NCSM Journal

of Mathematics Education Leadership

SPRING 2015 VOL. 16, NO. 1

National Council of Supervisors of Mathematics www.mathedleadership.org

Table of Contents

CCOMMENTS FROM THE EDITOR 1
Angela T. Barlow, *Middle Tennessee State University Murfreesboro, Tennessee*

MATH TEACHERS' CIRCLES AS A FORM OF PROFESSIONAL DEVELOPMENT: AN IN-DEPTH LOOK AT ONE MODEL 3
Diana White, *University of Colorado Denver*

NORMS FOR TEACHERS' DISCUSSIONS OF STUDENTS' MATHEMATICS IN PROFESSIONAL DEVELOPMENT SETTINGS 12
Cyndi Edgington, *North Carolina State University*
Paola Sztajn, *North Carolina State University*
P. Holt Wilson, *The University of North Carolina at Greensboro*
Marrielle Myers, *Kennesaw State University*
Jared Webb, *The University of North Carolina at Greensboro*

THE ESSENCE OF FORMATIVE ASSESSMENT IN PRACTICE: CLASSROOM EXAMPLES 19
Marjorie Petit, *Marge Petit Consulting, MPC*
Mary Bouck, *Michigan State University*

ENVISIONING THE ROLE OF THE MATHEMATICS TEACHER 29
Charles Munter, *University of Pittsburgh*

THE PERSPECTIVES OF TEACHER LEADERS ON MATHEMATICS, LEARNING, AND TEACHING: SUPPORTING REFORM-ORIENTED INSTRUCTION 41
Michelle T. Chamberlin, *University of Wyoming*
Melissa L. Troudt, *University of Northern Colorado*
Reshmi Nair, *Hood College*
Alisa Breitstein, *University of Northern Colorado*

INFORMATION FOR REVIEWERS **55**

NCSM MEMBERSHIP/ORDER FORM **56**

Math Teachers' Circles as a Form of Professional Development: An In-depth Look at One Model

Diana White, *University of Colorado Denver*

Abstract

In this paper, I discuss one chapter of a growing professional development program that aims to improve the quality of mathematics education for students by developing middle level mathematics teachers' content knowledge and problem-solving skills, as well as their facility with applying the Standards for Mathematical Practice. I provide context within the national program, then discuss how the local chapter emerged, describe in detail the summer workshop and associated academic year sessions, discuss outcomes from the program, and provide information regarding how to start a similar program.

Introduction

In the summer of 2009, the University of Colorado Denver and the St. Vrain Valley School District in Longmont, Colorado began a partnership designed to strengthen the problem-solving skills and mathematical habits of mind of middle school mathematics teachers. We wanted teachers to have a venue to work on their mathematical problem solving and to develop their mathematical habits of mind while simultaneously building their capacity to implement rich mathematical tasks in the classroom. After the initial summer workshop, the project expanded to include teachers from a variety of school districts, mostly in the Denver metropolitan area.

For over four years now, this program, known as the Rocky Mountain Math Teachers' Circle Program, has provided professional development for teachers in which they engage in the process of doing mathematics with guidance from university mathematicians. We recognize that an unfortunate side effect of how mathematics has traditionally been taught at both the K-12 and collegiate levels is that many teachers have never had the opportunity to truly explore mathematics using the same disciplinary-specific habits of mind that research mathematicians use on a daily basis. That is, they have not had the opportunity to explore, question, conjecture, create examples, generalize, and communicate mathematically (Conference Board of the Mathematical Sciences [CBMS], 2012).

Yet with the introduction of the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 2000) and the Common Core State Standards for Mathematics (Common Core State Standards Initiative [CCSSI], 2010), teachers are being asked to develop skills (e.g., the Standards of Mathematical Practice) in their students that they themselves may never have had the opportunity to develop (CBMS, 2012). As such, we take the approach that teachers will benefit in the classroom from focusing on their own development of these skills as learners. Since we know that teachers' instructional practices often reflect their own learning experiences, we provide "the opportunity to experience firsthand a form of teaching that facilitates and supports learning" (Smith, 2001, p. 43).

Since inception, the Rocky Mountain Math Teachers' Circle Program has served approximately 150 teachers from 25 districts, including numerous mathematics coaches and others with informal leadership roles (e.g., mathematics department chairs). Each summer, we hold a one-week summer immersion workshop. In addition, academic year sessions are held on Saturday mornings approximately once per month. Graduate credit or continuing education units are available to participants.

Background

The Rocky Mountain Math Teachers' Circle Program is part of a national network of Math Teachers' Circles organized through the American Institute of Mathematics (AIM), headquartered in Palo Alto, California. AIM is one of eight mathematics research organizations in the United States. While AIM predominantly provides a venue and structured format for research mathematicians to come together to further their own mathematical research, outreach via Math Teachers' Circles (MTCs) is a key part of AIM's efforts. The first MTC began at AIM in 2006, as an offshoot of a local math circle for K-12 students (American Institute of Mathematics, n.d.). Since then, MTCs have spread rapidly, thanks to the extensive efforts of AIM and a core group of dedicated mathematics professionals. There are now approximately 57 active MTCs across the country, with approximately 12 new teams attending training each summer at one of the two weeklong training workshops.

The first MTC was intended solely to provide a venue for teachers to explore exciting mathematics, much as they were observing their students do in the math circles for students (Donaldson, Nakayame, Umland, & White, 2014). This initial intent remains, but MTCs have evolved substantially over time, and they now align more closely with many fundamental tenets of mathematics professional development, including those of Desimone (2009) and Guskey (2003).

The Rocky Mountain Math Teachers' Circle Program began in the summer of 2009, when a five-member leadership team attended a weeklong training session entitled "How to Run a Math Teachers' Circle." It was organized by AIM, sponsored by the National Science Foundation, the National Security Agency, and the Mathematical Association of America (MAA), and hosted at the MAA's headquarters in Washington D.C. The leadership team was intentionally diverse, consisting of a mathematician and statistician from the University of Colorado Denver, and the district mathematics coordinator, a high school teacher, and a middle school teacher from the St. Vrain Valley School District.

Throughout the week, mornings were spent engaging in problem-solving activities (i.e., sample MTC activities and sessions) with seven different leadership teams, while afternoons were spent in structured planning for each team's local MTC. Although all MTCs share certain basic properties, each MTC is also tailored to meet the needs of the community that it serves.

There was ample time for the team to bond and get to know one another over meals and evening excursions in the D.C. area. This training was the first time that the entire team was together, and for some, the first time that they were meeting each other. In hindsight, this opportunity to spend an intense week immersed in training and preparing to run a MTC was pivotal in shaping the program and developing the professional and personal relationships to work effectively together and to develop a high quality MTC. Following this initial meeting, the team continues to meet for monthly planning over dinner at a central location to work further on fundraising, recruiting, specific activity planning, and other logistics.

The Rocky Mountain Math Teachers' Circle Program has evolved from this initial partnership with one district to a stand-alone professional development program open to teachers of mathematics from any district. While we focus primarily on middle-level teachers (i.e., grades 5-9), we have some dedicated high school teachers who attend, as well as an occasional elementary teacher. Rationale for attendance varies, with some teachers reporting that they attend because they feel like their district does not provide the math-specific professional development that they need or want. Others simply love to engage with and explore the mathematics. Several regular attendees teach in non-traditional settings (e.g., juvenile rehabilitation facilities, credit recovery alternative schools, charter schools). However, most teach in a traditional middle school setting.

Although the entire leadership team played important roles in developing the initial workshop and program, the high school teacher and the mathematician have emerged as the program co-directors. The high school teacher, with her expertise on assessment and on leading professional

development, has taken the lead on the overall structure of the workshop, to include community building, establishing group norms, and various logistics related to the physical set-up of the space. Given her role at the university and her content expertise, the mathematician has assumed overall leadership of the project as program director, handling local logistics such as graduate credit, fundraising outside the initial partner district, and overall content planning.

Summer Immersion Workshops

The summer immersion workshops, held each summer since 2010, last five full days and have each supported 12 to 25 teachers. Most have been held on the University of Colorado Denver campus, and have been fully funded through grants from the National Science Foundation and other foundations. With the exception of the first summer, all workshops have been widely advertised and open to teachers from any district. We estimate that approximately half of the participants attend a single summer workshop only, with the other half attending for at least one year, to include both academic year sessions and additional summer workshops.

Typically there are 3-5 facilitators who lead sessions throughout the week, providing participants with a variety of different styles of facilitation as well as a selection of diverse topics. These facilitators typically have significant experience working with teachers and a substantial mathematics content background. Most are mathematicians, though other members of the leadership team have led sessions throughout the week as well.

On the first day of the workshop, we openly acknowledge that the participants are in a dual role as both learners and teachers of mathematics. As Tassell and colleagues (2011) noted, they are engaged in "teacher learning through a bifocal lens" (p. 44). Specifically, we ask them to spend the first four days of the workshop focusing on their role as active learners of mathematics, and assure them that on the last day, we will connect what they have learned throughout the week to their role as teachers of mathematics. A general description of the week is provided in the sections that follow.

Day 1 - Morning

The first morning is spent setting the tone and developing the community for the week. We aim to create an atmosphere where all participants feel that their mathematical thinking is valued and that the other participants support and respect their personal learning. This is particularly important as participants' background can vary tremendously, from someone trained initially as an elementary teacher to someone with a bachelor's degree in mathematics.

To accomplish this, we complete two carefully constructed activities. First, after a brief introduction to the facilitators and announcements about logistics for the week, participants spend approximately 45 minutes creating community agreements for the week. These are developed through the following three questions:

> What are the characteristics of a problem solver?
>
> What are the characteristics of an effective group?
>
> What are the characteristics of an active listener?

After this, we begin our second activity that focuses on building community and trust amongst the participants. To accomplish this, we pose the first mathematical problem of the week. One problem that we have used repeatedly follows.

> In a crazy New York apartment building there are seven elevators, each stopping at no more than six floors. It is possible to get from any one floor to any other floor without changing elevators. What is the maximum number of floors in the building? (Konhauser, Velleman, & Wagon, 1996, p. 42).

In selecting the initial problem we are careful to choose one that is easy to state and understand, requires minimal mathematics background needed to begin to explore it, and allows for a variety of approaches that can lead to significant progress toward an answer. In the case of this particular problem, trial and error readily provides lower bounds for the number of floors, but finding the maximum number possible is considerably more challenging.

From the start of the workshop, we want group members to initially develop their own mathematical approaches and ideas, articulate them and have them heard by others, and examine various approaches. Thus, participants are asked to work individually for at least 20 minutes, before the facilitator has them share out within the others at their table using a round robin format. Each table group, generally consisting of four people, is asked to make a poster that includes the original thinking of all of the group members.

A random choice of presenter for each group then shows all participants that there is an expectation that each individual listen to and absorb the various ideas from others.

Day 1 Afternoon – Day 4: Main Workshop Sessions

Beginning with the first afternoon and continuing through Days 2-4, the workshop focuses on providing participants with experiences to engage with intense, cognitively demanding mathematics. To ensure that participants experience lots of different approaches to mathematics throughout the week, groups are randomly assigned each day.

Mornings are composed of one long mathematical exploration (3-3.5 hours), while afternoons are spent with two shorter sessions (1.5 hours each). At least one, sometimes two or more, topics are intentionally developed across multiple days. The reasons for this include: developing participants' comfort level with leaving problems unanswered; supporting participants in recognizing that often in mathematics there is no quick answer; and helping participants to realize that struggle and exploration are ongoing.

During these sessions, we intentionally choose topics and problems from across the mathematical spectrum, ensuring that we include diverse areas such as geometry, probability and statistics, discrete mathematics, and topics related to number systems. Often topics overlap several areas of mathematics. Sometimes sessions start with a specific question, such as one of the following:

1. Write numbers from 1 to 100 on the board. Select any two of the numbers, erase them, and write on the board the sum plus the product of the two numbers. For example, if you erased 2 and 5, the sum plus the product is 7 plus 10, or 17, and so you write a 17 on the board. Now there are two 17s, but that's OK. Repeat this process of selecting two numbers and replacing them with their sum plus their product. What are the possible outcomes?

2. A 3 x 3 x 3 cube is made up of 27 smaller 1 x 1 x 1 cubes. Each of the smaller cubes are painted such that the 27 cubes can be assembled to create an all blue larger cube. Then, they can be reassembled so that they can create an all red larger cube. Finally, the 27 little cubes can be taken apart and reassembled to create an all white larger cube. How could the 27 little cubes be painted in order for this to happen?

Other times, sessions surround a specific topic, for example, combinatorial games. A variety of games were introduced over several days, with participants given time to explore each game and work toward finding a so-called winning strategy (i.e., a strategy whereby if both they and their opponent make the best possible move at each turn, then they are guaranteed to win).

Another popular extended topic that has been used in several of the workshops is *Exploding Dots*, which investigates many of the basic ideas of place value and standard algorithms for arithmetic and algebra in a novel way. Tanton (n.d.) has a wonderful video exposition of this topic. Several shorter topics have investigated diverse topics such as symmetries of plane figures, ways to tile the plane, logic puzzles, and topics from probability and statistics.

Last Day

The last day of the summer immersion workshop begins with returning to the mathematical problem from the first day. Participants are provided additional time to work on it in their small groups for the day, and then report out to the large group. They are amazed to see how far they have progressed from that first day, and how much more versatile they are in their mathematical thinking.

As a way to encourage participants to reflect on their experience and learning from the week, the second and final activity of the morning is for the participants to make a poster, containing only symbols and pictures, which represents their journey for the week. Each small group presents their poster to the larger group.

The afternoon is dedicated entirely to connecting what participants have learned to their classrooms. Participants discuss the Standards for Mathematical Practice (CCSSI, 2010), which have been referenced throughout the week, as well as the 21st Century Skills (Partnership for 21st Century Skills, 2008). They watch a TEDxNYED video (Meyer, 2010) and read a chapter from the book, *The Courage to Teach* (Palmer, 1997).

Transitioning to concrete plans, participants are asked to choose three things from the week that they would most like to infuse in their classroom during the first semester of the upcoming academic year. Choosing one, they describe how it would look at the end of the semester, the end of the first quarter, and the first three weeks of the semester. They then make a to-do list of things that they

need for the plan. These could include a timeline, prompts to use, manipulatives, professional development, or support from others.

Summer Immersion Workshop Outcomes

Our hope was that, at the end of a weeklong immersion workshop, participants would have increased comfort level with open-ended mathematical problems, increased self-efficacy related to mathematical problem solving, increased content knowledge, increased mathematical knowledge for teaching, and a stronger desire to implement more student-centered mathematics into their classroom. Measuring all of this has proven to be a challenge. We have used three primary sources for evaluative data, which are described in the sections that follow.

Workshop Surveys

To measure the outcomes of the workshop, we have used end-of-workshop surveys. On these, self-report participant gains can be loosely separated into gains as a *learner* of mathematics and gains as a *teacher* of mathematics. In their role as learners, many commented that they were challenged by both the content and problem solving, and that they had not previously been asked to work collaboratively to this extent on mathematics. They also commented that they felt incredibly supported by the various facilitators throughout the week and that they see the value in observing how the various facilitators lead sessions, taking ideas or even specific mathematics problems back to their own classroom setting or to their mathematics club.

In their role as teachers, participants commented that they intend to require more justifications and explanations from students. They also plan to incorporate more group work, more open-ended problems and problems requiring exploration, and more mathematical discussions into their classrooms. They reported that they were able to learn teaching strategies such as effective questioning techniques by observing the instructional practices that the facilitators modeled. Although efforts to conduct case studies of participants' classrooms are ongoing (e.g., Donaldson et al., 2014), it should be noted that this self-report data alone is insufficient for drawing conclusions about the classroom teaching practices of MTC participants.

Content Assessment

For two years, we administered a pre-post assessment known as the Learning Math for Teaching assessment (Hill, Schilling, & Ball, 2004), which measures aspects of what is referred to as *mathematical knowledge for teaching* (Ball, Thames, & Phelps, 2008). This phrase refers to the mathematics specifically needed to teach mathematics, as opposed to the mathematics commonly needed in other professions that use mathematics, like science and engineering (Hill et al., 2004). Teacher performance on this instrument has been linked to student achievement (Hill, Rowan, & Ball, 2005). Both years, our participants showed statistically significant gains on the version that we gave, which focused on number concepts and operations at the middle level (White, Donaldson, Hodge, & Ruff 2013).

Facilitator Observations

Finally, the facilitators and co-directors debrief at the end of each day and at the end of the week. Our observations indicate that participants are developing perseverance, an openness to try problems that may have intimidated them before, communication skills, and a trust in their own mathematical reasoning.

Academic Year Workshops

Recognizing that teacher professional development needs to be sustained (Darling-Hammond & McLaughlin, 1995), we have offered approximately 7-9 sessions each academic year. Each meets on a Saturday morning for approximately 3.5 hours, with a free lunch immediately following. These sessions follow the same spirit as those of the summer, with a variety of different facilitators throughout the academic year.

Participants report that their ongoing participation helps keep them thinking mathematically throughout the year. There are several schools in which multiple teachers have committed to attending as a team for at least a semester. They then report co-planning and discussing what they have learned together at their schools. This is an area that we would like to study in more depth, as one of the tenants of effective professional development is collective participation (Desimone, 2009). Overall, most participants who attend more than one or two workshops attend regularly for approximately two years, with a few outliers having attended almost all four years of academic year sessions and summer workshops.

Effective Professional Development

The MTC model addresses the five criteria for effective professional development identified by Desimone (2009). A description of each follows.

Content Focus

MTC activities are centered on rich, open-ended problems with multiple entry points. Although the problems can be stated in such a way that a middle or high school student could understand them, some are rich enough that aspects of them are the subject of active mathematical research. Mathematicians are centrally involved in selecting problems and leading sessions to ensure participants' access to deep content contextualized within the mathematical process.

Active Learning

Participants are involved in active problem solving for the majority of each MTC session, with small group work and whole group discussions occupying the majority of each mathematics session.

Coherence

The activities of a MTC are designed to directly support participants' development of the habits of mind described in the Standards for Mathematical Practice (CCSSI, 2010). MTCs intentionally support participants in developing at least six of the eight standards, including the ability to (1) make sense of problems and persevere in solving them, (2) reason quantitatively and abstractly, (3) construct viable arguments and critique the reasoning of others, (5) use appropriate tools strategically, (6) attend to precision, and (7) look for and make use of structure. While the specific content addressed in any given MTC varies, the focus on one or more of these critical mathematical practices is always present.

Duration

Participants can attend MTCs for multiple years. Each participant engages in approximately 35 hours of professional development during each intensive summer workshop and between 21-28 hours during each academic year of participation.

Collective Participation

The MTC model builds a community among participants, provides a natural way for mathematicians to become involved in K-12 education and form meaningful long-term partnerships with teachers, and engages participants in the larger mathematical community.

Forming a Math Teachers' Circle – A National Community of Support

There is now a well-formed support network for those interested in starting a Math Teachers' Circle. AIM runs the national Math Teachers' Circle Network (mathteacherscircle.org) and provides resources both for existing MTCs and those interested in starting a new MTC. They can help connect interested district personnel with mathematicians at a local institution of higher education to explore forming a team to attend the weeklong "How to Run a Math Teachers' Circle" training workshop.

Each morning during this training workshop, experienced national MTC facilitators lead sample MTC sessions with the workshop participants acting as learners. In doing so, the teams explore a variety of problems that they could, in turn, use for their own MTC sessions, as well as see highly qualified MTC facilitators model sample sessions. This is especially important for those mathematicians who may have minimal, if any, prior experience working with teachers, as there can be a steep learning curve associated with learning to lead sessions effectively. Partnering with a teacher from the leadership team to co-develop sessions is one way in which some teams have worked together to develop and implement sessions effectively.

During the afternoons, teams work together to develop their own logistical plan for their MTC, including to define roles, learn about and plan for funding opportunities, write their own mission and vision statements, and make a concrete plan for starting their own MTC.

These training workshops have been quite successful, with over 85% of teams who have attended a training workshop successfully starting their own MTC. A variety of smaller seed grants ranging from $1500-$2000 have been available for the past few years to help new MTCs get started, and most have been able to find state or private foundation funding as well. In some states, Math-Science Partnership Grants or Improving Teacher Quality Grants ranging from $30,000 to $90,000 have been awarded for various MTC programs. The aforementioned website included a wide variety of materials that can be used to aid MTCs in securing funding, leading sessions, and gathering evaluation

data. Facilitators from various MTCs regularly visit other MTCs to act as guest facilitators, thereby further spreading knowledge and experience. There is also a national listserv open to all MTC leadership teams that is used as a forum for communication and sharing of information.

The national MTC community is welcoming and growing, with no shortage of interested and experienced people willing to help support new MTCs.

Conclusion

The purpose of the Rocky Mountain Math Teachers' Circle Program is to improve the quality of mathematics education for students, specifically by developing middle level mathematics teachers' content knowledge and problem-solving skills, as well as their facility with applying the Standards for Mathematical Practice (CCSSI, 2010). The program supports teachers by providing a variety of experiences for teachers to engage in learning mathematics though this active approach with authentic engagement in mathematical problems under the direction of professional mathematicians.

It is our hope that the impact of this program goes beyond participating teachers and their students, and that teachers take lessons learned back to their schools and districts to share with colleagues. In that way, they become informal teacher-leaders and the impact of the program is magnified. Research on the program at the national level is ongoing (e.g., Donaldson et al., 2014; White et al., 2013; White & Yow, in press), and other local Math Teachers' Circles are beginning to disseminate their programs and outcomes as well (e.g., Geddings, White, & Yow, 2015). Preliminary data analysis shows that the program, both at the local and national level, does have this effect on some teachers.

The program has evolved over time, and a variety of supplemental workshops have been developed to help further connect the mathematical learning with participant's classroom teaching. After four years, the Rocky Mountain Math Teachers' Circle program is still going strong, and we hope to report back in several years with more successes and lessons learned. ✪

References

American Institute of Mathematics. (n.d.) *Math Teachers' Circle network.* Retrieved on December 1, 2014, from http://www.mathteacherscircle.org/

Ball, D. L., Thames, M. H., & Phelps, G., (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59,* 389-407.

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics.* Washington, DC: National Governors Association Center for Best Practices and Council of Chief State School Officers. http://www.corestandards.org.

Conference Board of the Mathematical Sciences. (2012). *The mathematical education of teachers II.* Providence, RI: American Mathematical Society.

Darling-Hammond, L., & McLaughlin, M. (1995). Policies that support professional development in an era of reform. *Phi Delta Kappan*, *76*, 597-605.

Desimone, L. M. (2009). Improving impact studies of teachers' professional development: Toward better conceptualizations and measures. *Educational Researcher*, *38*, 181-199.

Donaldson, B., Nakayame, M., Umland, K, & White, D. (2014). Math teachers' circles: Partnerships between mathematicians and teachers. *Notices of the American Mathematical Society*, *61*(11), 1-8.

Geddings, D., White, D., & Yow, J. (2015). SCHEMaTC: The Beginnings of a Math Teachers' Circle, *Mathmate*, 37(1).

Guskey, T. R. (2003). What makes professional development effective? *Phi Delta Kappan*, *84*, 748-750.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, *42*, 371-406.

Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). *Developing measures of teachers mathematical knowledge for teaching.* Ann Arbor, MI: University of Michigan Study of Instructional Improvement.

Konhauser, J. D., Velleman, D., & Wagon, S. (1996). *Which way did the bicycle go?: And other intriguing mathematical mysteries.* Washington, DC: Mathematical Association of America.

Meyer. D. (2010). *Math class needs a makeover.* Retrieved December 29, 2013 from http://www.youtube.com/watch?v=NWUFjb8w9Ps.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

Palmer, P. (1997). *The courage to teach: Exploring the inner landscape of a teachers' life.* Indianapolis, IN: Wiley & Sons.

Partnership for 21st Century Skills (2008). *21st century skills, education & competitiveness: A resource and policy guide.* Tucson, AZ: Author.

Smith, M. S. (2001). *Practice-based professional development for teachers of mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Tanton, J. (n.d.) *Exploding dots.* Retrieved December 30, 2013 from http://gdaymath.com/courses/exploding-dots/

Tassell, J. L., Marchionda, H., Baker, S., Bemiss, A., Brewer, L., Read, K., . . . Woods, D. (2011). Transformational professional development: Teacher learning through a bifocal lens. *NCSM Journal of Mathematics Education Leadership, 13*(2), 44-51.

White, D., Donaldson, B., Hodge, A., & Ruff, A. (2013). Impact of math teachers' circles on teachers' mathematical knowledge for teaching, *International Journal for Mathematics Teaching and Learning*, available online at http://www.cimt.plymouth.ac.uk/journal/default.htm.

White, D., Umland, K., Donaldson, B., Nakayame, M., & Conrey, B. (2014). Effects of problem solving workshops on mathematical knowledge for teaching. *Proceedings of the 38th International Conference on Psychology of Mathematics Education.*

White, D., & Yow, J. (2015). Math teachers' circle: Connections to teacher leadership. *Proceedings of the 18th Annual Conference on Research in Undergraduate Mathematics Education.*