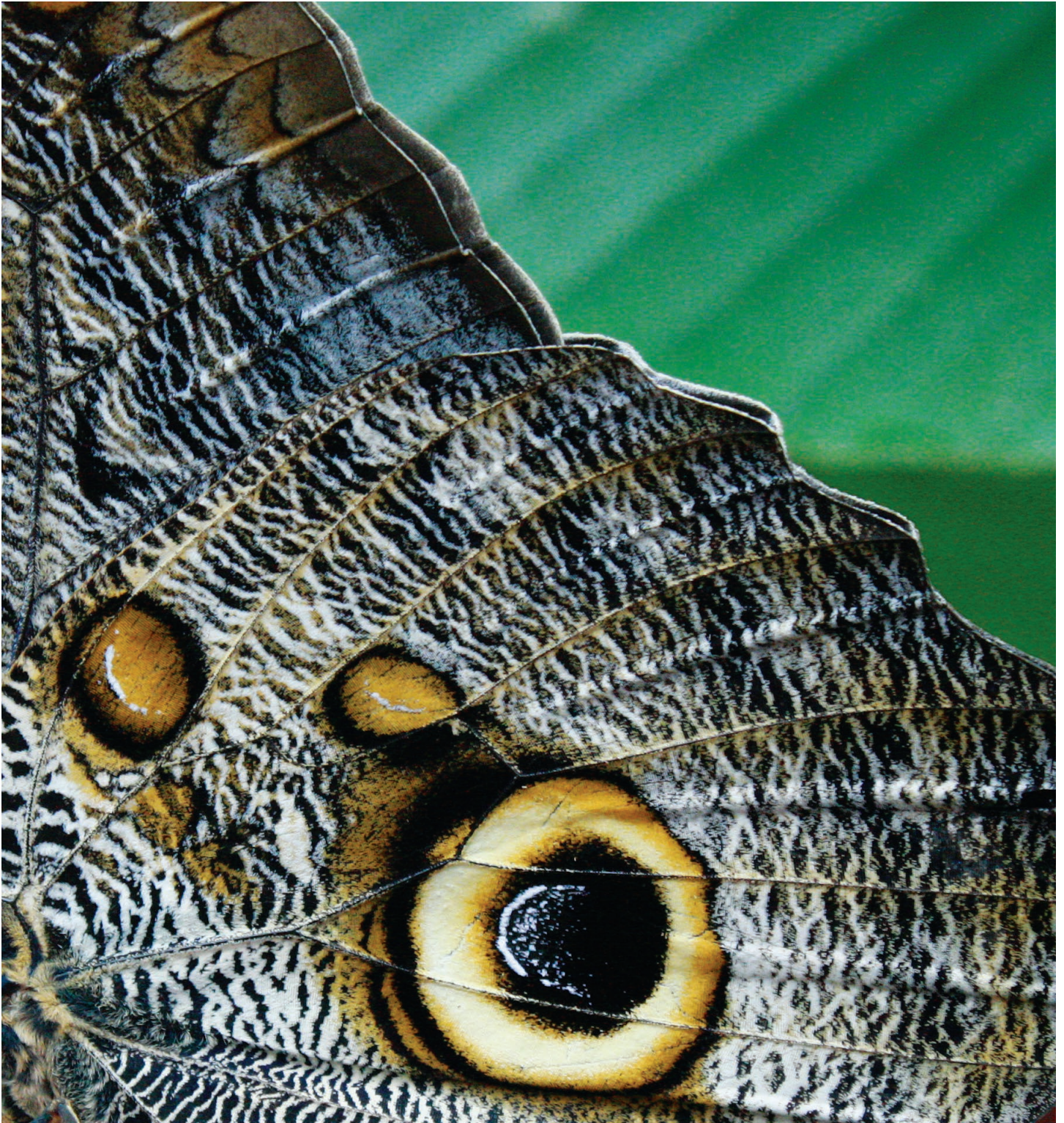


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Table of Contents

CCOMMENTS FROM THE EDITOR	1
<i>Angela T. Barlow, Middle Tennessee State University Murfreesboro, Tennessee</i>	
MATH TEACHERS’ CIRCLES AS A FORM OF PROFESSIONAL DEVELOPMENT: AN IN-DEPTH LOOK AT ONE MODEL	3
<i>Diana White, University of Colorado Denver</i>	
NORMS FOR TEACHERS’ DISCUSSIONS OF STUDENTS’ MATHEMATICS IN PROFESSIONAL DEVELOPMENT SETTINGS	12
<i>Cyndi Edgington, North Carolina State University</i>	
<i>Paola Sztajn, North Carolina State University</i>	
<i>P. Holt Wilson, The University of North Carolina at Greensboro</i>	
<i>Marrielle Myers, Kennesaw State University</i>	
<i>Jared Webb, The University of North Carolina at Greensboro</i>	
THE ESSENCE OF FORMATIVE ASSESSMENT IN PRACTICE: CLASSROOM EXAMPLES	19
<i>Marjorie Petit, Marge Petit Consulting, MPC</i>	
<i>Mary Bouck, Michigan State University</i>	
ENVISIONING THE ROLE OF THE MATHEMATICS TEACHER	29
<i>Charles Munter, University of Pittsburgh</i>	
THE PERSPECTIVES OF TEACHER LEADERS ON MATHEMATICS, LEARNING, AND TEACHING: SUPPORTING REFORM-ORIENTED INSTRUCTION	41
<i>Michelle T. Chamberlin, University of Wyoming</i>	
<i>Melissa L. Troudt, University of Northern Colorado</i>	
<i>Reshmi Nair, Hood College</i>	
<i>Alisa Breitstein, University of Northern Colorado</i>	
INFORMATION FOR REVIEWERS	55
NCSM MEMBERSHIP/ORDER FORM	56

Envisioning the Role of the Mathematics Teacher

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Abstract

This article presents a tool for identifying and tracking changes in teachers' (and others') evolving visions of the role of the mathematics teacher through five levels: motivator, deliverer of knowledge, monitor, facilitator, and more knowledgeable other. It includes a brief account of the rubric's development and a description of its use in a large-scale, longitudinal study of mathematics instructional reform efforts in four urban school districts, including an examination of relationships between the ways that teachers envision their roles and the quality of their instruction. The article concludes with a discussion of implications for mathematics education leaders, including a description of how the tool was used in a recent professional development effort.

Introduction

During the last two decades, mathematics educators have made considerable progress in describing the multiple facets and nuances of the role that effective mathematics teachers play in the classroom to support students in meaningfully participating in classroom mathematical activity (cf. Ball, Sleep, Boerst, & Bass, 2009; Chazan & Ball, 1999; Hiebert et al., 1997; Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013; Lobato, Clarke, & Ellis, 2005; Staples, 2007; Stein, Engle, Smith, & Hughes, 2008). Unfortunately, what has too often been omitted are descriptions of how teachers envision and enact their role

along the way to developing such professional proficiency. As Staples (2007) argued, research has focused on describing instruction “*only once the practices have been established*” (p. 164, italics in original), leaving those charged with supporting teachers' ongoing learning without a roadmap for doing so.

In this article, I present a rubric for identifying and tracking changes in teachers' (and others') evolving visions of the role of the mathematics teacher—from less to more sophisticated articulations of classroom practice. After describing the tool's origins, I report on its use in a large-scale, longitudinal study of mathematics instructional reform efforts in four urban school districts. Then, I describe the relationship between ways that teachers envision their roles and the quality of their instruction in order to (a) make a case for the relevance and importance of attending to instructional visions, and (b) provide insight to potential users of the tool. Last, I discuss implications of this work for mathematics education leaders, including a description of how it was used in a recent professional development effort.

Considering Instructional Vision

Underlying the work described in this article are three assumptions about teacher professional development. First, teacher professional development occurs in a variety of settings, with a variety of resources and possible foci, including, among other things, co-planning lessons, examining student work, reading and discussing books or articles, watching and discussing video of teaching, peer observation, coaching

cycles, and formal evaluation and feedback. Second, across most of these settings and foci, it is through talk that the bulk of teacher learning is expected to happen. Third, supporting individuals in developing more sophisticated ways of describing aspects of their practice can influence what they see and do in their classrooms. As Sfard (2007) suggested, “We need a discursive change to become aware of new possibilities and arrive at a new vision of things. We thus often need a change in how we talk before we can experience a change in what we see” (p. 575).

I refer to teachers’ and others’ dynamic conceptions and articulations of their (future) practice (Hammerness, 2001; Senge, 2006) as *instructional vision*. It is this notion of instructional vision that is the focus of this article, and the ways that teachers envision their role in the classroom in particular. If we expect teachers’ talk about mathematics instruction to precede their enactments, then we need to attend not only to what teachers do in their classrooms, but also the ways they articulate their role and the vision they have for what they are striving to accomplish. Knowing the end goals or best practices, however, is not sufficient; we need to be able to anticipate the trajectories of growth that teachers’ conceptions and enactments of high-quality mathematics instruction might follow (Sherin, 2001), and then support them in moving along that pathway. Of course, *growth* implies that progress is defined with respect to a particular vision of the teacher’s role, which I summarize next.

The Role of the Mathematics Teacher

Over the last several years, mathematics education research has documented cases of teachers striving to support students in learning mathematics with understanding (cf. Boaler & Humphreys, 2005; Hiebert et al., 1997; Staples, 2007) and in developing the kinds of mathematical practices identified in the Common Core State Standards for Mathematics (Common Core State Standards Initiative [CCSSI], 2010). Such work has often described the teacher as a *co-participant* in authentic mathematical activity (Cobb & Yackel, 1996; Rogoff, Matusov, & White, 1996). This does not mean that students are engaged in *pure discovery learning* or that the teacher’s job is to merely keep students on task as they spontaneously reinvent the mathematics curriculum. Rather, the teacher plays a crucial role in each phase of a lesson, as well as the planning and reflection that precede and follow the lesson. Although far

more complex than can be described here, the role of the mathematics teacher can be at least partially defined with respect to three dimensions drawn from the research literature.

Lesson Structure

The first dimension involves structuring a lesson’s activity by employing a three-phase classroom activity structure (Van de Walle, Karp, & Bay-Williams, 2012). Within this structure, the teacher begins by posing a problem and ensures that all students understand the context and expectations (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013). Next, students develop strategies and solutions, typically in collaboration with each other. Finally, through reflection and sharing, the teacher and students work together to clarify the mathematical concepts underlying the lesson’s problem (Stein & Smith, 2011; Stigler & Hiebert, 1999).

Classroom Discourse

In considering the role of the mathematics teacher, a second dimension includes influencing classroom discourse, in which the teacher proactively supports students in participating in mathematical conversations (Fraivillig, Murphy, & Fuson, 1999). This influencing includes: eliciting students’ explanations and questions and then using those contributions as lesson content (Lappan, 1993; Staples, 2007; Stein & Smith, 2011); engaging *with* students in mathematical argument (Lampert, 1990); and choosing appropriate moments to share essential information such as conventional rules or symbols and alternative methods (Hiebert et al., 1997).

Mathematical Authority

A third dimension to consider in the role of the mathematics teacher involves sharing mathematical authority with students. This can be evidenced by the teacher consistently treating students as thinkers and decision-makers (Staples, 2007). In addition, the teacher ensures that students share in the responsibility for determining whether mathematical ideas and strategies are valid, rather than relying solely on the teacher or textbook (Simon, 1994).

This vision of the role of the teacher represents the top level of the rubric described below. The rubric, which establishes the goal of instructional reform efforts, was originally developed within a research project involving four large urban school districts. A description of this research project follows.

Methods

The overall goal of the larger study, the Middle School Mathematics and Institutional Setting of Teaching (MIST) project, was to investigate, test, and refine a set of hypotheses and conjectures about organizational support structures that enhance the impact of professional development on middle-grades mathematics teachers' instruction and student achievement. Working for four years with four urban school districts with ambitious goals for reforming math instruction provided opportunities to investigate teachers' (and others') evolving conceptions of high quality mathematics instruction in settings in which leaders were promoting change. To do so, I developed a series of rubrics for assessing visions of high quality mathematics instruction (VHQMI), including teachers' articulations of high quality classroom discourse, mathematical tasks, student engagement, and the role of the teacher. The latter of these is the focus of the rubric presented in this article.

A more thorough description of the development and application of all of the VHQMI rubrics is provided elsewhere (Munter, 2014). Here, I will provide a brief account of the development of the role of the teacher rubric and its use in scoring interviews. Then, I will describe my methods for examining the importance of considering how teachers envision their role.

Rubric Development

I developed the rubric based on analyses of more than 100 interviews conducted during the first two years of the MIST project with middle-grades mathematics teachers, coaches, principals, and district leaders. In those interviews, we asked participants:

If you were asked to observe another teacher's math classroom, what would you look for to decide whether the mathematics instruction is high quality?

Why do you think it is important to use/do _____ in a math classroom?

Is there anything else you would look for? If so, what? Why?

If the participant had not already described the role of the teacher, we asked:

What are some of the things that the teacher should actually be doing in the classroom for instruction to be of high quality?

Taking the research-based description of the role of the teacher summarized previously as the top level, I interpreted each interview response against that benchmark, looking for patterns indicating potentially important qualitative distinctions that could help model a developmental trajectory of the ways that teachers' and others' visions of the role of the mathematics teacher might change over time in settings in which instructional reform is being supported.

Especially useful in this analysis was research that has identified important variations in form- and function-relationships within mathematics instructional reform efforts (Saxe, Gearhart, Franke, Howard, & Crockett, 1999; Spillane, 2000). For example, Saxe et al. (1999) described how teachers might employ *new forms* of assessment, such as more open-ended questions, to serve the *old function* of evaluating the correctness of answers, rather than using the questions to diagnose students' thinking. Likewise, Spillane (2000) found that district leaders might describe the need for real-world connections in terms of making mathematics more relevant and engaging for students, failing to emphasize the function of providing meaningful contexts for students' sense-making and mathematical reasoning. Such distinctions proved useful in differentiating between less and more sophisticated descriptions of the role of the teacher. For example, as described further below, I identified differences in articulated functions underlying assertions that the teacher should act as facilitator or refrain from lecturing, emphases on group work, and descriptions of the place and purpose of students' talk.

This analysis resulted in the rubric that is presented in Figure 1. All but the lowest level (Level 0) are defined with respect to the three dimensions identified above: *conception of typical activity structure*, *influencing classroom discourse*, and *attribution of mathematical authority*. Although reading all of the level descriptions is likely necessary in order to understand the qualitative differences that the rubric is intended to capture, a summary of the primary conceptual distinctions, accompanied by sample participant quotes from our interviews, may orient the reader to the rubric's intent.

Motivator. At the lowest level (i.e., Level 0), an individual's description of the role of the teacher is limited to an assertion that the teacher must be energetic and captivating so that students will be sufficiently motivated to learn. "It is more about being an entertainer than it is a teacher." But "making connections [to students]" does not mean that they will learn mathematics.

Deliverer of knowledge. Level 1 descriptions emphasize that the role of the teacher is to teach mathematics. Specifically, at Level 1 an individual’s description suggests that the teacher has mathematical knowledge that must be imparted unto students, which requires very clear explanation. For example, according to one participant the “teacher provides clear instructions, clear assignment, examples shown, students being walked through a problem.” Others noted that if students have questions, “they should feel free to ask,” and that the teacher “should answer all student questions,” including “explain[ing] why and how it’s used in everyday life, not just formulas.”

Monitor. At Level 2, individuals’ descriptions of the role of the teacher suggest that students play an active role in working together on mathematical tasks and that affording time to students for figuring out or, more likely, reproducing what the teacher has explained or demonstrated is important. A typical description at this level was that the teacher should “show [students] examples [of] how to do it and why are they doing it, what is the purpose of it. Then, do the facilitation, walk around, see the group work.” Whereas at Level 1 the image of students’ role is one of receiving knowledge, at Level 2 students play a role in mediating what the teacher has explained. Individuals

FIGURE 1. *VHQM Rubric: Role of the Teacher (continued on pg. 33)*

Level	Description	Potential ways of characterizing teacher’s role
4 Teacher as “more knowledgeable other”	Describes the role of the teacher as proactively supporting students’ learning through co-participation. Stresses the importance of designing learning environments that support problematizing mathematical ideas, giving students mathematical authority, holding students accountable to others and to shared disciplinary norms, and providing students with relevant resources (Engle & Conant, 2002).	Influencing classroom discourse: Suggests that the teacher should purposefully intervene in classroom discussions to elicit & scaffold students’ ideas, create a shared context, and maintain continuity over time (Staples, 2007).
		Attribution of mathematical authority: Suggests that the teacher should support students in sharing in authority (Lampert, 1990), problematizing content (Hiebert et al., 1996), working toward a shared goal (Hiebert et al, 1997), and ensuring that the responsibility for determining the validity of ideas resides with the classroom community (Simon, 1994).
		Conception of typical activity structure: Promotes a “launch-explore-summarize” lesson (Madsen-Nason & Lappan, 1987), in which (a) the teacher poses a problem and ensures that all students understand the context and expectations (Jackson et al., 2013), (b) students develop strategies and solutions (typically in collaboration with each other), and (c) through reflection and sharing, the teacher and students work together to explicate the mathematical concepts underlying the lesson’s problem (Stigler & Hiebert, 1999).
3 Teacher as “facilitator”	Focuses on the forms of “reform instruction” without a strong conception of the accompanying functions that underlie those forms: either (a) views the teacher’s role as passive, as students discover new mathematical insights as the result of collaborative problem solving (e.g. “romantic constructivism”), or (b) describes a transitional view that incorporates both teacher demonstration or introduction (e.g., at the beginning of the lesson) and “turning it over” to the students (who then make the remaining “discoveries”). Description likely stresses “rules” for structuring lessons, discussion, etc., or describes posing problems and asking students to describe their strategies but does not detail a proactive role in supporting students in engaging in genuine mathematical inquiry (Kazemi & Stipek, 2001).	Influencing classroom discourse: Describes the teacher facilitating student-to-student talk, but primarily in terms of students taking turns sharing their solutions; hesitates to “tell” too much for fear of interrupting the “discovery” process (Lobato et al, 2005).
		Attribution of mathematical authority: Supports a “no-tell policy”: Stresses that students should figure things out for themselves and play a role in “teaching.” Suggests that if students are pursuing an unfruitful path of inquiry or an inaccurate line of reasoning, the teacher should pose a question to help them find their mistake, but the reason for doing so focuses more on not telling than helping students develop mathematical authority. Is open to students developing their own mathematical problems, but these inquiries are not candidates for paths of classroom mathematical investigation.
		Conception of typical activity structure: Promotes a “launch-explore-summarize” lesson (Madsen-Nason & Lappan, 1987), in which (a) the teacher poses a problem and possibly completes the first step or two with the class or demonstrates how to solve similar problems, (b) students work (likely in groups) to complete the task(s), and (c) students take turns sharing their solutions and strategies and/or the teacher clarifies the primary mathematical concept of the day (i.e., how they “should have” solved the task).

who envision such a role of the teacher might suggest that students who “get it” should be invited to (re)teach their classmates. “Having a kid who’s really good at the math, but who’s still at their [peers’] level, sometimes they can explain it a little bit better [than the teacher].” Still, at this level, it is the teacher’s job to identify and correct students’ misconceptions by intervening directly. As one participant suggested, if students are pursuing a solution path that looks like a dead end, “the teacher needs to circle the wagons, regroup, ‘Oh guys this is not working out. We need to back up cause, cause we’re going the wrong way.’”

Facilitator. A Level 3 envisioning of the teacher’s role marks an important shift in who does the mathematical work in the classroom. At this level, an individual describes the teacher’s role as facilitating students’ sense making during at least part of the lesson, and this can be done in one of two ways. First, an individual may envision a *passive* role of the teacher, in which students collaborate to discover the lesson’s main ideas. “The kids are pretty much teaching themselves; the teacher’s just kind of up there facilitating and making sure that their light bulbs are turning on.” Alternatively, an individual may describe a

FIGURE 1. *VHQM Rubric: Role of the Teacher (cont. from pg. 32)*

Level	Description	Potential ways of characterizing teacher’s role
2 Teacher as “monitor”	Describes the teacher as the primary source of knowledge, but stresses the importance of providing time for students to work together, to try on their own and make sense of what the teacher has demonstrated, to (first) explain things to each other, and then get help from the teacher.	Influencing classroom discourse: Suggests the teacher should promote student-student discussion in group work.
		Attribution of mathematical authority: Suggests a view of teacher as an “adjudicator of correctness” (Hiebert et al, 1997). Students may participate in “teaching” but only as mediators of the teacher’s instruction, adding clarification, etc. If students are pursuing an unfruitful path of inquiry or an inaccurate line of reasoning, the teacher stops them and sets them on a “better” path.
		Conception of typical activity structure: Promotes a two phase, “acquisition and application” lesson (Stigler & Hiebert, 1999), in which (a) the teacher demonstrates or leads a discussion on how to solve a type of problem, and then (b) students are expected to work together (or “teach each other”) to use what has just been demonstrated to solve similar problems while the teacher circulates throughout the classroom, providing assistance when needed.
1 Teacher as “deliverer of knowledge”	Describes the teacher as the primary source of knowledge, focusing primarily on mathematical correctness and thoroughness of explanations (i.e., showing all steps). Description suggests that students are welcome to ask questions, but that there is no expectation that the teacher will facilitate student collaboration or discussion.	Influencing classroom discourse: Focuses exclusively on teacher-to-student discourse. Considers quality of teacher’s explanations in terms of clarity and mathematical correctness.
		Attribution of mathematical authority: Suggests that the responsibility for determining the validity of ideas resides with the teacher or is ascribed to the textbook (Simon, 1994). (This includes insistence that teachers be mathematically knowledgeable and correct.)
		Conception of typical activity structure: Promotes efficiently structured lessons (in terms of coverage) in which the teacher directly teaches how to solve problems. Periods might include time for practice while teacher checks students’ work and answers questions, but this is likely quiet & individually-based with no opportunity for whole-class discussion. Description suggests no qualms with exclusive lecture format.
0 Teacher as “motivator”	Suggests that the teacher must first and foremost be sufficiently captivating to attract and hold students’ attention.	

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transitional role in which, at most, the teacher introduces the task and does the first part or two with the class before “turning it over” to the students, and then keeps students on the “right path” by asking questions.

In either case, key ideas of the lesson are left to the students to figure out, rather than limiting students to reinterpreting what the teacher has already demonstrated or explained (Level 2). For example, one participant suggested that in a high-quality lesson, students are “not waiting all the time for the teacher [to] come and spoon feed them, but doing investigating on their own, coming up with ah-has on their own or coming up with ‘what if this?’” On its surface, such a description may represent a Level 4 envisioning of the teacher’s role. But what is key to consider is the function underlying that form. At Level 3, the rationale for actively engaging students in figuring problems out is not that it affords opportunities, for example, to become proficient in the mathematical practices specified in the Common Core (CCSSI, 2010), but rather that it “helps [students] remember it a little bit better than just a teacher up there talking about it.” Similarly, a Level 3 envisioning of the teacher’s role likely includes a commitment to “not telling” (e.g., the teacher should “answer questions with questions,” or, if students are headed “down the wrong path,” the teacher should “ask them something else to put them back on the right track”), but the rationale for which is not about supporting students in developing mathematical authority.

More knowledgeable other. At the highest level (described above), an individual describes the teacher’s role as proactive, co-participation with students, in which the teacher has a clear image of the instructional goals, and orchestrates, scaffolds, and builds on student contributions to achieve that goal. Distinct from Level 3, envisioning this kind of a role of the teacher requires acknowledging that students will likely not discover all of a lesson’s learning goals without purposeful work on the part of the teacher to support them in participating in solving problems and participating in productive discussions about their ideas, questions, and explanations. For example, one participant described how the teacher should play a proactive role in supporting and scaffolding students’ talk.

When [teachers] pose a question and a student answers, they don’t say “yes this is how it is always done.” They ask the kids to explain how they came up with the answer, ask for other students to explain how they came

up with the answer, present all the ideas to the student and ask them if these are good procedures for answering types of problems like this and talk about student preference—“Do you like one way more than another and does this way make sense?”—so that the kids can build their own frame of reference to the material.

Often accompanying such descriptions is a commitment to using students’ explanations, responses, questions, and problems as lesson content (Fraivillig et al., 1999): “Students should be involved in the learning process as far as asking questions and being able to maybe actually give examples and working them and talking to the teacher about them.” Such a perspective suggests that the teacher’s role includes keeping students positioned as thinkers and decision-makers (Staples, 2007), the underlying function of which is to support students’ engagement in mathematical practice. As one participant asked, “When kids are getting stuck, are you [the teacher] just pulling them out or are you asking those questions that press students to think even deeper so that they figure out the problem, that they become the problem-solvers?”

Interview Coding

The rubric for assessing individuals’ ways of envisioning the role of the teacher was applied to 932 transcripts of interviews conducted over the first four years of the MIST project, including 433 teacher interviews. Each relevant statement was scored according to the levels of the rubric, with a final score determined by the highest score that was assigned. For example, in an entire interview transcript, multiple statements might have been scored at level 2, but if just one statement was scored at level 3, the final role of the teacher score for the participant’s interview would have been a 3. The decision to score this way was both practically and conceptually motivated. The practical motivation stemmed from a need to establish rules for achieving sufficient reliability in coding nearly a thousand interviews. (Across all years combined, based on the 16% of transcripts that were double-scored, the overall rate of exact agreement in scoring with this rubric was 0.74.) More important, however, was the conceptual rationale. Because of the way the transcripts were coded, a score can be interpreted as representing the greatest level of sophistication with which a participant *was able to* describe the role of the teacher—not necessarily how the participant *typically* describes the teacher’s role (and likely not the role a teacher *actually plays* in her/his classroom).

Of course, this is not the *correct way* to use the tool, only how it was used in one large research study, the analyses of which are reported in the next section. In different settings, such as working to support the learning of a local group of teachers, it would likely be used differently. I return to this notion in the discussion section.

Statistical Analyses

The analyses that are the focus of this article were conducted in order to answer two research questions:

- 1) Do the ways that teachers (and others, including principals, coaches, and district leaders) envision the role of the mathematics teacher change over time in settings in which leaders are promoting models of instruction aligned with mathematics education research?
- 2) Is the sophistication with which teachers articulate the role of the teacher related to the quality of their instruction? If so, how?

Given that our participants were in districts in which leaders were actively pursuing change in ways aligned with the vision of mathematics instruction on which the rubric is based, my conjecture was that the sophistication with which teachers described their role in the classroom would increase over time. To determine whether this was the case, I examined both the average scores among all teachers combined for each year and, because any increases in scores could be attributed to changes in teaching staff or study participants, I also examined average scores among just the 44 participants (teachers and others) whose interviews were scored for role of the teacher in all four years.

To answer the second question above, using regression analysis I examined the relationships between teachers' scores on the role of the teacher rubric and an index of instructional quality. The quality of teachers' instruction was assessed with an adapted version of the Instructional Quality Assessment (Boston, 2012). The two primary sections of the IQA are designed to assess the cognitive demand of classroom activity over the course of the lesson (i.e., academic rigor) and specific aspects of discourse during the whole-class discussion after students have had a chance to work on solving the task (i.e., Accountable Talk®). Members of the research team used the instrument to score video-recordings of two consecutive

days of classroom instruction for each participating teacher in late winter of each year. Scores from eight IQA rubrics were combined to create two sub-scores, one pertaining to the cognitive demand of the mathematical task as posed and then as implemented, and one pertaining to class discussion. Additionally, these two sub-scores were averaged to create one annual, overall IQA score for each teacher. Each of these three scores—the task and discussion sub-scores, as well as the overall IQA—could range from 0 (low) to 4 (high), which, conceptually, maps roughly onto the range represented in the role of the teacher rubric.

Combining data across years, I calculated mean IQA scores for each level of the role of the teacher rubric. To do so, I used a two-level regression model to adjust for clustering within teachers. Including dummy variables for each level of the role of the teacher rubric and identifying each level as the base in multiple runs allowed me to test for significant differences in IQA scores between (consecutive) levels on the role of the teacher rubric. If the sophistication with which teachers describe their role in the classroom is associated with instructional quality, IQA means should increase with higher levels of the rubric.

Results

As listed in Table 1 (*see page 36*), the results of using the rubric to score interviews suggest that, on average, the sophistication with which teachers described their role in the classroom increased. In addition, average scores among just those 44 participants whose interviews were scored for role of the teacher in all four years increased as well, with some changes in consecutive years being statistically significant. This suggested that the increase was not attributable solely to fluctuations in district personnel or study participants.

Table 2 (*see page 36*) lists mean instructional quality scores (both overall IQA and IQA sub-scores) by level of the role of the teacher rubric, adjusted for clustering due to repeated observations across some teachers. Additionally, statistically significant increases between consecutive levels are noted. Generally, overall IQA scores increased as role of the teacher scores increased. The difference in IQA scores between teachers with a Level 2 role of the teacher score and those with a Level 3 score was statistically significant, as was the difference in IQA scores between teachers with role of the teacher scores of 3 and 4.

Table 1: Mean role of the teacher scores by year

Year	All teachers combined	Participants with role of the teacher score in all four years [n = 44]
1	1.85 (sd = 0.94) [n = 82]	1.98 (0.88)
2	2.23 (0.96) [111]	2.48** (0.95)
3	2.50 (0.71) [118]	2.75# (0.69)
4	2.63 (0.67) [122]	2.66 (0.75)

Wilcoxon signed rank tests comparing consecutive years' means for participants with role of the teacher score in all four years: ** p < 0.01; # p = 0.05

Examining the IQA sub-scores, however, provides at least two additional insights into this relationship. First, participants' task-related scores were, in general, higher than discussion-related scores. This is likely due, in part, to the fact that leaders in each of the districts had attempted to provide teachers with more inquiry-oriented curriculum materials, including the second edition of the Connected Mathematics Project series (CMP2; Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998) in three of the four districts. Simply using the tasks found in the district-provided curriculum would likely lead to higher task sub-scores. However, this does not explain the significantly higher scores among teachers with level 3 or 4 instructional visions of the teacher's role. As argued by Wilhelm (2014), this difference suggested that teachers with more sophisticated ways of envisioning their role were more likely to maintain a task's *potential* rigor in its *implementation* with students.

Second, although no difference in discussion sub-scores between consecutive levels on the role of the teacher rubric was statistically significant, there was an upward trend, including significant differences between teachers of Level 4 and Levels 1 and 2. Unlike the task sub-score, however, average discussion scores were higher among teachers who envisioned the role of the teachers as being primarily one of motivator (Level 0) than those of teachers with Level 1 or 2 instructional visions of the teacher's role. Although this finding should be treated cautiously, it does suggest

Table 2: Mean instructional quality (IQA) scores by role of the teacher level

Role of the teacher score	IQA Task	IQA Discussion	Overall IQA
0	2.50	1.76	2.09
1	2.50	1.62	2.07
2	2.58	1.63	2.10
3	2.69#	1.75	2.22*
4	2.97*	2.01	2.47*

Note: two-level regression analysis to test for differences between consecutive levels of role of the teacher rubric and adjust for clustering within teachers (425 observations across 223 teachers): *p < 0.05; # p = 0.07

that emphasizes on interpersonal and content-specific aspects of the teacher's role in teachers' instructional visions may relate differently to different aspects of their practice (i.e., those related to task choice and implementation, and those related to classroom discussion). In the following section, I discuss the implications of these results for potential users of the tool.

Discussion

In this article, I have presented a rubric that mathematics education leaders might use to identify and track changes in the ways that mathematics teachers envision their role in the classroom. It is intended to promote developmental approaches to supporting mathematics teachers' (and leaders') learning of high quality forms of practice (Stein & Matsumura, 2008), in that it provides a model for the pathways that the evolution in teachers' instructional visions (and possibly practice) might take. The findings reported in this article speak to the tool's validity. They suggest that the ways that individuals envision the role of the mathematics teacher in the classroom did change in settings in which such change was being promoted and supported. Also, the findings suggested that there is a positive relationship between the ways that teachers articulate their role and the quality of their instruction—an outcome of great importance to many stakeholders, considering the mounting evidence of the relationship between quality of teaching and student outcomes (cf. Nye, Konstantopoulos, & Hedges, 2004; Rockoff, 2004; Wilson et al., 2009). Before discussing the implications for mathematics education leaders, however, I wish to make two points of caution.

First, the rubric presented in this article was developed based on interviews with teachers in particular kinds of settings—four urban school districts whose leaders had formulated and begun implementing comprehensive initiatives for improving middle-grades mathematics instruction district-wide, including providing comprehensive professional development to teachers (and even principals) focused on placing students' reasoning at the center of instructional decision making, and adopting mathematics curricula aligned with such an agenda. The ways that teachers (re)envision their roles are likely highly influenced by the settings in which they work. The tool presented here was appropriately aligned for use in districts that participated in the MIST project; its levels may not align as well with different goals for instruction being promoted in other settings.

Second, the rubric was originally developed as a research tool, applied to transcripts of annual interviews with study participants. Although we observed increases in average scores, in many cases, we did not observe any change across multiple years. The rubric is likely very applicable for those working to support teachers' professional growth, but for those who interact more frequently and directly with teachers, it is important to remember that change takes time. That said, the levels should not be interpreted as rigid beliefs that teachers and others hold, or as developmental stages that instructional visions cleanly progress through one at a time. Instead they should be interpreted as a guide for what teachers currently consider important; are thinking about, looking for, or attempting to achieve in their classrooms; and what, of all of that, they are able to articulate. Finally, following the notion that talk might precede practice, users of the rubric should expect to find discrepancies between teachers' instructional visions and their instruction.

To that last point, the results presented above suggest that teachers' talk about mathematics instruction, indeed, often precedes their enactments. Even in classrooms of those who articulated the most sophisticated descriptions of the teacher's role (Level 4), IQA discussion scores were, on average, around a 2. Such a score represents instances in which students show and describe their work in solving a task, but discussion of that work is limited to procedures followed rather than connections to underlying concepts and/or other strategies. This likely does not come as a surprise to those charged with supporting teachers' learning; most of us who teach probably talk a better game than

we play. However, the findings reported here point to the potential for making productive use of such discrepancies—by framing teachers' descriptions as how they *envision* their role, rather than merely inaccurate (or worse, dishonest) descriptions of how they actually teach. In the following paragraphs, I discuss one example of such an approach.

Using the Tool in Work with Teachers

While the motivation for modeling developmental trajectories arose from a need to reliably document change in study participants' articulated visions of high-quality mathematics instruction, the instrument could potentially be useful for those working to support teachers' professional growth in a variety of settings. For example, diagnosing how individuals envision the role of the mathematics teachers could serve as a formative assessment of pre-service teachers' instruction and conceptions of practice; as a pre-post assessment of learning from professional development experiences; or as a means of determining where to begin professional development efforts and of identifying incremental goals over the course of that support.

As an illustration of the last possibility, in a professional development effort that I led with a small group of Algebra 2 teachers in an urban public high school in the northeast, my colleagues and I began by interviewing the teachers and their principal. In addition to inquiring about the setting in which they worked, we asked questions that pertained to their ideas about students and teaching, including the questions previously listed for eliciting their visions of the role of the mathematics teacher. Based on this initial diagnostic interview, we identified the instructional vision of the teacher leader with whom we worked the most as being a Level 3 on the role of the teacher rubric. In her interview, the teacher leader said she would want to see that:

[s]tudents are doing the work, not the teacher . . . the teacher is advancing the student's thinking by asking those higher level types of question by getting the kids to draw out connections between the math concepts, by getting kids to activate their prior knowledge to do the math . . . in an ideal situation of course, kids would be challenging each other's thinking and listening to each other. And I as a teacher would be monitoring, advancing thought when needed, often asking questions, but in an ideal world, kids would be pushing each other and listening to each other.

In initial observations of this teacher leader, we found that her envisioned role was not the role she was actually playing typically. In follow-up professional development sessions, she expressed her own frustration about this fact. She complained that students “constantly” asked her to tell them whether their answers were correct; that without immediate validation they would stop working. She did not blame the students, however. She suggested that “we as educators must have taught them to do it” through teacher-centered instruction and by making them feel insecure. “They feel like they’re always wrong and they don’t know what they’re doing.”

Thus, in our professional development efforts, we attempted to address the discrepancy between the teacher leader’s current classroom role and the role she envisioned for herself. She had an idea of what she wanted to see and experience with her students, and we worked to support her in achieving it. But we also aimed to support her in developing a more sophisticated vision for her role (i.e., Level 4), by beginning to identify ways that she might purposefully and proactively scaffold students in taking on responsibility for their learning. In this way, we were simultaneously working to meet her current (envisioned) goal, while supporting her in envisioning new goals for the future.

Conclusion

Recently, Hiebert (2013) argued that “the basic nature of teaching—presenting definitions and rules, demonstrating solution procedures on sample problems, and then asking students to practice the procedures on similar problems—

has remained remarkably consistent over the years,” and further suggested that “[t]he persistence of the way mathematics is taught in the face of numerous efforts to change it poses a serious and urgent problem for mathematics educators” (p. 45). This problem, he argued, is most productively approached as a problem of supporting teachers’ ongoing learning—a gradual process that takes time and requires consistent support. In this process, it is unlikely that all teachers learning paths will look the same (Fennema et al., 1996), or that they are all ready to transition to enacting the kind of role described as the goal in this article. Instead, transitions will likely be incremental, as teachers incorporate new practices into current repertoires.

The tool presented in this article provides a roadmap to what that transition might look like—at least in how teachers’ ways of envisioning their role change, if not their actual practice. It provides those charged with supporting teacher learning with a means of diagnosing teachers’ current ways of describing practice and then leveraging those instructional visions as both goals to reach and points on which to build.

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