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Structure vs. Pedagogy: The Impact of a Flipped Classroom Model of Instruction on Fifth-Grade Mathematics Students

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Abstract

The Flipped Classroom model of instruction is being implemented at all levels of schooling and academic areas; yet, there is very little research regarding its effectiveness. This study attempted to expand this body of research by looking at the Flipped Classroom model as it was implemented in fifth-grade mathematics classrooms. As enacted in this study, the model involved students watching a video lecture at home and then completing traditional homework in class the next day. The participants were 112 fifth-grade students from four classrooms in a Midwestern suburban school district. Qualitative and quantitative data were collected through classroom observations, interviews, and posttests. The Mathematics Teaching Practices were used as a framework to analyze the classroom instruction. Further, research on students' conceptual understanding of decimals and fractions formed the basis for understanding student thinking during interviews. The data suggested that the Flipped Classroom model, as enacted in this study, strongly supported the use of rules and procedures, not always accurately, to the detriment of developing conceptual understanding. Of equal concern was that low-achieving students had less access to the videos at home and more frequently found them frustrating or confusing. Implications for mathematics education leaders are provided.

Introduction

"All students should have the opportunity and the support necessary to learn significant mathematics with depth and understanding. There is no conflict between equity and excellence" (National Council of Teachers of Mathematics [NCTM], 2000, p. 5). This statement in conjunction with the prevailing achievement gap in mathematics (Boykin & Noguera, 2011) has given rise to innovations and research on teaching and learning mathematics in an effort to truly provide "high quality mathematics instruction for all students" (NCTM, 2000, p. 3). One of the many innovations that has become increasingly popular, the Flipped Classroom model of instruction, has now made its way from predominantly post-secondary classrooms into middle school and elementary classrooms (Bishop & Verleger, 2013; Hamden, McKnight, McKnight, & Arfstrom, 2013; Yarbo, Arfstrom, McKnight, & McKnight, 2014). This gives rise to two important questions. First, how does this model of instruction impact elementary-age students and their conceptual understanding and achievement in mathematics? Which then leads one to ask: based on these findings, how do we as mathematics education leaders continue to support high quality mathematics instruction when this model is implemented in elementary and middle school mathematics classrooms in order to promote student conceptual understanding and increased levels of achievement for all students?

The first question can be partially answered by this research study, which sought to examine the Flipped Classroom model of instruction as it was enacted in four fifth-grade mathematics classrooms. Previous research in the area of the Flipped Classroom model has been done at the secondary and post-secondary levels and typically in the areas of science and mathematics with the instructor serving the dual role as the researcher. Achievement on final exams and measurements of attitude based on course reviews have served as the major pieces of evaluation data in most of these studies (Bishop & Verleger, 2013; Hamden et al., 2013; Yarbo et al., 2014). Based on the current body of research, however, there is a significant need for research on the Flipped Classroom model at the elementary level specifically in mathematics with attention to teaching practices and the learning outcomes. Therefore, the purpose of this study was to examine how the Flipped Classroom model of instruction impacted fifth-grade students' achievement in mathematics with a particular focus on conceptual understanding versus procedural understanding. This study also examined teacher practices within the Flipped Classroom model enacted in the classrooms in this study and their alignment or misalignment to the Mathematics Teaching Practices (NCTM, 2014). Specifically, this study addressed the following questions intended to examine both the use of effective teaching practices and student achievement.

1. To what extent does the observed model of Flipped Classroom instruction align with the Mathematics Teaching Practices for high quality mathematics instruction in four fifth-grade classrooms? The specific practices addressed were:
   a. Implement tasks that promote reasoning and problem solving;
   b. Use and connect mathematical representations;
   c. Facilitate meaningful mathematical discourse;
   d. Build procedural fluency from conceptual understanding; and
   e. Elicit and use evidence of student thinking.

2. How is student achievement on the decimal and fraction units affected by the model of Flipped Classroom instruction in this study?
   a. Do the students meet the State Standards for decimal and fraction concepts as measured by the curriculum post-unit tests?
   b. To what extent do student understandings reflect conceptual knowledge of decimals and fractions based on research on student thinking in the areas of decimals and fractions?
   c. To what extent are there differences between high-achieving and low-achieving students' conceptual understanding and achievement in the Flipped Classroom model?

The significance of this study was its ability to inform mathematics education leaders with regard to areas to be addressed in professional development related to the use of class time in a flipped classroom model and issues of equity when enacting a flipped classroom model.

The Flipped Classroom Model

The definition of *Flipped Learning* or the *Flipped Classroom* used in this study was developed by members of the Flipped Learning Network (FLN, 2014) and stated on their website. It defines Flipped Learning as:

> A pedagogical approach in which direct instruction moves from the group learning space to the individual learning space, and the resulting group space is transformed into a dynamic, interactive learning environment where the educator guides students as they apply concepts and engage creatively in the subject matter. (FLN, 2014, "Definition of Flipped Learning")

In line with this definition, the students often watch a video lecture at home for homework and then work on problems or activities related to the video in class the next day. This can be enacted in a variety of ways with the most traditional model being that the students complete the typical pencil-and-paper homework in class (FLN, 2014; Hamden et al., 2013; Strayer, 2012). The teacher is then present to assist students with these practice problems.

This traditional model of the Flipped Classroom was the model observed in the classrooms in this study. The students watched a video each night made by district teachers and based on a lesson in the curriculum. On the next day in class, students worked on the corresponding lesson pages in a workbook. The idea of using the video instruction as homework made class time available to offer high quality, interactive, mathematical experiences to all students. Further, this model allowed the teacher opportunities to engage and interact with students and mathematics in significant ways that a traditional lecture model would not.

Conceptual Framework

Mathematics Teaching Practices

The eight Mathematics Teaching Practices, detailed in *Principles to Actions: Ensuring Mathematical Success for All*

(NCTM, 2014) "represent a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics" (p. 9). These eight Mathematics Teaching Practices are:

1. Establish mathematics goals to focus learning;
2. Implement tasks that promote reasoning and problem solving;
3. Use and connect mathematical representations;
4. Facilitate meaningful mathematical discourse;
5. Pose purposeful questions;
6. Build procedural fluency from conceptual understanding;
7. Support productive struggle in learning mathematics; and
8. Elicit and use evidence of student thinking. (p. 10)

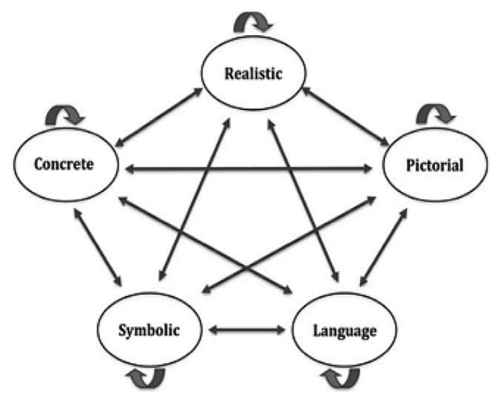
Teacher and student actions are outlined in this document to guide the development of these high-leverage practices and support the development of conceptual understanding of mathematics that students need to acquire. It was these practices, in conjunction with research on conceptual understanding specifically in the areas of fractions and decimals, which created the foundation for the conceptual framework of this study.

Conceptual Understanding

The idea of meaningful mathematics is generally connected to the work of Brownell (1935) who wrote extensively on the importance of teaching for understanding or meaning. Although a balance of meaning and skill is needed to be successful in mathematics (Brownell, 1956), what it means to truly understand needs to be defined. Skemp (1976) defined two types of understanding: relational understanding and instrumental understanding. Relational understanding involves knowing the why behind what one is doing whereas instrumental understanding involves knowing the rules. Relational understanding has been emphasized in curriculum documents (e.g., Common Core State Standards Initiative, 2010; NCTM, 2000) so that procedural or instrumental understanding is developed with accuracy and purpose.

Understanding relationships in mathematics comes from creating and internalizing mental models and making connections among these mental representations (Hiebert & Carpenter, 1992; Hiebert & Grouws, 2007). "Understanding occurs as representations get connected into increasingly structured and cohesive networks" (Hiebert & Carpenter, 1992, p. 69). These mental models or representations are created over time and through experiences. The Lesh Translation Model (Lesh, Post, & Behr, 1987) demonstrates the types of representations and translations that students must experience in order to support the development of conceptual understanding (see Figure 1). For example, when learning about the relative size of fractions, students can use fraction circles or fold paper strips to see them concretely. From there, students may draw pictures, describe them to their classmates, and finally record various equivalent fractions symbolically.

FIGURE 1.
Lesh Translation Model
Adapted from "Representations and Translations Among Representations in Mathematics Learning and Problem Solving," by R. Lesh, T. Post and M. Behr, 1987, In C. Janvier (Ed.), *Problems of Representations in the Teaching and Learning of Mathematics,* pp. 33-40. ©1987 by Routledge. Reprinted with permission.



Research explaining what it means to conceptually understand decimals and fractions, the mathematical focus of this study, includes the use of mental models and translations between models (e.g., Behr, Wachsmuth, Post, & Lesh, 1984; Cramer, Behr, Post, & Lesh, 1997; Cramer, Monson, Wyberg, Leavitt, & Whitney, 2009; Cramer, Post, & delMas, 2002; Hiebert & Wearne, 1986; Hiebert, Wearne, & Tabor, 1991; Roche & Clark, 2004). Researchers are concerned that in order to understand the relative size of fractions and decimals, as well as to compare, order, and compute accurately with fractions and decimals, students need to have many experiences with a variety of representations (Cramer et al., 2009; Hiebert & Wearne, 1986; Hiebert et al., 1991; Roche & Clark, 2004). In addition, connections among these representations are needed in order to develop a deep understanding of fractions and decimals. This research has also suggested that students struggle with interpreting symbolic representations of fractions and decimals. Much of students' difficulties result from their tendencies to employ whole number thinking to a variety of situations, which leads to inaccurate interpretations when comparing, ordering, and estimating with fractions and decimals (Cramer et al., 2009; Hiebert & Wearne, 1986; Hiebert et al., 1991; Roche & Clark, 2004).

Methods

This study was designed to examine what teaching practices existed in four elementary classrooms using a Flipped Classroom instructional model and how these practices affected the conceptual understanding and achievement in mathematics of the students in these classrooms. This study took place during two fifth-grade Math Expressions (Fuson, 2011) curriculum units of instruction (decimals and fractions), which occurred over eight weeks of time in four classrooms (117 students). All four classroom teachers used the Flipped Classroom model. The classroom teachers taught all of the lessons and administered all assessments that included the curriculum posttests. The posttests covered the mathematics content for each unit and were developed by the curriculum authors. This study was unique to the current body of research on the Flipped Classroom in that the majority of the other published work places the researcher in the role of the teacher. In contrast, the researcher in this study was an outside observer.

The context for this study was a suburban school district outside of a large Midwestern metropolitan area. At the time of the study, the district was in its fourth year of Flipped Classroom mathematics instruction at the fourth- and fifth-grade levels. The two schools featured in this study had relatively different demographics from each other although they were in the same district. Because the Flipped Classroom model was used throughout the district (10 elementary schools), the varying demographics of the selected schools allowed for a broader understanding of the impact this model of instruction had on students. Three classrooms were studied at Southside Elementary (pseudonym) because each fifth-grade teacher taught mathematics to his/her own students. One classroom was studied at Central Elementary (pseudonym) because this teacher taught mathematics to all of the students in this grade level. These four classrooms were also chosen based on the teachers' experiences with the Flipped Classroom model and their willingness to participate in the study. The demographics of the students in the study from the two schools as well as the district are shown in Table 1.

In order to document the actions of the teachers and students, classroom observations recorded as field notes were completed during 32 class periods. These observations were guided by the Teacher Actions and Student Actions identified for five of the eight Mathematics Teaching Practices in *Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2014). These five practices (see Research Question 1) were chosen because they were observable during the class periods. In considering the remaining three practices, two were considered difficult to observe. Although the third practice of *Pose Purposeful Questions* was observable, the researcher felt this practice could generate enough data to be a study on its own and would, therefore, distract from the purpose of this study. Therefore, this practice was excluded. In connection to

Table 1: Demographics of School Enrollment

Subgroup	School District N = 8,800	Southside Elementary N = 88	Central Elementary N = 29
Amer. Indian/Alaskan	.7%	0.8%	0.5%
Asian/Pacific Islander	5.2%	9.5%	7.2%
Hispanic	3.4%	9.5%	1%
Black, not of Hispanic origin	4.4%	13.8%	1.5%
White, not of Hispanic origin	86.2%	66.3%	89.7%
ELL	2.0%	13%	1.7%
Special Ed.	13.9%	13.6%	14.3%
Free/Reduced Lunch	16.5%	30.3%	5.2%

observing teacher and student actions through this lens, the elements needed for conceptual understanding, such as multiple representations of fractions and decimals, were noted whenever possible.

The field notes were coded first by activity and then by teaching practices and student actions observed. These actions were then matched, where possible, to the selected Mathematics Teaching Practices (NCTM, 2014). Themes developed that illustrated the routines and practices that were typical in the classrooms. These themes emerged as routine practices in each classroom and then across the four classrooms.

Students completed posttests after the conclusion of each unit. Additionally, 20 students participated in student interviews at the end of each unit (40 students total). These interviews included specific questions about both the students' experiences in the Flipped Classroom and their conceptual understandings of the content from each unit. The students were selected by their teacher and were identified as either high achieving or low achieving based on their test scores and the teacher's knowledge of the student. This was done purposefully to identify possible differences in thinking patterns between the two groups of students as well as how the Flipped Classroom model may or may not have impacted students differently. Questions related to the Flipped Classroom experience included students' opinions of the videos, how often they watched the videos, and their access to adequate internet devices at home. Specific questions involving decimals and fractions were asked so that students could demonstrate their conceptual understandings of comparing, ordering, estimating, and computing with decimals and fractions by explaining their thinking and their use of procedures.

This pragmatic approach of combining both qualitative and quantitative data to answer the research questions allowed for rich descriptions to be developed of what was taking place in the Flipped Classrooms in this study. This approach "attempts to provide evidence that meets the epistemological standard of what John Dewey called warranted assertability" (Johnson & Christensen, 2012, p. 432); that is, what can be a justified belief versus an opinion. The data generated from the themes found in the classroom observations and student interviews was put in concert with quantifiable data such as the frequency of various types of classroom activities and unit test scores to establish a more complete picture of these classrooms using the Flipped Classroom model of instruction and the resulting impact on students' mathematical understandings and achievement.

The results of the study follow in the next sections along with conclusions and recommendations. These conclusions and recommendations are based on the data collected in this study and supported by the research behind its conceptual framework and the research on conceptual understanding of decimals and fractions. However, this study has several limitations. First, the literature on the Flipped Classroom suggests that there are many ways that the model can be enacted. This study only observed one such model; therefore, other versions of the Flipped Classroom may offer different outcomes or results. Second, every student in each classroom was not interviewed so there may be perspectives from the average student not represented in these findings. Finally, the duration of the study was limited to approximately eight weeks of instruction and not every lesson in every classroom during those eight weeks was observed. It would be possible that over a longer period of time, different observations could lead to additional supportive or conflicting findings.

Findings Related to Classroom Activities and Teaching Practices

The classroom observations were conducted over the span of approximately eight weeks during two units of study: Unit 3 – Decimals and Unit 5 – Fractions. The classroom teachers used the district adopted Math Expressions (Fuson, 2011) curriculum for the majority of the students and an alternative sixth-grade textbook for those who passed the unit pretest with a score of 90% or better. In this section, results of the analyses from classroom observations are presented, followed by the alignment of these instructional practices with the selected Mathematics Teaching Practices.

Classroom Observations

Two variations of instructional models were observed (see Table 2). Most lessons began with warm-up problems and then a mini-lecture, which typically lasted 5 – 10 minutes and was based on the previous night's video. The rest of the class period was devoted to independent work time in the student workbooks (see Tables 2 and 3). In the homework videos, the teacher demonstrated the steps in

Table 2: Two Types of Observed Instructional Models

Instructional Model A	Instructional Model B
Students begin the class period with a warm-up or review problems.	Students work on workbook pages individually or with a partner (informal).
Teacher gives a 5 – 10 minute lecture based on the video from the previous night.	Teacher pulls a small group of students together for a short mini-lesson based on need.
Students work on workbook pages individually or with a partner (informal).	Teacher circulates the room assisting individual students.
Teacher circulates the room assisting individual students.	

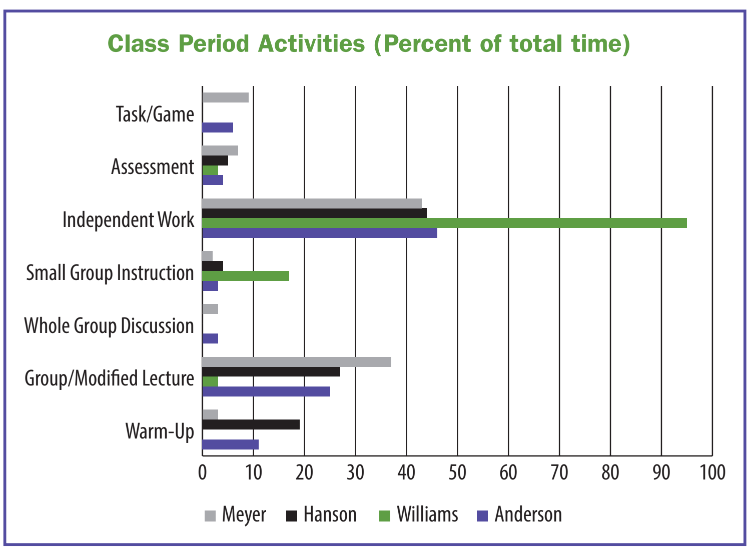
Table 3: Classroom Activity Descriptions

Activity	Activity Description
Task/Game	A whole class activity such as a problem-solving task, a game, or skills practice on a computer
Assessment	A quick quiz from the curriculum
Independent Work	The time that students are working out of their workbook or textbook
Small Group Instruction	A small group session of 3-8 purposefully selected students with the teacher to review specific content
Whole Group Discussion	A whole class session during which students are sharing their strategies, offering new strategies, and asking questions of each other and the teacher – conversation like
Whole Group/ Modified Lecture	A whole class session in which the teacher demonstrates a procedure and sometimes asks procedural questions in an IRE (initiate, respond, evaluate) type dialogue (e.g., "What is 3 x 4?" during the procedure to make common denominators)
Warm-up	Either a commercially made worksheet with one problem from each mathematics strand or several practice problems on the Smart Board connected to the video from the previous night

a procedure and then modeled several practice problems. In the mini-lectures, the teacher did the same thing using sample problems, put on the Smartboard, based on the problems and procedures in the video. Afterwards, students worked with a partner or by themselves on problems in their workbook, and the teacher circulated the room assisting individuals as needed. At times, small groups were pulled to work on a specific skill based on quiz scores or common student questions that had occurred in a previous class period. It appeared, and was also shared in teacher interviews, that the teachers depended on the video as the main vehicle to deliver the instruction.

Within both Instructional Models A and B (see Table 2), a variety of activities, such as whole group lecture, small group instruction and other instructional practices, took place. These activities are described in detail in Table 3. Based on the field notes, the duration of these activities were calculated (see Figure 2). The total percentage for each classroom exceeded 100% because some activities were going on simultaneously in the classroom such as small group instruction and independent work time.

FIGURE 2.
Classroom activities. (Teacher names are pseudonyms)



Alignment with Mathematics Teaching Practices

Overall, there was a weak alignment between the observed actions in the classrooms and the suggested NCTM actions. In general, the observed teacher and student actions rarely matched those identified with the Mathematics Teaching Practices (NCTM, 2014) as evidence of the practice occurring. In the sections that follow, results related to each of the selected Mathematics Teaching Practices will be presented.

Implement tasks that promote reasoning and problem solving. In this study, this practice referred to the teacher-selected work that the students completed in their workbook or notebook. The teacher chose this work from the textbook, as this was the main source for the student tasks. The selected problems aligned with the procedures taught in the video and reviewed in class.

The suggested actions for this practice (NCTM, 2014) call for engaging problems with multiple entry points. The featured problems were procedural-type questions used to practice what the students observed on the video and in class. Although there might have been multiple entry points, a variety of strategies were not observed being discussed and, therefore, were not likely used by the students. In addition, it was difficult to assess the types of reasoning and problem solving that the students used because this was not typically discussed in relation to their independent work. In general, the types of tasks recommended by the NCTM and the subsequent teacher and student actions were not typically observed in these classrooms.

Use and connect mathematical representations. The mathematical representations featured during instruction were all in pictorial form and appeared only when present in the curriculum materials. Typically, this occurred in the first few lessons of each unit. In addition, there was a fraction bar poster in each classroom that was occasionally referenced by the teacher. There was no evidence that any students used the posters as a tool. Further, there were no observations of connections being made among any of the pictorial representations. In part, this could be because of the types of conversation that were observed in these classrooms.

Facilitate meaningful mathematical discourse. When considering the practice of *Facilitate Meaningful Mathematical Discourse*, the majority of the dialogue heard involved the steps in procedures with short student responses. Students were typically asked to contribute the correct answer to the next step in the procedure. Alternatively, students occasionally shared how they answered a question, but this generally involved the steps used rather than the reasoning behind the steps. The *turn to your neighbor* protocol was frequently observed, although what was shared was a single answer or procedure to solve the problem. Students appeared comfortable sharing their ideas both with their partner and with the whole class and in several instances were observed modeling a procedure in front of the class in the role of the teacher. The observed sharing was generally focused on the steps of a procedure and the answer to the problem. An example of the type of discourse most frequently observed follows.

T: Who can tell me what an equivalent fraction is?

S1: Umm, I'm guessing but two fractions with the same denominator?

T: (calls on another student)

S2: Two fractions worth the same amount.

T: (Writes 1/2 and 3/6 on the board) These two fractions are equal – they show the same amount. Now we need to find the multiplier – the factor that we are going to multiply both the numerator and the denominator by to get the equivalent fraction. (Teacher writes a small x3 next to the numerator and denominator of 1/2)

T: (Writes 5/6 = 10/12 on the board) What do you multiply each number in 5/6 by to get 10/12?

S: (Chorally) 2

T: So if you have 15/18 = 5/6 (writes this on the board) what is the divisor?

S3: 3

* Teacher continues with two more examples this time having the students do this in their notebooks and then check with their neighbors about the multipliers. After a few minutes the teacher calls on a couple of students to give the answers – she writes the answers in on the board. (From Lesson 5.12 – Equivalent fractions)

This dialogue was typical of the type of discourse that occurred in these classrooms. That is, the teacher told the students what was needed to solve the problem and demonstrated how to solve the problem. Discussion about why the procedure made sense or how it related to the concept was generally not part of the discourse.

Build procedural fluency from conceptual understanding. Building procedural fluency was observed in all classrooms; however, *Building Procedural Fluency from Conceptual Understanding* was generally not observed. The emphasis was clearly on learning rules or procedures and then practicing these procedures. A great deal of class time was devoted to independent student practice, which stemmed from the instruction in the video and the mini-lecture at the beginning of each class period. Because of the limited use of multiple representations and connections through meaningful discourse in the classrooms, it was difficult to ascertain what level of conceptual understanding the students were using to do the work, compared to memorized rules and procedures.

Elicit and use evidence of student thinking. Two main actions were linked to the practice *Elicit and Use Evidence of Student Thinking*. First, in all classrooms, the most common action observed was that teachers spent a great deal of time talking with students individually. Usually this was driven by the student asking a question specific to a problem that he or she was working on in the workbook. Based on the question, the teacher gained an idea of what was likely misunderstood or confusing to the student. The second action occurred when, in some cases, the teacher pulled small groups of students together who needed similar support based on these individual conversations or previous quiz results from an earlier lesson. However, beyond talking with individual students, instructional decisions on the pacing or order of lessons appeared to be dictated by the curriculum. Every day the video for the next lesson was posted as homework and the in-class work the next day was the lesson workbook pages that went with it. The exception to this was the students working in the sixth-grade textbook who worked at their own pace so they could move ahead if they completed the work. Occasionally, the teacher announced that if a student had finished their assigned Math Expressions (Fuson, 2011) workbook pages they could go on to the next lesson as well or do some other worksheets that may or may not be more challenging. In general, eliciting student thinking centered on student questions or needs based on their ability to complete the questions in their workbooks or on quizzes accurately. Using evidence of student thinking was limited to pulling groups together or allowing students to work in the alternate textbook and work ahead.

Findings Related to Achievement and Conceptual vs. Procedural Understanding

Posttest Analyses

Each unit culminated with a posttest designed by the Math Expressions curriculum. The tests, as well as the lessons in each unit, were aligned to the state standards for fifth grade in the areas of decimals and fractions. All students in the classrooms in this study took the same posttest. This study used the score of 80% or greater as the cutoff to likely meet the state standards. This was a practical decision in that anything less than 80% clearly showed some understanding of the topic; however, misconceptions or errors were taking place which could limit the student's ability to meet the standards in that area at this time. The Unit 3 – Decimal test had a mean score of 91.45% with 94% of the students receiving a score of 80% or higher. The Unit 5 – Fraction test had a mean score of 81.31% with 63.4% of the students receiving a score of 80% or higher (see Tables 4 and 5). Possible insights regarding the difference in student achievement between Unit 3 and Unit 5 could be gained from the analysis of procedural versus conceptual understandings found during the student interviews after each unit. These results follow in the next section.

Table 4: Posttest Achievement Scores

Posttests	N	Mean (%)	SD	Min	Max
Unit 3 Posttest – Decimals	112	91.45	6.91	71.00	100.00
Unit 5 Posttest – Fractions	112	81.31	15.86	37.50	100.00

Table 5: Posttest Scores by Percentage Levels

Score	Unit 3 Posttest N = 112	Unit 5 Posttest N = 112
90 - 100%	75	48
80 – 89.9%	30	23
70 – 79.9%	7	19
60 – 69.9%		9
50 – 59.9%		7
40 – 49.9%		2
30 – 39.9%		4

Interview Analyses

The interviews revealed more detailed information as to how the students were actually thinking about decimals and fractions. During the decimal interviews, the use of whole number thinking was observed across the group of students, both high achieving and low achieving (see Tables 6 and 7). This type of procedural thinking (e.g., "0.7 is greater than 0.4 because seven is more than four" or "add zeros and line up the decimals") typically enabled the students to produce the correct answers while not necessarily understanding what they were doing. Only one student referred to a mental image of a grid and bar to explain how he got his answer.

Both high-achieving and low-achieving students struggled with the two estimating questions because their whole number thinking became an unreliable strategy. For example, the students were asked to think about the number 0.57. Then, they were asked if they were to take away 0.009, would they be left with a little more than a half (0.5) or a little less than a half (0.5). The relative size of the decimal, in the case of 0.009, was not generally thought of as being very small and, therefore, would cause little change to the original number of 0.57. Most students who correctly answered this question provided a procedural explanation, such as, "I imagined doing the problem in my head. I added a zero behind the 0.57 and then lined up the decimals." The purpose of this question, though, was to determine whether a student could use the relative size of a decimal number to make a correct estimation instead of using a procedure to get an answer. The unit posttest did not have any estimating questions on it; therefore, the

Table 6: Decimal Interview Responses by Type from Low-achieving Students

Low-achieving Students (N = 10)				
Questions	**Correct**	**Incorrect**	**Conceptual**	**Procedural**
0.7 or 0.4 Which is larger?	9	1	1	9
0.103 or 0.13 Which is larger?	5	5	1	9
Put these decimals in order from least to greatest: 0.245, 0.025, 0.249, 0.3	5	5	0	10
Estimate 0.37 + 0.4	1	9	0	10
Picture 0.57. If you took 0.009 away, would the amount left be more than a half or less than a half?	2	8	0	10
Solve 0.375 + 2.5	9	1	0	10
Solve 4.85 – 0.437	8	2	0	10

Table 7: Decimal Interview Responses by Type from High-achieving Students

High-achieving Students (N = 10)				
Questions	**Correct**	**Incorrect**	**Conceptual**	**Procedural**
0.7 or 0.4 Which is larger?	10	0	1	9
0.103 or 0.13 Which is larger?	10	0	0	9
Put these decimals in order from least to greatest: 0.245, 0.025, 0.249, 0.3	10		0	10
Estimate 0.37 + 0.4	8	2	0	10
Picture 0.57. If you took 0.009 away, would the amount left be more than a half or less than a half?	7	3	4	6
Solve 0.375 + 2.5	10	0	0	10
Solve 4.85 – 0.437	10	0	0	10

Table 8: Fraction Interview Responses by Type from Low-achieving Students

Low-achieving Students (N = 10)				
Questions	**Correct**	**Incorrect**	**Conceptual**	**Procedural**
Put these fractions in order; 1/5, 1/3, 1/4	6	4	6	4
Which fraction is larger 4/5 or 11/12?	0	10	4	6
Which fraction is smaller 1/20 or 1/17?	5	5	7	3
Are these fractions equal or is one less, 5/12 or ¾?	7	3	3	7
Are these fractions equal or is one less, 6/4 or 6/5?	6	4	2	8
2/5 + ¾ = 5/9 Do you agree?	9	1	0	10
Estimate: 7/8 + 12/13	1	9	1	9
Solve: 2 1/5 + 1 ¾ =	0	10	0	10
Solve: 4 1/8 – 2 2/4 =	0	10	0	10

Table 9: Fraction Interview Responses by Type from High-achieving Students

High-achieving Students (N = 10)				
Questions	**Correct**	**Incorrect**	**Conceptual**	**Procedural**
Put these fractions in order; 1/5, 1/3, 1/4	10	0	8	2
Which fraction is larger 4/5 or 11/12?	8	2	4	6
Which fraction is smaller 1/20 or 1/17?	10	0	7	3
Are these fractions equal or is one less, 5/12 or ¾?	9	1	1	9
Are these fractions equal or is one less, 6/4 or 6/5?	10	0	4	6
2/5 + ¾ = 5/9 Do you agree?	10	0	2	8
Estimate: 7/8 + 11/12	7	3	7	3
Solve: 2 1/5 + 1 ¾ =	10	0	0	10
Solve: 4 1/8 – 2 2/4 =	10	0	1	9

use of whole number thinking and following rules likely allowed many students to provide correct answers regardless of whether they had a conceptual understanding of the relative size of the decimal number.

The interviews after the fraction unit test showed more use of mental images or pictorial representations to explain some answers, such as working with unit fractions; however, they were not used consistently or to support estimation with fractions (see Tables 8 and 9). Further, all of the students interviewed could state the need for making common denominators prior to adding or subtracting fractions; however, very few were able to explain why they should do that and only 10 of the 20 students could do it accurately. Most students could explain how the denominator relates to the size of a piece of pizza or a candy bar when looking at unit fractions or fractions with a common numerator. Some students described this while others drew a simple picture. However, this same type of thinking tended to not be used when students were asked to compare fractions with unlike numerators. For example, when asked, "Which is greater 4/5 or 11/12?" common responses included, "They are equal because they are both one piece away from a whole," or, "The answer is 11/12 because the numbers are bigger." Regardless of the type of question, the students typically tried to find the common denominators before comparing, estimating, or computing with fractions. This frequently resulted in the wrong answer or a correct answer based on a procedure versus any demonstration of the conceptual understanding of the relative size or equivalence of a fraction.

The inconsistent demonstration of conceptual understanding and consistent, but frequently inaccurate, use of procedures potentially contributed to the wider range of test scores on the fraction unit test as well as the smaller number of students receiving a score of 80% or greater compared to the decimal posttest. Based on the student interviews, it would appear that a limited number of students had developed a conceptual understanding of fractions.

Findings Related to Overall Classroom Experience

All 40 students (20 high achieving and 20 low achieving) interviewed were asked the same five questions about their feelings toward mathematics and specific aspects of the Flipped Classroom model. Many students liked mathematics to some degree. In addition, they liked working with friends and having a video for homework instead of pencil-and-paper homework. The differences emerged, however, when asked specifically about the videos and their home computer and internet access (see Table 10). The high-achieving students generally liked how the videos told the student what to do. In contrast, the low-achieving students frequently reported the videos to be confusing. Many of these students also reported frustration with not being able to ask their teacher a question during the video and typically did not re-watch a video as often as the high-achieving students. This difference in re-watching the videos could be linked to the fact that some of the low-achieving students had to watch the videos at school because they did not have computer access at home. A few shared that they did not like to miss class to watch the video therefore re-watching the video could make this a worse situation. In general, the high-achieving students reported the use of multiple home devices to watch the videos and good internet connections. Alternatively, the low-achieving students typically had one device at home with mixed comments on their internet connections. The interview data suggested that there were discrepancies in access to computers and the internet as well as in experiences with the videos between low-achieving and high-achieving students.

Discussion

The definition of the Flipped Classroom used in this study began with the language, "a pedagogical approach" (FLN, 2014, "Definition of Flipped Learning"). This implies that what the teacher does within the model is critical to the success or failure of the model and that of the students. Further, the definition described a classroom that is a "dynamic, interactive learning environment where the educator guides students as they apply concepts and engage creatively in the subject matter" (FLN, 2014, "Definition of Flipped Learning"). These ideas would

Table 10: Student Interview Responses Regarding Video and Technology Access

	Low-achieving Students	High-achieving Students
Positive Feedback	• Videos are helpful • Liked video homework better than workbook homework	• Liked video homework • Videos tell you how to do it
Negative Feedback	• Videos are too long • Videos are confusing and go too fast • Prefers lesson in class so you can ask questions • Misses having a teacher • Didn't like missing class to watch the video	• Videos are boring
Re-watching Videos	• 8 out of 20 had re-watched a video	• 11 out of 20 had re-watched a video
Computer Access	• Most have only one device in their home to watch the videos • 6 out of 20 students reported that they do not have internet access at home • About half reported a slow connection	• Most have multiple devices to watch the videos • Most report that they have a good internet connection

appear to align with the expectations for high quality mathematics instruction for all students (NCTM, 2014). The teacher is responsible for intentionally and purposefully selecting engaging tasks with multiple entry points, offering many experiences with multiple representations, making connections among the representations, and then making instructional decisions based on elicited student thinking. The purpose in these actions, based on research, supports the deep learning of mathematics both conceptually and procedurally. The qualitative data in this study, however, suggested that the observed Flipped Classroom model supported the teaching of rules and procedures and did not necessarily align with the expectations that support deep learning.

During the student interviews, the use of rules or procedures dominated the processes used by the students, although not always accurately. When merging qualitative findings with the quantitative posttest data, it suggested that students were able to demonstrate their ability to meet the state standards more frequently in the area of decimals when taking a test based on the use of procedures. Conversely, when the students were less able to utilize the procedures and had limited conceptual understandings, they did not perform as well, as in the case of the posttest on fractions in which fewer students were likely to meet the state standards at that time. Further, the data from the student interviews suggested that lower-achieving students tended to be more frustrated by the videos, did not re-watch the videos as often, and had more access issues to computers and the internet compared to their high-achieving classmates.

Research-based practices were generally not employed in the Flipped Classroom model examined in this study. Further, the FLN description, stated at the beginning of this section, did not seem to describe the classrooms observed. Teacher beliefs about teaching and learning mathematics can be productive or unproductive (NCTM, 2014) and greatly influence what happens in the classroom. The importance of doing this study from an outside observer perspective brought these conflicts to light.

Of equal concern were the issues surrounding the differences between high-achieving students and low-achieving students in regards to their reactions to the videos and their access to computers at home along with the internet connection. From an adult perspective, including secondary and post-secondary students in other studies (Bishop & Verleger, 2013; Hamden et al., 2013; Yarbo et al., 2014), the opportunity to be able to watch a video repeatedly is very appealing when working with challenging material. Elementary-age students in this study, however, did not appear to share this same thought. Likely due to computer access issues, this may be especially true for those low-achieving students in need of the most support mathematically. The use of videos at home may be supporting the disparity in achievement between high-achieving and low-achieving students in this study instead of being a useful tool for learning, as perceived by adults. As the NCTM Equity Principle states, "Access to technology must not become yet another dimension of educational inequity" (NCTM, 2000, p. 14).

Recommendations

The idea of flipping the classroom has become very popular across all levels of education and many content areas (FLN, 2014). This study demonstrated that teachers who choose to implement this model in their classrooms need to be very intentional with their pedagogy within this model just as they would within the standard classroom model. The idea or structure of flipping the classroom does not necessarily support students any more than the traditional classroom model. The intentional use of effective practices is one critical element to the success of the students. Based on this research, three recommendations are offered.

First, teachers utilizing a Flipped Classroom model need the opportunity to explore how to use the classroom time that is freed from lecture and turn it into productive activity that supports the significant understanding of mathematics. Mathematics education leaders cannot assume that because a teacher has adopted a new instructional model in his/her classroom that the instruction in the classroom will change or that students will automatically benefit.

Second, alternative methods to support students' access to the videos in a Flipped Classroom model need to be developed. Using other class time during the school day is not an equitable approach to solving this problem. Communication with families, while potentially challenging, could play an important role in working to resolve this issue.

Third, support is needed for teachers who want to implement a Flipped Classroom model that is consistent with the Mathematics Teaching Principles (NCTM, 2014). Collaborative planning or coaching that focuses on using the newly available classroom time for encouraging productive discussions and engaging students in high quality tasks is essential. A possible model could be the Four-Phase Process designed by Strayer, Hart, and Bleiler-Baxter (2016), which involves using the homework video as a jumpstart to the in-class lesson. Students come to class having had the opportunity to think about a problem ahead of time, based on some background information, and then use the class time to engage in rich discussion and problem-solving activities to learn the mathematics content. This is a new space in professional development that would be valuable for teachers interested in using a Flipped Classroom model at all levels.

Concluding Remarks

Implementing the Mathematics Teaching Practices (NCTM, 2014), changing pedagogy, and creating a new learning structure or environment are very complex tasks that teachers are undertaking. The intent is to provide students with excellent instruction so that all students have the opportunity to succeed. This study used the research behind the Mathematics Teaching Practices and conceptual understanding of fractions and decimals to examine the Flipped Classroom model. In doing so, it offers insight into a very popular, yet minimally researched, instructional model. The results of this study highlight the importance of considering how changes in class structure influence not only student learning but also the learning experiences of specific groups of students. The Flipped Classroom model will continue to be implemented across the United States; therefore, it is critically important to continue to support the development of research-based teaching practices as well as encourage an acute awareness of newly created issues of equity based on the use of technology. Research-based practices that support high-quality mathematics instruction for all students as well as equitable learning environments are necessary regardless of the teaching model, if we are going to close the achievement gap. ✪

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