Evaluate*, and *Create*. Many of the student actions embedded within the activity types are drawn directly from the NCTM standards themselves. Each of the seven genres is presented in a separate table below that names the activity types included in that genre, defines them briefly, then provides some example technologies that could strategically be used to support students' learning within each activity.

THE "CONSIDER" ACTIVITY TYPES

When learning mathematics, students are often asked to consider and make sense of new information. This request is a familiar one to both students and teachers. Yet, although such learning activities can be very important contributors to student understanding, the "Consider" activity types also often produce some of the lowest levels of student engagement, and are manifested typically using a comparatively direct presentation of foundational knowledge.

THE "PRACTICE" ACTIVITY TYPES

In learning mathematics, it is often important for students to be able to practice computational techniques or other algorithm-based strategies so that fluency with these skills can be developed for later and higher-level mathematical application. Some educational technologies can be used

Table 1

THE "CONSIDER" ACTIVITY TYPES		
Activity Type	**Brief Description**	**Example Technologies**
Attend to a Demonstration	The student gains information from a teacher or student presentation, videoclip, animation, interactive whiteboard or other display media.	Powerpoint, YouTube, document camera, interactive whiteboard, videoconferencing, or other display media
Read Text	The student extracts information from textbooks, or other written materials, in either print or digital form.	Electronic textbooks, websites (i.e. the Math Forum), informational .pdfs
Discuss	The student discusses a concept or process with a teacher, other students, or an external expert.	Ask-an-expert sites (e.g., Ask Dr. Math), online discussion groups, videoconferencing
Recognize a Pattern	The student examines a pattern presented and attempts to more fully understand the pattern.	Graphing calculators, virtual manipulative sites (e.g., the National Library of Virtual Manipulatives), spreadsheets
Investigate a Concept	The student explores or investigates a concept (such as fractals), perhaps by use of the Internet or other research-related resources.	Web searching, informational databases (Wikipedia), virtual worlds (Second Life), simulations
Understand or Define a Problem	The student strives to understand the context of a stated problem or to define the mathematical characteristics of a problem.	Web searching, concept mapping software, ill-structured problem media (i.e., Jasper Woodbury)

Table 2

THE "PRACTICE" ACTIVITY TYPES		
Activity Type	**Brief Description**	**Example Technologies**
Do Computation	The student undertakes computation-based strategies using numeric or symbolic processing.	Scientific calculators, graphing calculators, spreadsheets, Mathematica
Drill and Practice	The student rehearses a mathematical strategy or technique and perhaps uses computer-aided repetition and feedback in the practice process.	Mathblaster drill and practice software, online textbook supplements, online homework help websites (WebMath).
Solve a Puzzle	The student carries out a mathematical strategy or technique within the context of solving an engaging puzzle that may be facilitated or posed by the technology.	Virtual manipulatives, Web-based puzzles (magic squares), brainteaser Web sites (CoolMath)

to assist these processes. The table above offers both the range of practice-based learning activities and example technologies that can assist their implementation.

THE "INTERPRET" ACTIVITY TYPES

In the discipline of mathematics, concepts and relationships can be quite abstract, and can sometimes seem to be a bit of a mystery to students. Students often need to spend time exploring these relationships in order to understand them more deeply. Educational technologies can be used to help students investigate concepts and relationships more actively and assist with interpretation of what they observe. Table 3 displays activity types that can support such interpretive processes and provides examples of available technologies that can be used to support their formation.

Table 3

THE "INTERPRET" ACTIVITY TYPES		
Activity Type	**Brief Description**	**Example Technologies**
Pose a Conjecture	The student poses a conjecture, perhaps using dynamic software to display relationships.	Dynamic geometry software (Geometer's Sketchpad), widgets (Explore Learning), e-mail
Develop an Argument	The student develops a mathematical argument related to why they think that something is true. Technology may help to form and to display that argument (e.g., a proof).	Concept mapping software (Inspiration), presentation software, blogs, specialized word processing software (Theorist), e-mail
Categorize	The student attempts to examine a concept or relationship in order to place it into a set of known categories.	Database software (Microsoft Access), online databases, concept mapping software, drawing software
Interpret a Representation	The student explains the relationships apparent from a mathematical representation (e.g., table, formula, chart, diagram, graph, picture, model, animation).	Data visualization software (Inspire Data), 2D and 3D animations, video (iMovie), Global Positioning Devices (GPS), engineering visualization software (MathCad)
Estimate	The student attempts to approximate some mathematical value by further examining relationships using supportive technologies.	Scientific calculator, graphing calculator, spreadsheets, student response systems (Clickers)
Interpret a Phenomenon Mathematically	The student, assisted by technology as needed, examines a mathematics-related phenomenon (e.g., velocity, acceleration, the Golden Ratio, gravity).	Digital cameras, video, computer-aided laboratory equipment, interactive graphing program, specialized word processing, robotics, electronics kits

THE "PRODUCE" ACTIVITY TYPES

When students are actively engaged in the study of mathematics, they can become motivated producers of mathematical documents rather than just passive consumers of prepared materials. Educational technologies can serve as excellent "partners" in this production process, aiding in the refinement and formalization of student products as well as helping students share the fruits of their mathematical labors. The activity types listed include suggestions for technology that can assist these efforts.

Table 4

THE "PRODUCE" ACTIVITY TYPES		
Activity Type	**Brief Description**	**Example Technologies**
Do a Demonstration	The student demonstrates a topic or concept to show their understanding of a mathematical idea or process. Technology may assist in the development or presentation of the product.	Interactive whiteboard, video (YouTube), document camera, presentation software, podcasts
Generate Text	The student produces a report, annotation, explanation, journal entry or document, to illustrate their understanding.	Specialized word processing (Math Type), collaborative documents (Google docs), blogs, online discussion groups
Describe an Object or Concept Mathematically	Technology may assist in the description or documentation process, as the student produces a mathematical explanation of an object or concept.	Engineering visualization software, concept mapping software, specialized word processing, Mathematica
Produce a Representation	The student develops a mathematical representation (table, formula, chart, diagram, graph, picture, model, animation, etc.) using technology for production assistance, if necessary.	Spreadsheet, virtual manipulatives (digital geoboard), spreadsheets, Inspire Data, concept mapping software, graphing calculator
Develop a Problem	The student poses a mathematical problem that is illustrative of some mathematical concept, relationship, or investigative question.	Word processing, online discussion groups, Wikipedia, Web searching, e-mail

THE "APPLY" ACTIVITY TYPES

The utility of mathematics in the physical world can be found in its authentic applications. Educational technologies can be used to help students apply mathematics in the world and link mathematical concepts to real-world phenomena. The technologies essentially become students' assistants in their mathematical work, helping them connect mathematical concepts to the realities in which they live.

Table 5

THE "APPLY" ACTIVITY TYPES		
Activity Type	**Brief Description**	**Example Technologies**
Choose a Strategy	The student reviews or selects a mathematics related strategy for a particular context or application.	Online help sites (WebMath, Math Forum), Inspire Data, dynamic geometry/algebra software (Geometry Expressions), Mathematica, MathCAD
Take a Test	The student applies their mathematical knowledge within the context of a testing environment, such as with computer-assisted testing software.	Test-taking software, Blackboard, survey software, student response systems
Apply a Representation	The student applies a mathematical representation to a real life situation (table, formula, chart, diagram, graph, picture, model, animation, etc.).	Spreadsheet, robotics, graphing calculator, computer-aided laboratories, virtual manipulatives (algebra tiles)

THE "EVALUATE" ACTIVITY TYPES

When students evaluate the mathematical work of others or reflect on their own work, they have an opportunity to develop more sophisticated understandings of mathematical concepts and processes. Educational technologies can become valuable allies in this effort by assisting students in the evaluation process, helping them compare concepts, test solutions or conjectures, and integrate feedback from other individuals into revisions of their own work. The following table lists the range of evaluation-related mathematics learning activities.

Table 6

THE "EVALUATE" ACTIVITY TYPES		
Activity Type	**Brief Description**	**Example Technologies**
Compare and Contrast	The student compares and contrasts different mathematical strategies or concepts to determine which might be more appropriate for a particular situation and why.	Inspiration, Web searches, Mathematica, MathCad
Test a Solution	The student systematically tests a solution and examines whether it makes sense based upon systematic feedback, and which might be assisted by technology.	Scientific calculator, graphing calculator, spreadsheet, Mathematica, Geometry Expressions
Test a Conjecture	The student poses a specific conjecture and then examines the feedback of any interactive results to potentially refine the conjecture.	Geometer Sketchpad, statistical packages (e.g., SPSS, Fathom), online calculators, robotics
Evaluate Mathematical Work	The student evaluates a body of mathematical work through the use of peer- or technology-aided feedback.	Online discussion groups, blogs, Mathematica, MathCad, Inspire Data

THE "CREATE" ACTIVITY TYPES

When students are involved in some of the highest levels of mathematics learning, they are often engaged in very creative and imaginative thinking processes. Albert Einstein once implied that imagination was as important as knowledge in mathematics (Priwer & Phillips, 2003). It is said that this idea was consistent with his strong belief that mathematics is a very inventive, inspired, and imaginative

Table 7

THE "CREATE" ACTIVITY TYPES		
Activity Type	**Brief Description**	**Example Technologies**
Teach a Lesson	The student develops and delivers a lesson on a particular mathematics concept, strategy, or problem.	Presentation software, interactive video, video, podcasts
Create a Plan	The student develops a systematic plan to address some mathematical problem or task.	Concept mapping software, collaborative writing software, MathCad, Mathematica
Create a Product	The student imaginatively engages in the development of a student project, invention, or artifact such as a new fractal, tessellation, or other creative product.	Word processor, animation tools, MathCad, Mathematica, Geometer Sketchpad
Create a Process	The student creates a mathematical process that others might use, test, or replicate, essentially engaging in mathematical creativity.	Computer programming, robotics, Mathematica, MathCad, Inspire Data, iMovie

endeavor. Educational technologies can be used to help students to be creative in their mathematical work. The activity types following represent these creative elements and processes in students' mathematical learning and interaction.

Combinations of Activity Types

A creative lesson or learning plan by a teacher usually combines two or more activity types into a varied and engaging learning experience. In fact, when learning activities are combined and integrated, they may better resemble the complexity of real-life applications of mathematics, creating opportunities for students to encounter and solve rich mathematical problems that are often more realistic than the often more artificial problems often found in textbooks (Checkly, 2006; Fuson, Kalchman, & Bransford, 2005). Combining activity types may also provide opportunities for students to develop more divergent ways of thinking (Aris, 1994; Gershenfeld, 1998). Below are a several examples of how activity types might be combined, including a simple combination and two more complex combinations of mathematics activity types.

Example 1: Recognizing and Researching the Fibonacci Series

A common mathematics topic for teachers to assign for student research in middle school is the remarkable Fibonacci Series. This series, where each term is created by summing the two terms that appear before it (e.g., 1, 2, 3, 5, 8, 13, 21, 34) is found quite commonly in such items as the spiraled skin of pineapples, the stems of conifer trees, the curved edges in sea shells, and even the family trees of honeybees (Cook, 1979). A simple combination of activity types that might be used to build student understanding of the Fibonacci Series involves first asking students to "recognize [the] pattern" (from the "Consider" activity types) by asking them to display it on a chalkboard or spreadsheet in order to ensure that students are constructing the sequence correctly and then asking students to "investigate [the] concept" (also from the "Consider" activity types) by doing a Web search on the Fibonacci series to explore where it might be represented in the physical world. Students are often amazed at the many diverse examples of this series that can be found. These activities can become a context for exploring patterns within the Fibonacci sequence and how they can be expressed.

Example 2: Defining, Representing, and Solving a Paper Folding Problem

An interesting problem that is sometimes posed to elementary students who are studying exponential numbers to explore what happens to the thickness of a piece of paper if it is folded in half a total of 10 times. The increasing thickness of the folded paper soon creates an impossible situation and students find that they need to move to computational strategies to solve the problem. At this point, a teacher might encourage students to use a spreadsheet to mathematically "represent" (from the "Produce" activity types) what is happening in the problem and look for patterns. Students might then be encouraged to "test a conjecture" (from the "Evaluate" activity types) about these patterns having to do with powers of 2 and the notion of exponential growth. If the teacher realizes this same problem has been showcased on the television program "MythBusters," where the hosts jokingly use a sheet of paper the size of a football field and modern construction equipment to see if the size of the paper makes any difference in how many folds they are able to make while exploring this problem, students could be asked to "attend to" (from the "Consider" activity types) the related Mythbusters clip and then discuss it in relation to their explorations.

Example 3: Interpreting, Producing, and Testing a Garbage Pickup Model

In high school discrete mathematics, mathematical modeling activities often include the question of how a garbage truck might efficiently move through a system of streets to pick up the garbage each week. A teacher might encourage students to "interpret a representation" (from the "Interpret" activity types) by examining maps of local streets, or perhaps viewing a satellite image of their area using Google Maps. The students could then be asked to "understand or define a problem" and decide upon the parameters for efficient garbage pickup, such as the need to conserve gas by not retracing a route once the truck has already traveled a street. The students could then be encouraged to "produce a representation" (from the "Produce" activity types) of the streets as a network of line segments for the streets and nodes for the street intersections. They could then be asked to "create a plan" (from the "Create" activity types) for an efficient garbage pickup route using this mathematical representation of their neighborhood. Often students prefer to use some sort of computer-assisted drawing program, such as the drawing utilities in Microsoft Word or the more sophisticated MathCad, to depict a system of nodes and connecting line segments and to formalize their planning. Soon students realize that "odd or even" nodes (named according to the number of line segments coming together at a street intersection) are important considerations for planning the most efficient route. Finally, students might be asked to "compare and contrast" their routes (from the "Evaluate" activity types) by creating some sort of numerical index for their route (perhaps with a spreadsheet chart) that might compute the number of miles traveled or the amount of gasoline used. As they do so, they are encouraged to "evaluate [their own and others'] mathematical work" (from the "Evaluate" activity types) to create maximally efficient routes. This particular mathematical challenge illustrates the use of mathematical modeling while also entailing a combination of mathematical learning activity types that encourage flexibility, creativity, and pedagogically appropriate technology use.

Final Thoughts

As leaders of mathematics education, we know that "doing mathematics" is a very creative, exciting and dynamic endeavor. It "involves observing, representing, and investigating patterns and relationships in social and physical phenomena and between mathematical objects themselves" (Steen, 1998, page 16). We hope the mathematics learning activity types presented in this article might help teachers better engage and motivate students in their classrooms, involving them more fully in the creativity of doing real-life mathematics, acquainting them with the growing number of technology tools available to explore that mathematics, and helping them appreciate the role of mathematics in understanding our natural world.

If we are to help teachers to develop their TPACK so they might be better prepared to integrate mathematics content, pedagogy, and technology successfully in their classrooms, we will no doubt need a range of instructionally sound strategies and examples. When we separate mathematics content, pedagogy, and technology instruction in our pre-service teacher education programs or in our professional development efforts with practicing teachers, we risk giving teachers a very superficial understanding of the instructional power of their successful combination, resulting at times in less-than-optimal mathematics lessons. Instead, we need to carefully and consciously scaffold the development of teachers' TPACK, so they can make thoughtful and maximally effective instructional choices that combine mathematics content, pedagogy, and technology and more authentically engage students in "doing mathematics" together in classrooms.

Such integration can be done, and done well, if we give teachers the support and encouragement they need to be creative designers of classroom instruction. Ben Franklin would no doubt be in agreement with this approach, since he had an uncommon passion for applying knowledge to the world in which he lived, as well as generating new knowledge from the successes and failures of the experiments he conducted. The taxonomy of mathematics learning activity types shared here is an attempt to provide a vehicle to support teachers who are trying to generate similar passions among their students, building interest and motivation through diverse, engaging, and technology-rich learning activities while also deepening and extending their learning of important mathematics. If we are successful in such efforts—in the words of Mr. Franklin—the students in our schools will be less likely to "buy a cow to ride on" and will instead be well prepared to do and see mathematics in the world around them.

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