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Moving Beyond One-Size-Fits-All PD: A Model for Differentiating Professional Learning for Teachers

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Abstract

This article describes an innovative model for differentiating professional development to address teachers' wide range of content knowledge, experiences, and interests. The model has three components: core activities *that all participants experience;* choice points *that allow teachers to choose options to individualize their learning; and* self-assessment opportunities *to help teachers reflect on their knowledge and identify areas to strengthen. To elucidate the workings of each component and the overall model, we present design principles and examples from a differentiated professional development sequence on fraction multiplication. To support the application of the model, we share implementation findings and offer suggestions to help mathematics education leaders plan and facilitate professional development that is differentiated for their teachers' needs.*

Introduction

> Raising achievement in mathematics for every student and effectively implementing the CCSSM in every classroom requires extensive and ongoing opportunities for teachers to enhance their own professional learning and build their capacity to reach all students. (National Council of Supervisors of Mathematics [NCSM], 2014, p. 44)

As districts strive to implement the Common Core State Standards for Mathematics (CCSSM) (Common Core State Standards Initiative [CCSSI], 2010) with rigor and equity for all students, mathematics education leaders play a critical role in providing teachers with much-needed professional development (PD) and support. The challenge for teachers is not only to enact an ambitious set of mathematics standards and practices, but also to do so in ways that are accessible, meaningful, and effective for a wide range of students. Just as students have diverse learning needs, teachers themselves vary greatly in their own prior content knowledge, experiences, and strategies for meeting this challenge, and yet PD programs often use a one-size-fits-all approach.

In this article, we describe a model for differentiated professional development (DPD) that enables teachers to play an active role in tailoring PD to meet their varied

professional learning needs. We created the model using an iterative process of design, testing, and revision, and applied it to different mathematics topics and teacher audiences from the upper elementary and middle grades. We begin our discussion with a brief rationale for differentiating professional learning, followed by an overview of the DPD model. To elucidate how the model works, we offer an in-depth look at the model's three components, accompanied by examples and a summary of implementation findings. We conclude with suggestions and planning tools to help mathematics education leaders, PD developers, and facilitators use our DPD model to address their teachers' needs and goals. These suggestions draw on three authors' perspectives as developers/facilitators of differentiated PD and one author's role as a researcher.

Why Differentiate PD?

The case that has been made for differentiating instruction for students (Huebner, 2010; Tomlinson, 2001) also applies to teachers. According to Tomlinson (2005), staff development needs to be differentiated to address the "reality that teachers themselves differ in readiness, interest, and learning profile, [and] will do so throughout their professional lives" (p. 12). In their professional contexts, educators also differ in their roles and in the uses they expect to make of their takeaways from PD. Teachers, however, have limited opportunities to make choices about which PD to attend (Bill & Melinda Gates Foundation, 2014) or to individualize their experiences within PD. One study that surveyed over 10,000 teachers reported, "Many teachers' complaints about their professional development appear to stem from a sense that it is not customized to fit their needs" (TNTP, 2015, p. 26). It is concerning that teachers are typically expected to differentiate instruction for their students but rarely experience differentiation firsthand in their own professional learning.

The need to differentiate PD for teachers is supported by research on adult learners. According to Knowles, adults want what they learn to be directly relevant to their work situations, roles, goals, and interests (as cited in Kenner & Weinerman, 2011). Because they have limited time available for dedicated learning experiences, adult learners want to have choices in what and how they learn. Giving teachers choices in PD helps increase their ownership and investment in their own learning, which are critical components for adult learners. Increasing teachers' investment in *career-long professional growth* reflects the recommendations of the Professionalism Principle of the National Council of Teachers of Mathematics (NCTM, 2014).

> As professionals, mathematics teachers recognize that their own learning is never finished and continually seek to improve and enhance their mathematical knowledge for teaching, their knowledge of mathematical pedagogy, and their knowledge of students as learners of mathematics. (p. 99)

The benefits of differentiating PD extend to the district level. With the implementation of CCSSM, districts have an increased need for PD, but the time available has typically remained the same or decreased. This lack of time for PD has been identified by both teachers and administrators as one of the top barriers to effective PD (Bill & Melinda Gates Foundation, 2014). Differentiating professional learning provides a way for districts to maximize the available time by allowing teachers to make choices to customize the PD to directly respond to their individual needs.

Overview of DPD Model

We designed our DPD model during a five-year research and development project funded by the National Science Foundation (NSF) (DRL-1020163), *Differentiated Professional Development: Building Mathematics Knowledge for Teaching Struggling Learners*. Our central design challenge was how to create PD that would achieve our goal of building *all* participating teachers' mathematics content knowledge, diagnostic approaches, and instructional practices, while differentiating the learning experience so that *each* teacher would have opportunities to meet his or her professional learning needs. To address this challenge, we built three main components into the DPD model: core activities, choice points, and self-assessment opportunities. *Core activities* are expected of all participants because they cover essential content and provide a common ground for building a learning community. *Choice points* provide the opportunity for teachers to choose options to customize their learning. For example, a choice point might invite teachers to select from a set of activities, choose their own starting point or path within a given activity, or select the level of challenge of mathematics problems to solve. *Self-assessment opportunities* help teachers assess their level of understanding, reflect on their progress towards the learning goals, and identify areas to strengthen. This information, in turn, helps teachers to select topics or activities on which to focus in the choice points. The model's three

components work together to provide a comprehensive and flexible PD approach that allows teachers to collaborate on common goals while also making choices to individualize their learning. In the sections that follow, we provide a closer look at these components.

Our DPD Model in Action

In this section, we present examples from a PD session on fraction multiplication to illustrate how we use the DPD model to build an understanding of key topics from the CCSSM standards. We have found that teachers vary in their levels of prior experience exploring fraction multiplication at a conceptual level that goes deeper than performing the algorithm. Therefore, this session engages teachers in a variety of activities to strengthen their understanding of what it means to multiply fractions and to build flexibility with representing the operation visually, verbally, and numerically. Teachers also learn about common student difficulties and misconceptions, such as *multiplication always makes larger* (i.e., the incorrect assumption that the product is always larger than the factors), as well as ways to address these misconceptions. For this PD session, we differentiate by creating a sequence of core activities and choice points, which are described in the following sections. Although this example is from a face-to-face PD session, we later describe ways to create choice points for online settings.

Core Activities

To launch the topic of representing the multiplication of fractions, we use a series of core activities to engage teachers in common experiences that motivate further exploration and serve as a shared reference point for later activities (see Figure 1). Our initial goal is for teachers to make sense of a fraction multiplication situation by creating their own ways to represent it visually. All of the teachers work on the same word problem (see Figure 2) and individually create a picture or diagram to represent the problem. Then, they share their representations with the group, discussing the similarities and differences. This core activity showcases a variety of approaches and provides us with formative information on teachers' prior knowledge on which to build in the subsequent activities.

FIGURE 1.
Core activities.

Overview of Core Activities	
1.	Teachers come up with their own visual representations for a word problem and share approaches.
2.	Teachers watch and discuss a classroom video in which students draw representations for the same word problem that the teachers worked on. The video shows a few common difficulties.
3.	Facilitators demonstrate how to use one area model approach for fraction multiplication, and they connect it to teachers' approaches in the initial activity.
4.	Teachers solve a few problems using this area model approach.

After sharing their approaches, teachers watch a video of a fifth grade classroom in which pairs of students work on the same word problem that the teachers completed. The video shows a few examples of students having difficulty making sense of and representing the problem. As teachers watch, they take notes and then discuss their observations with the group. Because the video is new to all participants, making this a core activity is a straightforward decision. This shared experience builds awareness of student difficulties with fraction multiplication and motivates teachers' interest in learning ways to provide support. One instructional strategy is to help students use an area model to represent a variety of problems and see firsthand how the size of the product relates to the size of the factors. This visual approach helps address a common student misconception that products are always larger than the factors, by showing that "multiplying a given number by a fraction

FIGURE 2.
Sample problem from Core Activity 1.

Make Drawings to Represent and Solve Problems

1. Jodi is decorating a cake for a party with her friends. She knows that her friends have different tastes and she wants everyone to get what they like. She frosts 1/2 of the cake with chocolate frosting and the other 1/2 of the cake with vanilla frosting. Then, she puts rainbow sprinkles only on 1/3 of the chocolate-frosted part. What fraction of the whole cake will have chocolate frosting AND rainbow sprinkles?

   Make a drawing to represent and solve this problem.

FIGURE 3.

Sample problem from Core Activity 3.

Use An Area Model for Fraction Multiplication

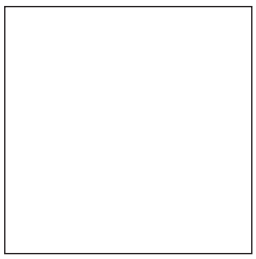
1. Celia got a block of clay to use for a school project. After she finish the project, she had 1/4 of the block of clay left over. She gave 2/3 of the leftover clay to her brother. What fraction of the whole block of clay did Celia give to her brother?

   A) Represent the situation with an area model.

   B) Answer: Celia gave _____ of the whole block of clay to her brother.

   C) How would you represent the situation with words and a number sentence?

   - Words: _____ group of _____
   - Number sentence: _____



less than 1 results in a product smaller than the given number" (CCSSI, 2010, p. 36).

Next, we demonstrate an area model for fraction multiplication and ask participants to explore it themselves in the role of learners. Although the model is new to some teachers and familiar to others, we feel that it is important for everyone to see the same demonstration to provide a shared reference point for further work and discussion. After the demonstration, teachers use the model to represent and solve several problems (see Figure 3). Because of teachers' varied prior knowledge of the model, we keep this section relatively short and follow it with a choice point to offer teachers an additional opportunity for support, exploration, or challenge.

Choice Point Options

We want teachers who are new to the area model approach for fraction multiplication to have the opportunity to immerse themselves in using the representation and for experienced teachers to be able to stretch their knowledge. Therefore, we next provide a variety of choice points that allow teachers to customize their learning experience. When we designed these choice points, we considered what professional learning needs teachers might have after completing the core activities and what learning experiences would address those needs. Figure 4 presents the list of teachers' varied needs and the options that we brainstormed for the choice points.

FIGURE 4.

Planning choice points by identifying teachers' varied needs and possible options.

Consider Teachers' Varied Needs ***What might teachers be thinking after the core activities?***	**Brainstorm Choice Point Options** ***What are ways to address these needs?***
This is new to me. I never thought about fraction multiplication except with the algorithm. I need more time getting to know the model.	Provide these teachers with more opportunities to work with the model by starting with problems like the ones in the demonstration. Then provide a variety of problems that progress in difficulty.
Representing mathematical ideas visually is hard for me. I'm having difficulty with the model.	Provide these teachers with more instruction on the model by having a facilitator work with a small group. Give teachers the opportunity to use an applet that helps set up the model and gives immediate feedback.
I feel like I have a good grasp of how to use the model myself. I'm ready for more challenge.	Provide more challenging problems that involve more difficult fractions and creating products that are larger or smaller than a given number.
I have experience teaching this model. I want to think about ways to improve how I use this model.	Ask teachers to create their own problems for using the model with their students.

Working from our brainstormed list, we selected three options to offer at the choice point (see Figure 5). For face-to-face sessions, we find that limiting the number of options to two or three works well, because having more choices can be overwhelming for teachers to decide among and challenging for facilitators to implement. To help teachers make a choice, we describe the options and instruct participants to reflect on their prior knowledge/experience. Teachers move to different tables for their selected option and work individually or in pairs.

At the Option B table, one facilitator provides a short introduction to the applet to get teachers started with using it. Then both facilitators circulate among the tables to provide support as needed. We encourage teachers to focus in-depth on their chosen option and work at their own pace. We also provide teachers with copies and/or links to all of the options for later use, so that they do not try to rush through all options at the session for fear of missing out. Figures 5a, 5b, and 5c show sample problems for each of the options.

FIGURE 5.
Overview of choice point options in the fraction multiplication sequence.

Choice Point: Area Model		
Directions: Reflect on your prior knowledge/experience with the model. Based on your experience and self-assessment, choose an option to move your learning forward.		
Option A	**Option B**	**Option C**
If you want to focus on getting to know the model: Work on a series of problems designed to build your understanding and fluency with using the model. Tip: Choose this option if you have no or little prior experience teaching the model.	*If you want to try an applet approach to setting up the visual representation:* Use an applet, Field of Fractions, that provides support in using the approach by drawing and dividing the parts.	*If you feel ready for more challenge:* Work on more challenging problems and create your own problems.

FIGURE 5a.
Sample problems for Option A.

Get to Know an Area Model

Problem	Describe in Words	Draw an Area Model	Product
1. $\frac{1}{5} \times \frac{1}{3}$	_____ group of _____		Product: _____
2. $\frac{5}{6} \times \frac{1}{2}$	_____ group of _____		Product: _____
3. $\frac{2}{3} \times \frac{4}{5}$	_____ group of _____		Product: _____

4. Write a problem that has a product **greater than** the product in #3 but is less than 1.

Represent Sequences of Fraction Multiplication Problems

1. Multiplying proper fractions: Represent each problem by drawing an area model.

a) $\frac{2}{3} \times \frac{3}{4}$ b) $\frac{3}{4} \times \frac{3}{4}$ c) $\frac{4}{5} \times \frac{3}{4}$ d) $\frac{4}{5} \times \frac{4}{5}$

2. Look at the area models for the different problems in **#1**. How is the size of the products related to the size of the factors?

FIGURE 5b.
Screenshots from the Field of Fractions *Applet (http://tube.geogebra.org/m/40736) for Option B.*

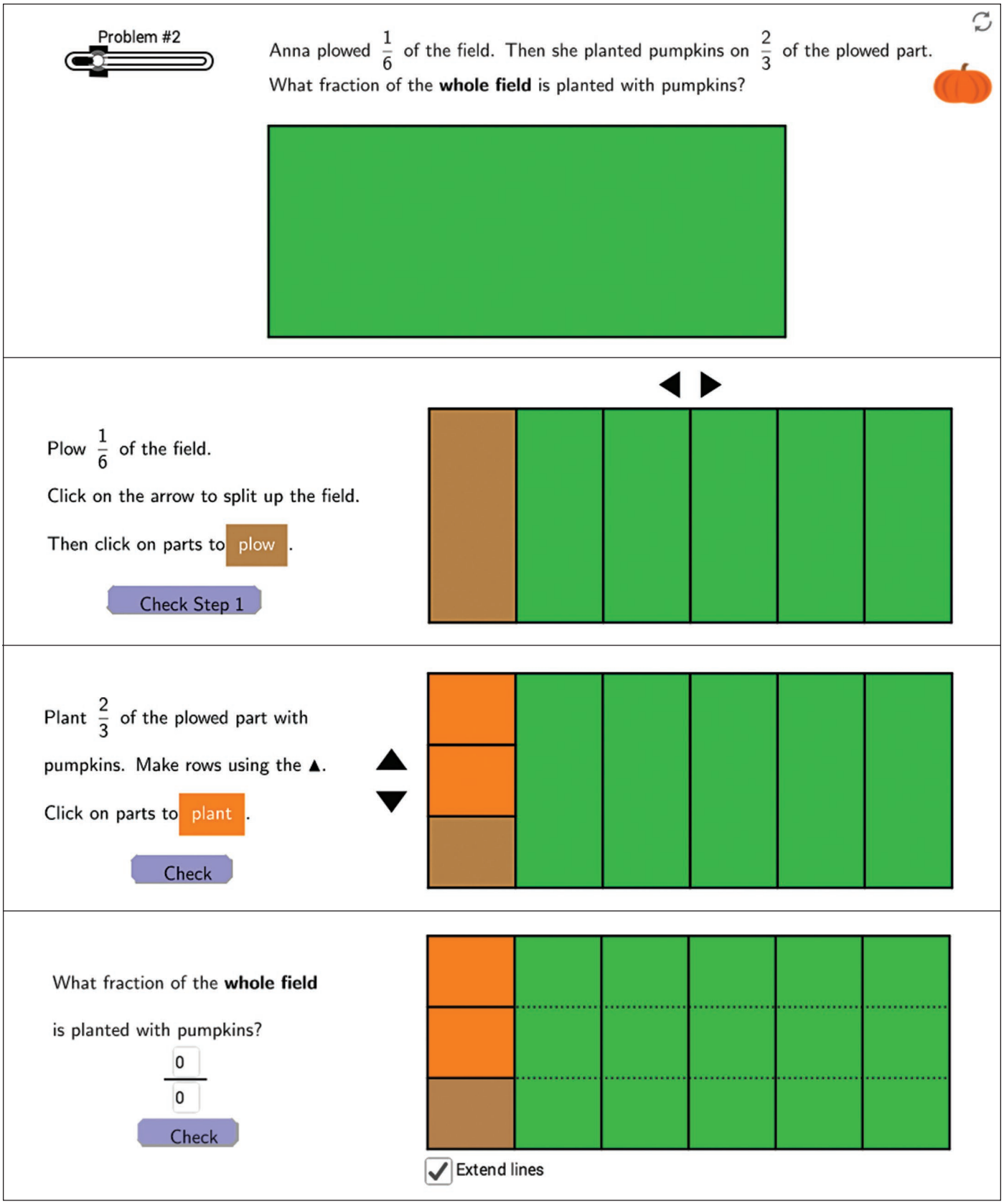


FIGURE 5c.
Sample problems for Option C.

Create Larger and Smaller Products

1. Task: Find all the ways to get products that are **greater than 1/2 but less than 1**

Possible digits: Use 2, 3, 4, 5 or 6 to fill in the blanks.

a) $\frac{?}{5}$ of $\frac{2}{?} > \frac{1}{2}$ but < 1

Show all the ways by drawing area models or explain why it is not possible.

b) $\frac{5}{?}$ of $\frac{?}{2} > \frac{1}{2}$ but < 1

Show all the ways by drawing area models or explain why it is not possible.

Create Your Own Problems

2. Create your own problem by using a similar format.

a. Decide which type of problem you want to create:

_____ a problem with two or more ways to get a product less than $\frac{1}{2}$.

_____ a problem for which it is not possible to get a product less than $\frac{1}{2}$.

b. Fill out the starting information for the problem.

Task: Find all the ways to make products that are less than $\frac{1}{2}$.

Available Digits: _____

Starting Expression. (Put 1 or 2 digits in the blank boxes to start.)

$\frac{\square}{\square}$ *of* $\frac{\square}{\square} < \frac{1}{2}$

c. Prepare the solution for your problem by using area models to show all the possible ways to get the target product. Explain how you know that you found all the ways. Or explain why the problem is impossible.

d. Reflect on your experience. What important mathematical ideas did you use to create and solve the problem?

To wrap up the choice point section, we bring all teachers back together for a shared discussion about themes that cut across all three options. One challenge of having teachers work on different activities is designing and facilitating discussions to bring together ideas from their different experiences. We strive to create discussion questions that are applicable to each option and allow all participants to contribute. In this example, we engage teachers in first discussing the area model from their experiences as learners and then from a teaching perspective. We ask them to share considerations and suggestions for using the area model to build their students' understanding of fraction multiplication, with particular attention to addressing the common misconception described above. Sample discussion questions include:

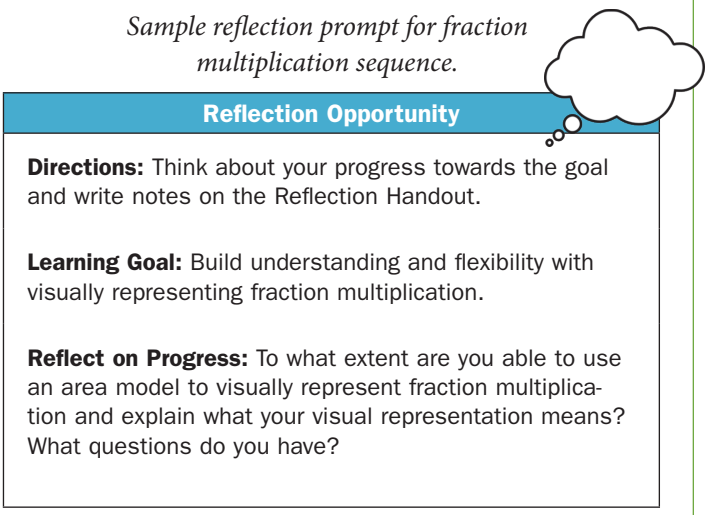
- What was your experience like using the area model as a learner? What important ideas did it bring out about fraction multiplication?
- What are the strengths and limitations of this model for building understanding of what it means to multiply fractions? What are the model's strengths and limitations for solving problems?
- One potential pitfall is that the model could be used in only a procedural way. What are ways to use the model to build conceptual understanding of fraction multiplication?

For this choice point, we use a culminating whole-group discussion because all of the options focus on the area model. When the options are more disparate, we may use separate small-group discussions and then ask each group to share a few ideas with the whole group. For some choice points, we include a few common mathematics problems in each option to facilitate the sharing of approaches in the subsequent whole-group discussion. In designing choice points, it is important to consider not only how to create each option but also how to bring together ideas across options to move learning forward for the whole group as a learning community.

Self-assessment Opportunities

We present the learning goals, such as *build understanding of and flexibility with visual representations for fraction multiplication*, at the beginning of a face-to-face session. Then, during the sequence of core and choice point activities, we pause periodically to ask teachers to reflect on and self-assess their learning. Figure 6 is a PowerPoint slide from the sample session, showing the kind of prompts we use to engage teachers in this process.

FIGURE 6.
Sample reflection prompt for fraction multiplication sequence.



During these intervals of reflection and self-assessment, teachers write their thoughts on a Reflection Handout. As they continue to work on fraction multiplication, they refer back to the handout to identify areas to strengthen and inform their decision making at subsequent choice points.

Two Design Decisions for the DPD Model

The three DPD model components that we have demonstrated here (i.e., core activities, choice points, and self-assessment opportunities) work together to create a robust, flexible approach to differentiating professional learning. In designing this model, we made two key decisions about the differentiation:

1) not everything would be a choice; and when choices were offered, teachers would decide for themselves what options to select. As we have described, the reason for the first decision was that we wanted some content to be required for all teachers in order to provide a shared experience with these topics/activities. These core activities serve as a foundation for further learning and help to build a community of learners. For our differentiated PD courses, we create a combination of activities that is about 60% core activities and 40% choice points.
2) The second decision was to allow teachers to choose an option rather than having the differentiated activity selected for them, such as basing the assignment on their test results. We believe that giving teachers choices promotes ownership and investment in their own

learning and respects their professionalism. A potential downside of this decision is that, as teachers are learning to make these choices for their professional learning, they may overestimate their own understanding or, conversely, choose topics that they are already comfortable with instead of ones that they need. We strove to mitigate these issues and support teachers in their decision-making through the use of reflection and self-assessment opportunities as well as through the facilitator's role in building a supportive learning community that encourages participants to take risks and stretch themselves in their learning.

A Closer Look at the Model's Components

The fraction multiplication sequence above illustrates how our DPD model's three components work together to foster differentiated and group learning in our PD courses. Here we offer a closer look at each component and share the decision-making process we use to interweave the components to meet the professional learning needs of teachers.

Core Activities

To decide which topics should be addressed in core activities, we consider the learning goals and participants' professional learning needs, including their prior knowledge and experiences with the topic, as well as its relevance to their roles and work with students. If the topic is new to all participants, our decision to create a core activity is straightforward. When there is a lot of variation in participants' prior knowledge, such as large groups of new and experienced participants, we tend to use choice point formats. For other situations, we weigh the pros and cons of offering options to differentiate the learning experience versus keeping the group together for a shared activity.

When we choose a core activity format, we consider ways to design a common experience that takes into account participants' varied needs. Even when all participants are new to a topic, they may vary in other ways, including comfort with doing mathematics. It is important to design activities to be accessible and engaging to a range of learners, such as by using a low threshold, high ceiling approach and by giving teachers the opportunity to use multiple strategies. We strive to provide entry points that allow all participants to get started and immerse themselves in the tasks. In addition, we plan ways to draw in and motivate teachers who may have low interest in a topic because they are not responsible for teaching it at their grade level.

In some cases, we decide to use a core activity format for topics in which participants vary greatly in prior knowledge because we want to provide a shared experience on which to build in future activities. In doing so, however, we risk the potential of frustrating participants who are new to the topic by moving too quickly or those who are experienced with the topic by moving too slowly or spending a long time on a familiar topic. In light of these concerns, we aim to design a streamlined core activity that engages all participants and then moves quickly to a choice point.

Choice Points

Choice points are our model's central vehicle for differentiation. In developing the model, we explored a variety of ways to create choice points to allow teachers to make decisions based on different factors: prior knowledge or experience, mathematics topic, type of mathematics problem, desired level of challenge, preferred mode of getting information, and type of activity. We describe each type of choice point and provide examples in Figure 7. Our intention is not to suggest that someone use all of the different types in one PD program. Instead, we encourage the selection of one or more that best match participating teachers' needs.

Prior knowledge and experience. Teachers, who are new to a topic or approach, benefit from introductory activities and from moving at a slower pace with more support than those who have extensive experience teaching the topic to students. Experienced participants need opportunities to build on their prior experiences, stretch their knowledge, and view the topic in new ways. In this type of choice point, we ask teachers to reflect on and self-assess their prior knowledge and experience and choose accordingly.

Mathematics topic. These choice points allow teachers to choose a topic on which to focus in more depth and extend their learning from the core activities. Teachers may select a topic to strengthen their own knowledge and/or focus on content that is applicable to their grade level standards. This type of choice point is helpful for designing PD for teachers from different grade levels.

Type of mathematics problem. We give teachers options to work on different types of problems, such as word problems, numeric (non-word) problems, or estimation problems for the same mathematics topic, such as fraction multiplication. They might select a type of problem that they find more challenging themselves or that they want to strengthen in their work with students.

FIGURE 7.
Types of choice points.

Make Choice Based On:	Examples
Prior Knowledge and Experience	***Use prior experience with fraction circle manipulatives to choose a starting point:*** A. If you have *no or little prior experience*, start on page 1 with an introductory exploration of fraction circle manipulatives and unit fractions. B. If you have *some prior experience*, start on page 3 to use fraction circles to find equivalent fractions. C. If you have *a lot of prior experience*, start on page 5 to use fraction circles to compare fractions. ***Use prior experience to choose to focus on one manipulative or compare two:*** A. *If you are new to using manipulatives for fraction addition*, choose *one* of the following to explore and analyze: Fraction Circles, Fraction Bars, or Pattern Blocks. B. *If you have experience using manipulatives for fraction addition*, choose *two* manipulatives. Compare the strengths and limitations of the manipulatives for building understanding of fraction addition. Consider the ways in which using the manipulatives might support mathematics practices 2 and 7.
Mathematics Topic	***Choose problems based on specific mathematics content:*** A. Multiplication problems with whole number times fraction (Grade 4 standard). B. Multiplication problems with fraction times fraction (Grade 5 standard). C. Mix of multiplication problems. ***After focusing on core fraction division activities, choose to focus in more depth on one of the following topics:*** A. Solve word problems for building understanding of division of whole numbers by fractions and vice versa. B. Use visual models for representing and solving fraction division problems with remainders. C. Further investigate why the fraction division algorithm works.
Type of Mathematics Problem	***Choose type of problem:*** A. Word problems. B. Numeric/symbolic (non-word) problems. C. Estimation problems.
Level of Challenge	***After solving a set of core problems, choose to:*** A. Continue solving problems at the same level. B. Solve problems at an easier or more foundational level. C. Solve more challenging problems.
Preferred Mode for Getting Information	***Build background knowledge of the number line representation for fractions by choosing to:*** A. Watch a video. B. Read an article. C. Explore resources on a website.
Type of Activity	***Choose what kinds of mathematics activities to do:*** A. Paper-and-pencil mathematics activity. B. Interactive mathematics applet with online feedback. C. Collaborative mathematics game to play with colleagues.

Desired level of challenge. This format begins with all participants working on the same problems to gain a sense of the level of difficulty. After finishing the initial problem set, they have the opportunity to adjust the level of challenge for the subsequent problems by continuing to work on problems at a similar level, moving to more foundational problems, or skipping to more challenging problems. Teachers might want to work on more foundational problems because they are having difficulty themselves or because they would like to provide those types of problems to their struggling students. In a face-to-face session, teachers make their choices by moving to different pages in a packet of handouts. In the online environment, participants use interactive menus to branch to their chosen level of challenge.

Preferred mode for getting information. We offer participants choices of getting background information in various formats, such as watching a video or reading an article. They select the format based on their learning preferences (e.g., visual, auditory, verbal) or to serve different purposes. For example, a teacher may prefer to watch a video to first learn about student misconceptions with the number line representation and then use a reading as a reference later on.

Type of activity. These choice points offer a selection of instructional activities, such as a collaborative game, computer applet, or paper-pencil activity. Our goal is to help teachers expand their repertoires of instructional activities for use with their students. We encourage teachers to try new activities and approaches or consider new ways to apply them in their classroom practice.

Choice Point Formats for Face-to-Face and Online Settings

We have created and tested choice points for both face-to-face and online sessions; our intent was to explore ways to leverage the unique features of each environment to support differentiation. We use several strategies to differentiate PD in the *face-to-face* setting. For example, we offer participants a packet of mathematics activities that has a *choose your own adventure* format. That is, as teachers work on the activity handouts in the packet, they come to choice points with options, such as skipping to more challenging problems or moving to a different type of problem (see Figure 8). Teachers work on different activities that are all related to the same mathematics topic so that connections can be made in subsequent whole group discussions. Another differentiation strategy is to use centers, stations, or breakout rooms to allow teachers to move to the topics or activities on which they want to focus. In addition, we offer options for different ways to work on mathematics problems, such as a choice of manipulatives or models.

FIGURE 8.
Example of face-to-face choice point: Choose Your Own Adventure format.

Choice Point

Directions: Reflect on your experience solving fraction addition and subtraction word problems on pages 1-2. Choose an option to move your learning forward.

A. To solve more word problems, work on pages 3-4.
B. To write and solve your own word problems, go to page 5.
C. To use pattern blocks to solve problems (not word problems), go to page 7.

In the *online* sessions, we offer participants similar opportunities to explore representations and build upon the core content by choosing options from choice point menus (see top example of Figure 9). In addition, the power and flexibility of the online environment provides different ways for teachers to customize their learning. Because the online environment allows easy access to materials in various media, online choice points allow teachers to build background knowledge of the same topic by watching a video, reading an article, or working with an applet (see bottom example in Figure 9). To foster teachers' use of diagnostic approaches, we give them a choice of examining student work samples or watching videos of mathematics interviews so they can decide on which type of evidence to focus. For the videos of the mathematics interviews and other topics, teachers can choose to view them multiple times to take a closer look at students' approaches.

FIGURE 9.
Two examples of online choice point menus.

Choice Point: Compare Decimals by Using Different Representations

Directions: Choose two representations. We encourage you to choose representations that are less familiar to you.

A. Use base-ten blocks to build decimals and compare them.
B. Shade grids to represent and compare decimals.
C. Use number lines to locate and compare decimals.

Choice Point: Build Background Knowledge of Number Line Representation

Directions: Choose at least *one* option to build background knowledge on these key questions.

A. Why is the number line an important representation for fractions? (Video)
B. How does the number line representation help students build understanding of fractions as numbers? (Reading)
C. What are key fraction/number line concepts from the Common Core State Standards? (Reading)

Self-assessment Opportunities

For teachers to make choices that are a good match to their learning needs, they need to clearly understand the goals for which they are aiming and have opportunities to regularly self-assess their understanding, pinpoint their strengths and weaknesses, and gauge their progress toward achieving the goals. Articulating learning goals, including success criteria for meeting the goals, and providing reflection opportunities are essential parts of our DPD model. As described in the example above, during face-to-face sessions, we begin with the learning goals and then pause at key points during the day to ask teachers to reflect on their progress towards the goals. Teachers also have opportunities to consider their learning during discussions and to write down "ideas to take away." Similarly, in the online sessions, we incorporate *reflect on progress prompts* (like those in Figure 6) to engage teachers in taking stock of their learning, as well as opportunities to discuss their experiences in the discussion forums. In addition, we provide online *self-check and reflect* activities that include questions about the central mathematics concepts, accompanied by immediate feedback (see Figure 10). These activities are designed for self-assessment purposes and are non-evaluative. Their main purpose is to help teachers identify areas to strengthen and to inform their selection of options at the choice points.

FIGURE 10.
Sample question from a Self-Check and Reflect *activity.*

Self-Check and Reflect Activity

Directions: Read the two problems. Can each problem be solved by using the calculation 1/4 x 2/3?

Problem I: After the party, Sue brought home 2/3 of a cake. She ate 1/4 of the leftover cake. What fraction of the whole cake did she eat?

Problem II: Tomas made an apple pie for the picnic. He ate 1/4 of the pie and Chris ate 2/3 of it. What fraction of the whole pie did they eat?

Select one:
○ **a.** Problem I only
○ **b.** Problem II only
◉ **c.** Both problems I and II
○ **d.** Neither problem

Your answer is incorrect.

Problem I is a fraction multiplication situation but Problem II is not. In Problem I, you need to find 1/4 of 2/3 to determine what part of the whole cake was eaten, so it makes sense to multiply. For Problem II, 1/4 and 2/3 should be added to determine what part of the whole pie was eaten altogether.

Suggestion: If you want to solve more word problems with mixed operations, go to Session 6, Tab 5.

Implementation Findings

A team of researchers gathered information on the implementation of the DPD model as part of extensive field tests for three differentiated courses developed during our NSF-funded project. Overall, 148 mathematics teachers, general educators, and special educators from 21 school districts completed one or more of the courses. Teachers were asked to complete several instruments that explored their experience with the DPD model, including course evaluation surveys and telephone interviews. Here, we share findings from the fractions course because it was the largest, with 104 participants from 16 districts.

Teachers' Perceptions of Usefulness

There were many indications that participants found the PD to be useful, high quality, and a good match for their

professional learning needs. For example, using a scale from 1–not useful to 5–very useful, participants gave the overall PD experience a mean rating of 4.8. There were no significant differences in usefulness ratings of the fractions course by participants with different types of certification (i.e., general education/mathematics, special education, or dual), different roles (i.e., general educator, mathematics teacher, special educator), or those with and without post-secondary study of mathematics or mathematics education. These findings demonstrated that the PD was viewed as useful by participants with different professional backgrounds, years of teaching experience, and roles, which reflected positively on the DPD model.

Teachers' Feedback on the Choice Points

Participants gave high ratings to the choice points in both the face-to-face and the online sessions. The research team asked participants how well the choice points met their needs, and the average rating was 3.6 out of 4 for the face-to-face and 3.5 for the online sessions (rating scale: 1–not at all; 2–a little; 3–some; and 4–a lot). When asked to explain their ratings, participants' most common reason for a high rating was the opportunity to customize the learning experience for one's needs, including permission to start at one's current level of knowledge or comfort, challenge oneself, or concentrate on specific topics. As one teacher wrote:

> In courses/workshops I am always feeling others know more than me so at times I am uncomfortable with the idea of everyone working on the same assignment. I was relieved when I could pick my level which lessened my performance anxiety and at times was pleasantly surprised that I could choose a more challenging assignment.

Another teacher wrote, "I loved that we could work at our own pace. [It] really allowed me the opportunity to use the manipulatives and gain an understanding of the concepts."

Other reasons for high ratings included feeling ownership and appreciating the freedom to make choices. In the words of one teacher, "[Choice points] gave me the freedom to pick things that I knew I either needed some practice/support with, or things that I knew I would be able to use in the classroom. It was nice to have that freedom."

Many teachers said they chose options that were new to them or would expand their own learning. As one participant said, "If you are going to take a course like this I think it is important to really dig into what you don't know or have not had much exposure to." Although the PD placed a strong emphasis on having teachers build their own content knowledge, many participants' decisions of which activities to select were heavily influenced by the potential for classroom application. Like other adult learners, teachers want their learning to be relevant, so it is natural that the participants would choose options that they felt they could ultimately use with students. Many participants were interested in learning effective ways to teach concepts and skills that they considered particularly important or difficult for students, or methods suitable to specific needs of their students. As one teacher wrote, "I wanted to work with word problems as my students have language-based learning disabilities."

Although the primary intent of the choice points was to meet participants' varied needs, experiencing differentiation firsthand gave some participants a sense of its benefits for students. One teacher wrote, "I liked the choices because they engaged me. If I enjoyed the choices, my kids will enjoy them. . . . We still covered the agenda we had to get through. We can now provide choices for our kids." Another commented, "I thought you really exemplified HOW to differentiate through using these [choice points]."

Reasons for Low Ratings of Choice Points

Although the majority of participants gave the choice points high ratings, it is important to consider the explanations given for a small number of low ratings. The most common reason was the desire to complete *all* of the activities. Some participants who were new to the content wanted to work on everything; therefore, having options did not increase the usefulness of the program for them. One participant wrote, "Basically, I wanted to try all the problems. So I always started at the beginning rather than jumping into the middle or end." Some participants seemed to have difficulty trusting that they could make choices and still learn all they needed. As one put it, "I tried to do all of the choices because I felt like I was missing out if I didn't." Our perception is that many participants were new to making choices in professional development, and thus it was not surprising that some would feel unsure about the process.

Issues with Choice Point Decisions

For the most part, choice points allowed participants to customize their learning experience in beneficial ways. In a few cases, participants' interviews and survey responses

indicated that they made choices for expedience or to avoid taking risks in learning. Examples included: deciding not to select activities with a particular visual model because they found it confusing themselves and thus would not use it with students; choosing an activity they already knew because they were hesitant to step outside of their comfort zone; or choosing the first activity in the list just to fulfill the requirements, without considering all the different options.

Suggestions for Mathematics Education Leaders, PD Providers, and Facilitators

Just as differentiating instruction for students is more complex than teaching everyone the same way, differentiating PD requires a different type of planning and facilitation on the part of mathematics education leaders, PD developers, and facilitators. In the following sections, we first provide an overview of the planning process and then offer suggestions for designing, implementing, and facilitating differentiated PD using our model.

Overview of the Planning Process

The planning process begins with guiding questions that are essential for designing any PD program: *What are the professional learning goals?* and *What are participants' needs?* In the DPD model, deciding where and how to differentiate involves asking additional questions: *What content will be core for all participants?* and *What content will be differentiated to address teachers' varied needs?* Figure 11 incorporates these questions in the second and third columns to provide a differentiation lens for the planning process.

This expanded set of guiding questions helps strengthen the overall PD design by closely aligning the goals and activities with participants' needs. Considering what will be core and what will be differentiated provides a useful lens for clarifying what is most important for all participants to learn and experience, and identifying where there are openings for individualization.

We have also created a *Differentiated PD Planning Tool* that incorporates the main guiding questions into an agenda format (see Figure 12). To use the tool, we suggest filling out the first two rows and columns of the agenda as you would for any PD program. Then, examine the agenda several times through a differentiation lens. In the second column, star (*) the activities/topics for which teachers have particularly varied needs. Look over topics/activities to make an initial decision about which activities might be core and which might be differentiated; label them with a "C" or "D" in the third column. Next, consider all topics with "D's" to decide which ones are the top priorities to differentiate and write down ideas for creating choice point options in the fourth column. This approach can be adapted for use with existing PD agendas; start with the prior agenda and add columns with the questions on differentiation.

FIGURE 11.
Guiding questions for DPD Model.

PD Planning Questions	What content will be *core* for all participants?	What content will be *differentiated?*
What are the professional learning goals?	What is essential for everyone to learn?	In what ways do the goals vary for different groups of teachers (by role, grade level, etc.)?
What are participants' professional learning needs?	For which areas do participants have a lot of consistency in their professional learning needs?	For which areas do participants have a lot of variation in their professional learning needs? What is the distribution of needs?
What activities will you use to address the learning goals and participants' needs?	How important is it for all teachers to experience this activity for building knowledge and/or providing a shared experience?	What are ways to differentiate the activity to address teachers' varied needs? What choices might you offer?

FIGURE 12.
Differentiated PD Planning Tool.

What are the professional learning goals?			
What are participants' learning needs?			
Time	**What are the topics & activities?** Fill out as you would for any agenda. Then star (*) topics/activities for which participants have particularly varied needs.	**What might be core (C) or differentiated (D)?**	**What will you differentiate? How?** Look over the topics/activities that you marked 'D'. Which of these are high priorities to differentiate? Choose a few and brainstorm ways to differentiate by using a choice point or other methods. Write down ideas below.

Suggestions for Designing and Implementing Differentiated PD

Prior to the PD, the following steps are recommended.

1. Identify the professional learning goals.
2. Conduct a needs assessment.
3. Analyze the findings to identify areas of variation.
4. Decide what will be a core activity and what will be differentiated. *Tip:* Start small by choosing one section of an agenda to differentiate.
5. Plan ways to address logistical constraints, such as available space for dividing into groups and the number of instructors available to lead simultaneous activities.

During the PD, consider the following steps.

6. Gather ongoing information from participants to fine-tune the differentiation.
7. Gradually add choice points to multi-session PD programs.

Finally, after the PD, evaluate and revise the differentiation.

Suggestions for Facilitating Differentiated PD

If you will be facilitating differentiated PD, the following suggestions will help alert you to challenges you may encounter and give you strategies to overcome potential obstacles.

Help teachers make choices and feel comfortable with them. As discussed in the Implementation Findings section, some teachers may feel unsure about which choices to pick or may select "safer" options because they are reluctant to move outside their comfort zones. To address these issues, facilitators need to be careful to describe all of the choices clearly and equitably, without placing value on one over another, and to set a comfortable tone for making the decisions.

In face-to-face sessions, teachers may feel more self-conscious about their choices. In planning sessions, consider how different formats for choice points might affect teachers' comfort in making choices. One option is to have teachers stay at the same table and choose to work on different but related activities in a packet of handouts (using the *choose your own adventure* format previously described). Because teachers do not need to move to a new location, this option lets them choose in greater privacy

and also allows them to change direction more inconspicuously if a choice does not meet their needs. Alternatively, there are many benefits to having teachers move so that everyone at a table is working on the same activity.

The anonymity of the online environment reduces some of the concerns described above for the face-to-face sessions; teachers may feel more comfortable making choices and taking risks away from the eyes of their colleagues. Teachers can preview choices to decide if an activity will meet their learning needs and can easily switch activities. Another benefit of the online environment is that the number of choices offered is not constrained by the availability of meeting rooms and instructors.

Although the online environment offers great potential for differentiating learning, it also poses some challenges. Because it is easy to switch from activity to activity online, teachers may skim the options at a choice point without delving into them. Also, while teachers may feel more comfortable in the anonymity of making choices online, they may also feel less motivated, connected, and accountable because they are not working with colleagues. As an online facilitator, you can help by placing a high priority on fostering interaction, building community, and making connections across participants who are working on different activities. These design and facilitation principles are integral to implementing online professional development in general and have particular importance for differentiated programs.

Foster collaboration. A potential downside of differentiation is that the experience may become too individualized. Facilitators need to be attentive to differentiating in ways that support, rather than detract, from building a learning community. We recommend starting differentiated sessions with core activities to provide shared experiences and community building to lay the groundwork for ongoing collaboration. For choice point sections, plan and facilitate discussions to engage teachers in sharing ideas from their different experiences and bringing out crosscutting themes.

Set clear expectations and build in accountability. While establishing expectations and accountability is essential for all PD programs, there are specific issues that need to be addressed in facilitating differentiated programs. One potential issue is that teachers may think of the choice point activities as optional, or as less important than the core activities, and thus decide to skip or skim them.

Another concern is that teachers may feel less accountable because different people are working on different activities as opposed to everyone working on the same one. It is important to explain that everyone is responsible for focusing in-depth on his/her selected activity and to clarify expectations by setting an end goal, such as being prepared to share ideas with the whole group.

Implications for Mathematics Education Leaders

Differentiating professional development offers important benefits to both teachers and their school districts. Our DPD model, with its combination of core activities, choice points, and reflection opportunities, allows teachers to work together on common goals while also making choices to individualize their learning. It gives teachers greater ownership of their professional learning, as each practitioner chooses to focus on the knowledge or skills that he or she needs to strengthen. For districts, this approach for customizing offerings to teachers' varied needs helps to optimize the limited amount of time available for professional development. Although the examples in this article focus on fractions, the DPD model lends itself to mathematics topics across the standards and grade levels. We invite mathematics education leaders, PD providers, and facilitators to use the model to differentiate professional development to address their districts' specific mathematics goals and their teachers' diverse learning needs. As the model is applied, we encourage the exploration of new directions, the sharing of approaches, and research that investigates the impact of differentiated PD on teacher and student learning. ✪

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