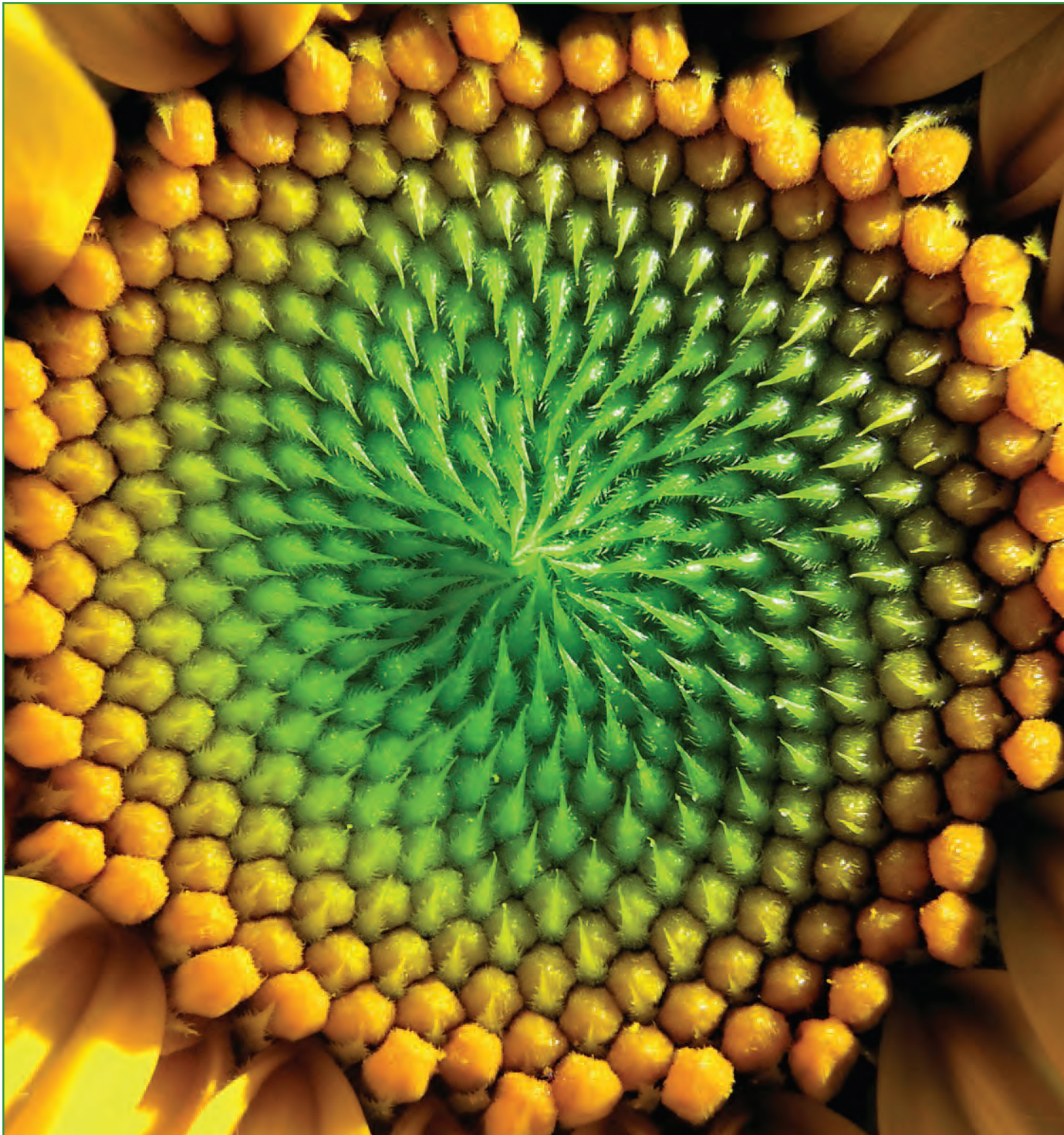


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Early Numeracy Intervention: One State's Response to Improving Mathematics Achievement

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A significant number of children struggle with quantitative ideas during the first few years of their academic careers and fail to construct a meaningful sense of number. These early struggles, if not addressed, can limit students' mathematics performance as they move through the grade levels. If these struggling students are fortunate enough to receive intervention services, most often they are provided too late and are provided by teachers with little knowledge of numeracy development. Duncan et al (2006) identified the predictive power of early mathematics knowledge and found that knowledge of numbers and ordinality were the most powerful predictor of later learning. The study recommends future research to identify "promising early math interventions" (p. 21).

Research on the factors contributing to student achievement discovers again and again that teacher expertise is one of the most important variables in determining student achievement (Darling-Hammond & Ball, 2004). Research also suggests that many elementary school teachers in the United States lack essential knowledge for teaching mathematics and this lack of knowledge directly impacts how well they teach mathematics (Ball, 1990; Ma, 1999). In fact, many elementary teachers report that they do not have sufficient conceptual understanding of mathematics and rely on rote computations and algorithms for instruction (Gerretson, Bosnick, & Schofield, 2008).

There seems to be agreement that mathematical performance is unlikely to improve without serious attention to the ongoing professional development of elementary

teachers of mathematics. The research findings and student achievement data reflect the compelling need for high-quality professional development opportunities for elementary teachers of mathematics that focuses on developing knowledge of the conceptual foundations of elementary mathematics, the features of effective mathematics instruction, how to use curriculum materials to support instruction, and strategies for using assessment data to inform that instruction. There is a critical need to identify effective early intervention programs to enable more students to be successful mathematicians.

Review of Literature

Engaging teachers in identifying which concepts and skills they want students to learn, anticipating students' challenges, and understanding the nature of students' misconceptions improves teachers' instructional practices and results in more positive and significant student outcomes (Blank, de las Alas, & Smith, 2007; Carpenter, Feneman, Peterson, Chiang, & Loef, 1989; Cohen & Hill, 2001; Lieberman & Wood, 2001; Saxe, Gearhart & Nasir, 2001). Kennedy (1998) conducted a literature review to identify the features of effective professional development programs and found that "programs whose content focused mainly on teachers' behaviors demonstrated smaller influences on student learning than did programs whose content focused on teachers' knowledge of the subject, on the curriculum, or on how students learn the subject" (p. 18). Kennedy's literature review suggests an important role for content-emphasis in high-quality professional development. The most useful professional development directly relates to the teachers' work and involves a cycle of assessment,

active teaching, observation and reflection (Darling-Hammond & McLaughlin, 1995).

Ongoing and sustained professional learning that provides teachers with opportunities to collaborate together allows teachers to learn together, apply their learning to their classroom context, and reflect on what is effective and why (Loucks-Horsley, Hewson, Love, & Stiles, 2009) and also promotes the creation of a shared understanding of what constitutes effective mathematics teaching and learning. The process of learning in small, supportive groups with colleagues promotes the likelihood of teachers changing their instructional practices (Dunne, Nave & Lewis, 2000). Situating collaborative conversations in dilemmas that teachers experience in their teaching creates meaningful and authentic opportunities for teachers to examine their instructional practices. Little (1990) found that collaboration focused on authentic work resulted in high-quality solutions to instructional challenges, increased teacher confidence and resulted in significant gains in student achievement. Findings from the National Commission on Teaching and America's Future (2010) indicate this kind of collegial interchange is a requirement of professional learning designed to strengthen instruction.

Context of the Study

In spring 2005, the Kentucky Legislature passed House Bill 93 that called for the development of a multi-faceted strategic plan to improve K-12 student achievement in mathematics. One important focus of this bill was the need to provide developmentally appropriate and research-based diagnostic and interventions services to kindergarten through third grade students through a newly created Kentucky Center for Mathematics (KCM). The KCM chose as part of its mission to develop expertise among the Mathematics Intervention Teachers (MIT) community in order to affect significant positive changes in student learning of mathematics at the primary grades.

Typically, students participated in 30-60 minute intervention sessions daily (in addition to the regular mathematics instruction) and these sessions might involve either individual students or small groups of students. The KCM purposefully allowed some flexibility in this area so that schools might construct an individualized model for intervention to meet the needs of varying school contexts/structures. The only 'non-negotiables' were that intervention sessions did not conflict with the students' classroom

mathematics instructional time and that MITs' time must be spent conducting mathematics intervention with children. Students who no longer needed additional support, based on assessments administered by the MITs, were released from the program.

Given the considerable evidence supporting the effectiveness of sustained and job-embedded professional development (Loucks-Horsley, Hewson, Love, & Stiles, 2009), the KCM required that MITs involved in this project participate in ongoing job-embedded professional development provided by the KCM. This consisted of a program of professional development that grew to include an intensive 5-day summer institutes, periodic 2-3 day within-year institutes, weekly online team meetings, and periodic face-to-face collegial team meetings.

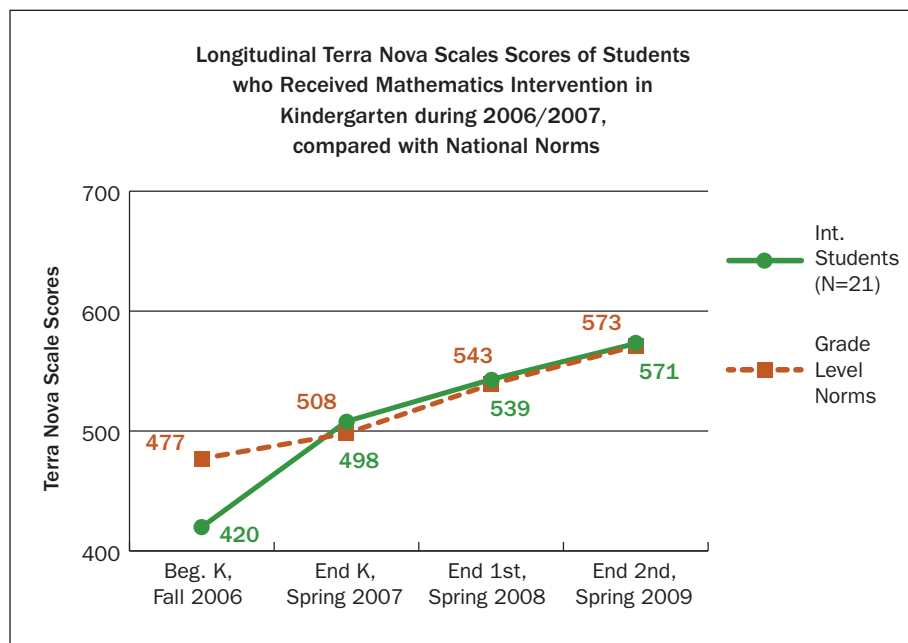
Initially, the MIT professional development was associated with either Number Worlds (Griffin, 2004; Sarama & Clements, 2004; SRA, 2007) or Math Recovery (U.S. Math Recovery Council, 2006, 2008; Wright, Martland, & Stafford, 2000; Wright, Stanger, Stafford & Martland, 2006) intervention programs. The professional development focused on developing MITs' knowledge of the complexity of numeracy development. MITs learned about the stages of numeracy, characteristics of the various stages in the learning trajectory, and the instructional strategies appropriate for advancing student development along the learning continuum during the summer sessions. The weekly online meetings and the collegial team meetings provided a forum for MITs to discuss their individual students, professional challenges, and new professional insights with other MITs. Beginning in 2007, many MITs who were using Number Worlds also chose to participate in professional development associated with Math Recovery or Add+VantageMR (U.S. Math Recovery Council, 2008) and used a combination of the approaches in the intervention teaching. Add+VantageMR is, foundationally, very similar to Math Recovery in that both programs use similar professional development frameworks and progressions to map children's mathematical development. Indeed, Add+VantageMR and Math Recovery both rely upon learning frameworks developed by Wright et al. (2000; 2002; 2006); however, one key difference is that Add+VantageMR was designed for small group and whole class interventions while Math Recovery was designed for more intensive one-on-one interventions. This difference is articulated in professional development that emphasizes instructional experiences groups or individuals respectively.

External evaluators conducted a randomized study comparing the pre- and post-achievement levels of participating students and students eligible for participation but unable to receive services due to the MITs capacity limit. The external evaluators also identified similar sites without an MIT and tracked the progress of students deemed eligible for intervention at those sites. The end of the first year and every year thereafter on the Terra Nova assessment noted significant gains in student achievement among students receiving support from these MITs for students in kindergarten through third grade. More specifically, pre- and post-test student achievement results on the Terra Nova at the end of the first

year of implementation indicate that first grade intervention students supported by the Number Worlds intervention (1000+ students) achieved, on average, an increase of more than one year and the first grade intervention students supported by Math Recovery gained on average more than two years growth. On average, the intervention students made gains that exceeded their peers eligible for the intervention who were not provided with these services. Sustained impact was evidenced by longitudinal Terra Nova data demonstrating that intervention students were performing at or near grade level a year or more after exiting the program (Figure A). Average first grade achievement results over four years demonstrated results at or near grade level expectancy after one year of intervention despite beginning average scores well below those expected of entering kindergarten (Figure B).

The student achievement results suggest that the KCM was successful in its efforts to strengthen the mathematics achievement of low-performing students' in kindergarten through grade three for those students supported by the interventions offered by MITs participating in the professional development provided through this initiative. The purpose of this study is to identify the factors that contributed to the success of the professional development initiative.

FIGURE A



Methodology

PARTICIPANTS

The primary participants in this study were the MITs who participated in the professional development initiative and provided instructional interventions based on individual student understanding of and fluency with number. The program started with 46 MITs in the summer of 2006, an additional 41 MITs in the summer of 2007, and an additional 27 MITs in the summer of 2008. Interviews were also conducted with building and district administrators and classroom teachers whose students were serviced by the MITs.

DATA SOURCES

The study used data from end of the year surveys, semi-structured interviews, and field notes of observations that allowed for a convergence triangulation of data across perspectives. Surveys were administered following each year of participation in professional development to assess MIT perceptions of the nature of mathematics, beliefs about teaching and learning mathematics, self-efficacy regarding their own proficiencies in mathematics, and the usefulness of the professional development in improving student learning. Semi-structured interviews and observations of MITs engaged in teaching, collaborative planning, and professional development sessions were conducted beginning in July 2007 and concluding in May 2010.

During the 2007-2009 school years, observations and informal interviews were conducted in professional development sessions during the summer and in collegial team meetings with regular education classroom during site visits to seven schools (three urban, two rural and two suburban schools) during the school year. During the fall of 2009, participant observations and interviews were also conducted during four days of Math Recovery training. Additional interviews were conducted and recorded via telephone.

A total of 112 semi-structured interviews were conducted with 47 MITs, 56 regular education teachers of participating students, and 9 administrators from July 2008 through May 2010. The purposes of second and third year semi-structured individual and focus group interviews and observations were to learn: 1) how participation in program activities contributed to teacher growth, 2) what learning was most transformative, 3) what changes in instructional practices resulted from participation in the program, and 4) how these instructional changes impacted their students.

Qualitative data was analyzed using the constant comparative method of grounded theory (Glaser and Strauss, 2009) and involved the constant interplay between the researcher, the data, and the developing theory. All interviews and field observations were transcribed and analyzed using three cycles of analysis: open coding, axial coding and selective coding. On this basis, a theory was developed that enabled a rich description of the components that contributed to MITs' professional knowledge and aspects

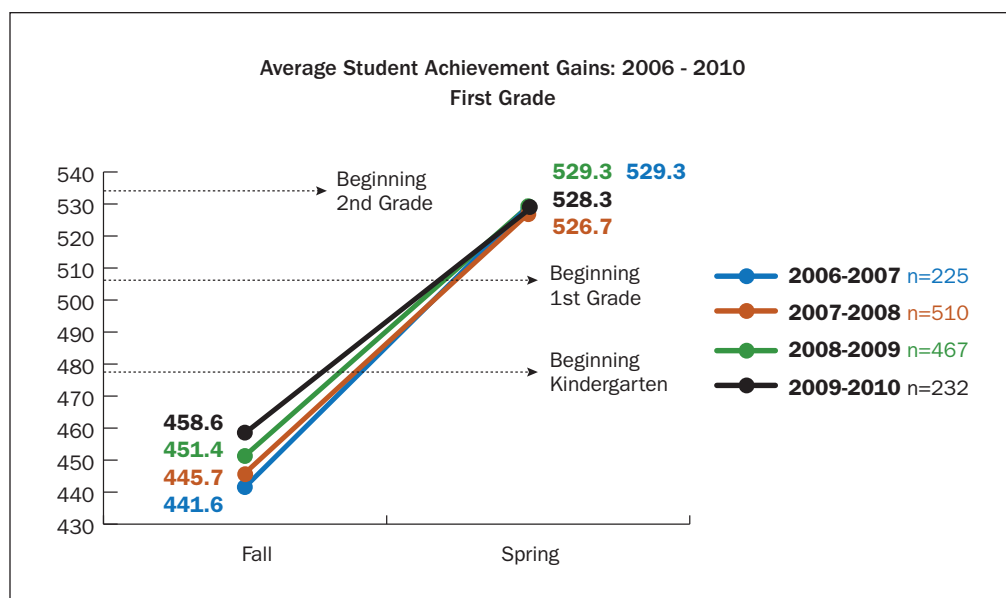
of their interventions that contributed to increased student achievement.

Results

MATHEMATICS BELIEFS SURVEY

The Mathematics Beliefs Survey results MITs were generally confident of their knowledge of mathematics and generally enjoyed mathematics. A high percentage of MITs across all three years of the program indicated they liked doing mathematics and were interested in mathematics. Changes in MITs' attitudes towards mathematics from the pre-participation survey to the post-participation survey demonstrated significant changes in the MITs' attitudes towards mathematics in a positive direction. As a result of participation in this professional development, an increased percentage of MITs also indicated that they looked at underlying reasoning, application, and use of hands-on activities and that anyone can learn mathematics and that they know they understand a concept when they successfully explained it to another person. For example, MITs were less likely to agree or strongly agree that, "To understand mathematics, students must solve many problems following examples provided." Significant declines were also present with the following questions: "Doing mathematics consists mainly of using rules." and "Knowing step-by-step procedures is necessary to solve mathematical problems" (University of Cincinnati Evaluation Services Center, 2009). Complete evaluation reports are posted on the Kentucky Center for Mathematics website at <http://www.kentuckymathematics.org/research.asp>.

FIGURE B: Program Consistency



SEMI-STRUCTURED INTERVIEWS AND OBSERVATION DATA

Three main themes emerged from the analysis of interviews and observations of MITs' responses during professional development sessions attended. These themes were:

- 1) conditions and culture
- 2) professional competencies, and
- 3) changes in practices and beliefs.

Conditions and culture describes the requirements of the MIT role and the level of collaboration.

Professional knowledge describes the professional knowledge of teaching including knowledge of mathematical content and pedagogy. Change describes the changes in MITs' instructional practices and beliefs about how children learn mathematics. While these themes had distinct qualities to them, there was much overlap and interplay between these. These themes will be presented separately for the purpose of reporting.

Conditions and culture. A majority of MITs were classroom teachers selected by their building or district administrator to serve in this role. Participation in ongoing job-embedded professional development was a job requirement. During the professional development, MITs deepened their conceptual understanding of early numeracy, exploring mathematical tasks from many different perspectives, and exploring those different perspectives together. They spent many hours reviewing, analyzing, and discussing video clips of students responding to similar mathematical tasks using the Stages of Early Arithmetic Learning (Steffe, von Glasersfeld, Richards, & Cobb, 1983; Steffe, Cobb, & Glasersfeld, 1988; Steffe, 1992) and other constructivist researchers to make sense of what they saw students doing and thinking. A culture of inquiry was modeled throughout the professional development so MITs could experience what it meant to participate in meaningful learning together. It is important to note the professional development focused on developing mathematical content knowledge and pedagogical knowledge, not just how to teach a particular intervention program.

MITs reported that they had never been involved with any professional learning that was as challenging and rigorous. As one MIT stated, "The training for this program is intense, but all of us have learned so much about the research behind good math education. . ." Another MIT expressed the importance of the affirmation she felt while also revisiting her beliefs and practices:

And when I hear some of the things I'm already doing, it affirms me that I am doing the right thing. I need to know if I am doing the right thing and if I am not, I need to know what I should be doing. Without the support, I would get frustrated and teach the way I always have. I know I can call my regional coordinator and talk through my challenges. The collegial meetings give me the chance to solve issues and get reassurance that what I am doing is what I should be doing. In this way of teaching, you have to rely on what you know and what

the student is doing. It's not like, read the manual and do the next lesson. It's like, really trying to understand what the child is thinking and what settings [instructional tasks] will move him forward.

The MITs also appreciated the opportunity to participate in the culture of inquiry created in the professional development and were able to connect it to their own instructional practices. As one MIT remarked, "The leader ran the meeting in the same manner that it was supposed to be implemented in the classroom. It was not simply knowledge that was passed along, it was modeled."

Professional competencies. The professional development activities engaged the MITs in developing knowledge of early numeracy progression, diagnostic and formative assessment strategies, and strategies for designing instruction based on assessment data. While many of the MITs were selected because of their teaching expertise, every participating MIT interviewed reported that they did not have the necessary knowledge of early numeracy development. The following is representative of many MITs' responses, "I guess I knew there was a numeracy foundation. I just did not know what made up that foundation—what they specifically needed to know." They described developing deep insights about the components of early numeracy development and clearly articulated their knowledge of early numeracy as a result of their participation in the professional development.

It helped me to understand the development progression of early numeracy. It takes you from children that can't count by ones, those who have no number correspondence, and it teaches you how to help them develop a solid numeracy foundation with forward and backward number sequences, structuring numbers with five, ten and twenty.

A significant number of MITs also shared that they learned the value of observing how individual students solved mathematics problems. The following is one example of such sentiment, "I am thinking a lot about how the kids are getting the answer. I watch their thinking more than I ever have before. This training helped me think about the kids individually." Many MITs reported that the practice of sharing and discussing video clips of students solving mathematical tasks demonstrated the value of analyzing student thinking as a tool for focusing instruction on the individual needs of students.

Changes in practices and beliefs. MITs' approach to teaching mathematics changed as a result of their participation in the professional development activities. MITs reported they were engaging students in more discussions about how students approached and solved mathematical tasks and they were asking students many more probing questions about the students' thinking. As one MIT remarked, "Did you understand that?" used to be the most probing question I asked. But now it's, "Why did you think to do it that way, I wouldn't have thought about that?" The asking of these kinds of questions provided MITs with a deeper understanding of how their students were making sense of the mathematics. "I have learned how important it is to ask probing questions. The questions can both guide the students to think deeper about the math and they help me to understand their thinking so I can better guide their learning."

Many MITs reported an increase in their use of manipulatives designed to support the development of early numeracy (e.g., five frames, ten frames, empty number lines, covered counters). Many MITs expressed the importance of students constructing mathematical understanding and how different this was from their prior understanding of the importance of procedures and memorization in learning mathematics. The following response represents a common theme expressed by every participating MIT interviewed.

It is kind of sad to think the way we were taught to teach. No wonder my children had learning gaps. When I was ready to teach tens and ones, I now realize that half of them probably could not identify 12 and 20 or they would confuse these. I remember telling parents that their children just needed to memorize the facts. Well, no wonder my children didn't get it.

The changed perspective of children constructing knowledge effected changes in MITs planning for instruction and the pacing of instruction.

I have taught math for 24 years. I used to just follow the manual. I really didn't know if the kids had the basics before I started teaching something new. You were so limited, I had so much to teach in such a short time. Now I use the assessments and that guides my instruction. This year I take as long as it takes to make sure they get it.

Many of the MITs recognized that their earlier limited knowledge of how children developed early numeracy limited their instructional practices to rote memorization and modeling. Overwhelming, MITs reported teaching with a great emphasis on developing understanding and less on the surface features of "doing as I show you."

Many MITs reported that their participation in the professional development increased their self-efficacy. "I feel like I did not know what I was doing with the math. And I thought I was a good math teacher. Now I feel I am more capable of working with the struggling students and I have more confidence now." It was a common occurrence for MITs with 20 or more years of teaching experience express how changes in their teaching were resulting in an increased sense of efficacy and renewed enthusiasm for teaching.

Discussion

The professional development associated with the KCM initiative went beyond "adding" knowledge and skill to transforming MITs' knowledge, beliefs, and instructional practices about mathematics teaching and learning with struggling students from kindergarten through third grade. This was achieved through the design of professional learning that drew on the research literature on effective professional development—a focus on mathematical knowledge for teaching related to the MITs job responsibilities that was ongoing and sustained and situated in a culture of collaboration and support. This resulted in increased MIT competencies, changes in instructional beliefs and practices, and increased student achievement.

MITs acquired knowledge of the stages of numeracy, characteristics of the various stages in the learning trajectory, and the instructional strategies appropriate for advancing student development along the learning continuum. The training and ongoing support activities provided an authentic lens for understanding their students' learning challenges and deepened their understandings of content and pedagogy. By situating analysis and planning in classroom practice, teachers were able to connect and implement ideas from current research in their instructional practices. The gap between professional development sessions provided the teachers an opportunity to reflect on the professional literature and the authentic cases presented in their teaching. In many ways, this gap served as a bridge to what they learned in their training sessions to what

their students were actually doing. This resulted in MITs who were students of numeracy content and pedagogy and could structure learning experiences based on assessment of student thinking. The combination of deepened knowledge of how children develop early numeracy and an understanding of how formative assessment, discussions, and manipulatives support student learning resulted in significant changes in the MITs approach to teaching early numeracy. They moved from reliance on the textbook to reliance on professional knowledge and student thinking and became “teacher engineers.”

The requirement of ongoing participation in professional learning as a part of the MITs’ work enabled *all* MITs to deepen their knowledge and understanding over a period of three years while being supported as they implemented changes in teaching. These supports provided MITs the opportunity to discuss ongoing questions and challenges they had in changing their approach to teaching early numeracy.

MITs and facilitators of the professional development worked collaboratively and the lines between expert and novice were blurred. They created a culture wherein professional relationships were valued and promoted the principles of: 1) collegiality and collaboration; 2) everyone engaged as active learners; 3) learning is ongoing. As a result, MITs felt secure enough to share individual struggles and reflect on their learning with collegial support. This created a community who became ongoing learners of student thinking and numeracy.

Implications

This study suggests implications for implementing broad scale reform efforts designed to strengthen mathematics teaching and learning in primary grades through intervention strategies. It provides insights on how MITs can strengthen the mathematics achievement of struggling students through early and focused interventions and collaboration with the classroom teacher. It provides further and compelling evidence that job-embedded sustained learning, a culture of collaboration and exploration, and focus on deepening teachers’ understanding of the specific mathematics content and pedagogy related to one’s teaching are critical features of professional development designed to achieve these goals. It also provides insights that these interventionists need strong knowledge of how children develop early numeracy and the opportunity to develop a strong practice as interventionists in order to have an impact.

The study provides insights that may increase the efficacy of other school-wide and district-wide professional development initiatives. Careful consideration and planning are needed to identify the conditions and culture that provide the necessary structure and support for success. The content focus of the training sessions needs to ensure depth of knowledge growth and flexible application of the knowledge to meet the diverse needs of students. The results of the professional learning should result in transformative practices and beliefs and empower the teacher to be the architect and engineer of student learning.

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