NCSM Journal

of Mathematics Education Leadership

SPRING 2017 VOL. 18, NO. 1

National Council of Supervisors of Mathematics www.mathedleadership.org

Table of Contents

Meeting the Needs Expressed by Teachers: Adaptations of the Traditional Model for Demonstration Lessons

Jeremy F. Strayer, *Middle Tennessee State University*
Angela T. Barlow, *Middle Tennessee State University*
Alyson E. Lischka, *Middle Tennessee State University*
Natasha E. Gerstenschlager, *Western Kentucky University*
D. Christopher Stephens, *Middle Tennessee State University*
J. Christopher Willingham, *James Madison University*
Kristin S. Hartland, *Middle Tennessee State University*

Abstract

Demonstration lessons are one means for providing teachers with opportunities to reflect on instruction. Although different models for demonstration lessons are described in the literature, the Implementing Mathematical Practices And Content into Teaching Project, or Project IMPACT, developed two additional models of demonstration lessons in response to the expressed needs of project participants. In this article, we introduce these two models with the goal of supporting mathematics education leaders in enacting these models, or further adapting them, in their own work. Further, we aim to demonstrate how these models were developed in response to project participants' needs.

Introduction

With increased expectations regarding mathematics learning (e.g, Common Core State Standards Initiative [CCSSI], 2010), there is a strong need to support teachers as they "envision and implement classrooms in which students are effectively engaged in learning mathematics and understand the instructional decisions that they need to make in order to create this environment" (National Council of Supervisors of Mathematics [NCSM], 2014, p. 1). This type of mathematics teaching is complex (National Council of Teachers of Mathematics [NCTM], 2014) and requires that mathematics education leaders "model effective instructional strategies" (NCSM, 2014, p. 16) as a way to encourage teachers to professionally reflect on instruction (NCSM, 2014). One way to provide teachers with the opportunity for such reflection is through the use of demonstration lessons (Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2010).

Demonstration lessons represent one type of public teaching[1], where an instructor conducts a lesson with students and

[1] We recognize that there are many ways to use public teaching for professional development, such as lesson study and model lessons. In this paper we focus solely on demonstration lessons.

invites other teachers and colleagues into the classroom to observe and reflect upon that lesson (Loucks-Horsley et al., 2010). The traditional demonstration lesson model combines briefing (prior to the lesson) and debriefing (after the lesson) discussions with a lesson observation to provide rich opportunities for teacher learning and is recommended as an effective tool to facilitate professional development activities for teachers (Conference Board of the Mathematical Sciences, 2012). There are many purposes for which demonstration lessons can be employed, therefore different recommendations exist regarding how to conduct demonstration lessons (Casey, 2011). In our professional development project, however, the purpose and design of demonstration lessons emerged in direct response to the needs of participating teachers. In this article, we discuss different models of demonstration lessons for professional development in the literature and then share our refinements of demonstration lessons to better meet the professional learning needs of project participants. In doing so, our intent is two-fold. First, we aim to introduce two new models of demonstration lessons and support the reader's understanding of these models so that they may be employed in other settings. Second, we seek to demonstrate how these models of demonstration lessons emerged based on the project goals and participants' needs.

Demonstration Lessons in the Literature

Most professional development providers who use demonstration lessons do so to create a space for teachers to critically reflect upon the practice of teaching by observing the overall classroom environment and the teacher's actions during a lesson (Clarke et al., 2013). To accomplish this, demonstration lessons are generally structured to include: a briefing that focuses observing teachers' attention on selected mathematical or pedagogical features of the lesson; the observation of the demonstration lesson where observers record notes on the features of interest; and a debriefing where the observing teachers' observations are discussed along with implications for future instruction (Clarke et al., 2013; Loucks-Horsley et al., 2010). This structure allows a demonstration lesson to be conducted in one sitting.

When reviewing the literature, we noticed specific reasons for which demonstration lessons were implemented. The most common purpose was to cast a vision for what mathematics instruction that focuses on student thinking can look like and to invite observing teachers to consider how they might change their practice to align with this approach. We refer to this model of demonstration lessons as *exemplar demonstration lessons*. For example, Clarke and colleagues (2013) used exemplar demonstration lessons in a large (over 650 teachers), multi-year professional development project aimed at supporting teachers as they transitioned to incorporating reform-oriented teaching practices in their classrooms. Throughout the project, teachers attended one or more demonstration lessons. During a demonstration lesson briefing, teachers were given the freedom to choose their own focus areas for the observation with regard to both teaching and student learning. As teachers observed the demonstration lesson, they recorded what they noticed on an observation form that encouraged observing teachers to consider the connections between teacher actions and student responses. During the debriefing, teachers reported what they had observed. After the debriefing, teachers reflected on the experience and shared anything that occurred that they believed would contribute to a change in their own teaching practices. Teachers were also asked to describe any intended changes in their practices. In general, observing teachers often initially struggle to focus on anything other than the teacher during a demonstration lesson. However, as a result of this work, Clarke and colleagues concluded that their structure for a demonstration lesson resulted in observing teachers having a greater focus on both student thinking and teacher actions. Also, many of the observing teachers in this study intended to change their practice to include greater opportunities for students to articulate their thinking and to increase their use of hands-on resources to support student thinking. These results are typical of successful uses of exemplar demonstration lessons.

Other professional development projects have conducted exemplar demonstration lessons with the added step of having the observing teachers return to their classrooms and teach the exact same lesson with their own students. We call this model *replicated demonstration lessons*. In one such study, Herbert, Vale, Bragg, Loong, and Widjaja (2015) chose teachers' ability to notice and attend to students' mathematical reasoning as their focus. The briefing prepared participating teachers for this focus. Then teachers observed a demonstration lesson and, during the debriefing, discussed what they noticed about students' mathematical reasoning throughout the lesson. Next, each teacher taught the exact same lesson in his/her own classroom. After the lesson replication, the researchers

interviewed teachers to gain insight into their developing abilities to notice and respond to students' mathematical reasoning. Later in the project, the teachers participated in another replicated demonstration lesson. They observed a second demonstration lesson, debriefed with other teachers, taught the exact second lesson with their own students, and participated in a second interview. Herbert and colleagues analyzed data collected throughout the study and classified the various ways in which teachers perceived what constitutes mathematical reasoning, which included: thinking; communicating thinking; problem solving; validating thinking; forming conjectures; using logical arguments for validating conjectures; and connecting aspects of mathematics. During this study, the replicated demonstration lesson model provided teachers with multiple vantage points from which to notice student mathematical reasoning during a lesson: an outsider's view as observer and an insider's view as the teacher of the lesson.

Both exemplar demonstration lessons and replicated demonstration lessons engage teachers in meaningful reflection on instructional practices. The lessons observed in exemplar demonstration lessons aim to provide a vision of the type of mathematics instruction needed to engage students in learning meaningful mathematics. This vision is extended to include implementation within the classroom in replicated demonstration lessons. During our professional development project, we wondered if other models of demonstration lessons might be useful for moving teachers beyond envisioning and implementing effective mathematics instruction towards understanding the instructional decisions made in this regard. Therefore, the following section presents an overall description of our project followed by descriptions of our models for demonstration lessons.

Demonstration Lessons in Project IMPACT

The Implementing Mathematical Practices And Content into Teaching Project, or Project IMPACT, is an ongoing professional development effort that serves over 150 K-8 mathematics teachers, primarily drawn from five partner districts. The project seeks to promote teacher growth in four critical areas: building mathematical knowledge and employing it in the work of teaching; utilizing student thinking during instruction; developing productive habits of mind; and building collegial relationships to support continued learning (NCTM, 2010). The work of this five-year project has entailed classroom observations of a sample of teachers, two-week intensive summer institutes that incorporate immersion and practice-based experiences (Loucks-Horsley et al., 2010), fall and spring sessions that continue to provide immersion and practice-based experiences during the school year, and multiple fall and spring demonstration lessons.

During Project IMPACT, demonstration lessons have been key to supporting participants' continuous professional learning. So that participants can observe the demonstration lessons live, the project pays for substitute teachers and participant mileage. Participants travel to a selected school where the demonstration lesson is conducted in a large room, such as a library or gym. Between 30 and 60 participants participate in any given IMPACT demonstration lesson. Including the briefing and debriefing sessions, one demonstration lesson is typically completed during a three-hour block of the school day.

From the beginning, our broad goal has been to use demonstration lessons to help participating teachers move from a practitioner's stance to professional development (Farmer, Gerretson, & Lassak, 2003), which focuses on taking ideas from professional development and using them with little modification in the participant's own classroom, toward an inquiry stance, which focuses on using professional development as an opportunity to investigate the teaching process. To help participants embrace an inquiry stance, we knew we needed to seek to impact participants' knowledge and beliefs, which have been shown to influence their instructional practices (Ernest, 1989).

As the IMPACT team planned, implemented, studied, and revised the work of the project, we developed different models of demonstration lessons in response to the needs of participating teachers. In the paragraphs that follow, we describe how the team has used demonstration lessons to support participants' professional growth during the project.

Initial Demonstration Lessons

At the onset of the project, IMPACT staff conducted classroom observations in pairs of a subset of teachers that represented approximately 25% of the participants and was drawn from its two primary partner districts. The Reformed Teaching Observation Protocol (Arizona Board of Regents, 2002) was utilized during these observations. In utilizing this protocol, each observer developed a written record of the lesson that included statements and questions offered by students and teachers as well as pictures of

FIGURE 1.
Exemplar demonstration lesson structure.

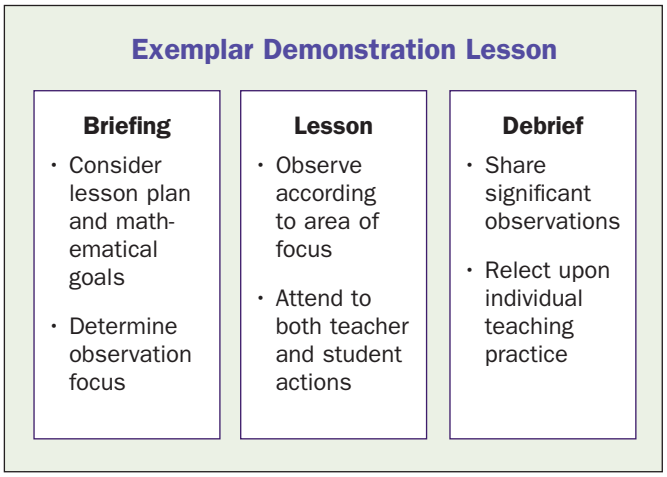


FIGURE 2.
Summary of the Acrobat Task (Burns, 1996).

In round 1 of a tug-of-war, four acrobats tied with five grandmas.

In round 2 of a tug-of-war, one dog tied with two grandmas and an acrobat.

In round 3 of a tug-of-war, three grandmas and the dog are pulling against four acrobats. Who will win?

student-generated artifacts from the lesson. An analysis of these written records revealed two common instructional features that were grounded in participants' knowledge and beliefs. First, participants did not scaffold student engagement in the problem-solving process. Therefore, students did not engage in productive struggle. Second, participants demonstrated mathematical procedures first and then asked students to apply these procedures to solve problems. This pattern of instructional practice did not support deep learning of mathematics (NCTM, 2014).

In response to these observations, the IMPACT team planned an initial round of exemplar demonstration lessons to model teaching through problem solving. The goals of these demonstration lessons were: first, to set a vision of effective mathematics instruction; and second, to gain traction with teachers and inspire them to change their practice to align with the vision. In this way, these initial demonstration lessons aligned with the literature, both in terms of purpose and design (Loucks-Horsley et al., 2010). The vision for instruction in the demonstration lessons was grounded in the Standards for Mathematical Practice (CCSSI, 2010) and the Mathematics Teaching Practices (NCTM, 2014). Figure 1 provides a visual of these exemplar demonstration lessons with the briefing and debriefing occurring immediately prior to and after the demonstration lesson, respectively.

As an example, the initial demonstration lesson in Project IMPACT featured the Acrobat Task (Burns, 1996), which is summarized in Figure 2. Note that the original task uses pictures to communicate what happens in each of the tug-of-war rounds. During the briefing, participants reviewed this task and expressed concern. They imagined that the IMPACT instructor would give students the task and ask them to work independently for 10 minutes before sharing their thinking with others. Participants communicated that students' unfamiliarity with such a task would lead to an inability to successfully find an entry point to solving the problem. These concerns disappeared, however, once the lesson plan was distributed and participants gained insight into the scaffolding that was provided to support students' engagement in the problem. Figure 3 (next page) provides the opening portion of the lesson plan, with the intended scaffolding represented in the bolded statements. The enacted lesson demonstrated this scaffolding, which led to students successfully engaging in problem solving and producing their solutions. In this way, the lesson provided a vision for how to support students' engagement in productive struggle along with a vision for instruction that did not follow the common teaching practice of demonstrating procedures to be applied by students.

During subsequent project activities, some participants shared that they had successfully implemented the demonstration lessons in their own classrooms. In this way, although the IMPACT team did not intend for these initial demonstration lessons to be replication demonstration lessons, some participants sought to make initial changes to their practice by replicating the demonstration lessons in their own classrooms.

Day Two Demonstration Lessons

During project IMPACT's second year, the team continued to implement exemplar demonstration lessons. At the debriefing sessions, participants were encouraged to consider ways in which they might change their own practice to align with research-based instructional practices. As participants shared during these sessions, though, it became clear that demonstration lessons exposed students' mathematical

FIGURE 3.
Opening portion of the lesson plan for the Acrobat Task (Burns, 1996) with scaffolding aspects in bold.

Warm-up (5 minutes)

Display the following question on the document camera.

What do you know about the game of Tug of War?

Allow 30 seconds for independent think time, 30 seconds of pair time, and 3 minutes of share out time. Utilize index cards to call on groups to share out. At this time, try to bring out strength as the key factor in winning a tug of war.

Acrobat, Grandmas, and Ivan Task

Understanding the Problem (15 minutes)

Display the initial problem sheet. Introduce the people who will be featured in the problem.

Display the Round 1 picture and context on the document camera. Read Round 1 aloud.

Think-pair-share: **Based on this information, what is something that we know about the grandmas and acrobats?** As students share their ideas, record these on a piece of chart paper with "Round 1" as the heading.

Display the Round 2 picture and context on the document camera. Read Round 2 aloud.

Think-pair-share: **Based on this information, what is something that we know about the grandmas, acrobats, and Ivan (the dog)?** As students share their ideas, record these on a piece of chart paper with "Round 2" as the heading.

misunderstandings yet left little time to resolve them in a single lesson. This was problematic for participants, and they would often ask during debriefing discussions, "What would you do the next day?" In response, we structured *day two demonstration lessons* for the third year of the project to answer this question. A general description of this model is provided in the next section, followed by an example taken from Project IMPACT.

Description. *Day two demonstration lessons* aim to help teachers develop lessons that build from one day to the next based on students' thinking. In this model (see Figure 4), an instructor first teaches a lesson to a teacher's class. The lesson is video recorded. On the next day, a group of teachers gather for the day two demonstration lesson. In this setting, the briefing involves the group examining the lesson plan for the previous day's lesson along with edited video of the lesson. This discussion includes reviewing the original day two lesson and any modifications to the lesson that resulted from students' conceptions and misconceptions that surfaced during the previous lesson. Then, teachers observe the day two demonstration lesson, followed by a debriefing that considers implications for future instruction. In addition, teachers discuss ways in which they might design instructional experiences based on the understandings and misunderstandings of students in their own classrooms.

Example. To describe how the day two demonstration lesson process unfolded in Project IMPACT, we share an example that is based on the L Problem (Watanabe, 2008). The L Problem (see Figure 5 on next page) is intended to engage students in finding the area of a non-typical shape, with a goal of developing strategies that can later be utilized to generate area formulae for shapes (e.g., parallelograms, triangles, trapezoids). An IMPACT instructor taught this lesson in one participant's classroom, while other staff members observed and video recorded the lesson. During this lesson, many students decomposed the shape into rectangles, found the area of each rectangle, and then incorrectly multiplied the different subareas to find the total area.

FIGURE 4.
Day two demonstration lesson structure.

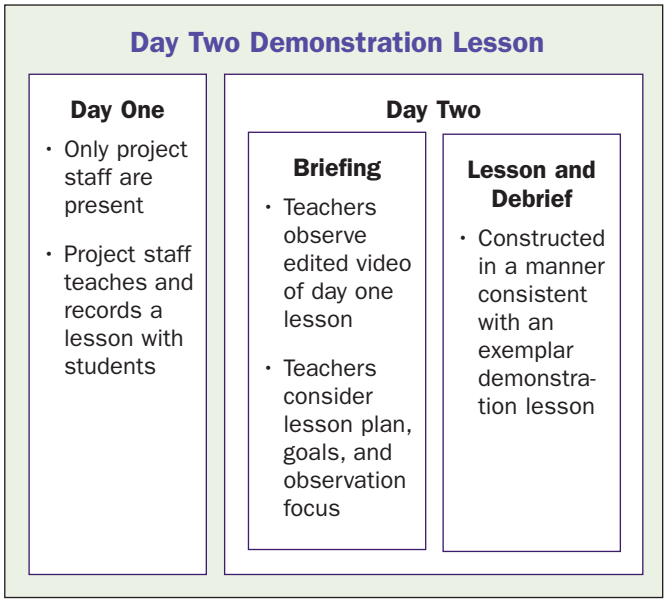
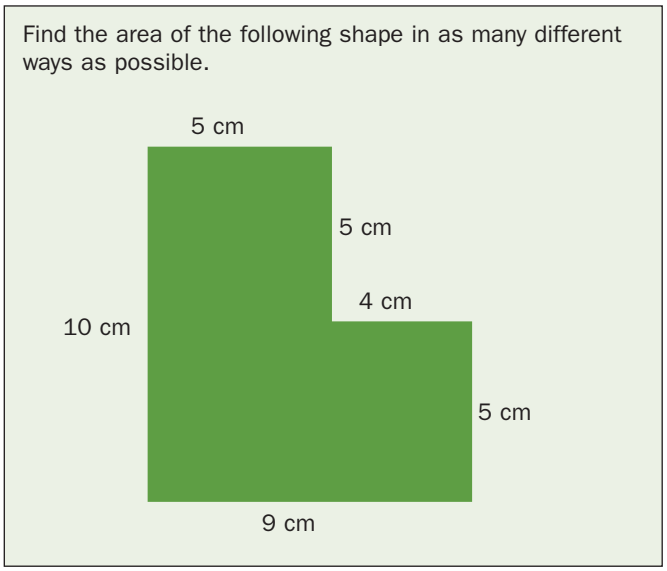


FIGURE 5.
The L Problem (Watanabe, 2008).
Note that the figure should be drawn to scale.



During the briefing of the day two demonstration lesson the next day, participants examined the lesson plan for the L Problem along with the edited video of the lesson. Participants then reviewed the student work from this lesson to better understand the students' views of area, which appeared to be limited to length times width and relied on multiplication as the operation without justification as to why this might (or might not) be appropriate. Next, IMPACT staff described the need for students to consider counting squares as a means for thinking differently about area and for verifying and/or making sense of solutions. To accomplish this, the lesson instructor explained that the upcoming lesson would include asking students to hold centimeter grid paper behind the L-shape in order to count and determine the area. Then, students would be directed to find the area of the F-shape (see Figure 6a), which provided an opportunity to apply this square counting strategy or other strategies. In working with the F-shape, students would be given one-inch graph paper and rulers. Figure 6b shows what the F-shape looks like when replicated onto graph paper.

FIGURE 6a.
The F-Shape as distributed to students in the day two demonstration lesson. Note that the figure was drawn to scale and students had rulers with which to work.

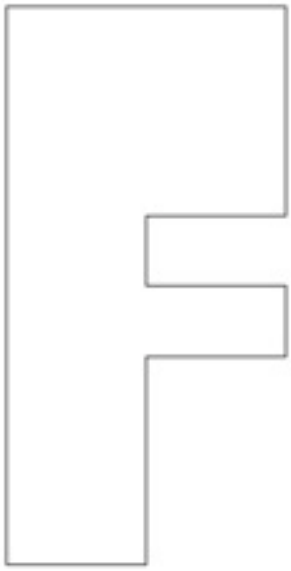
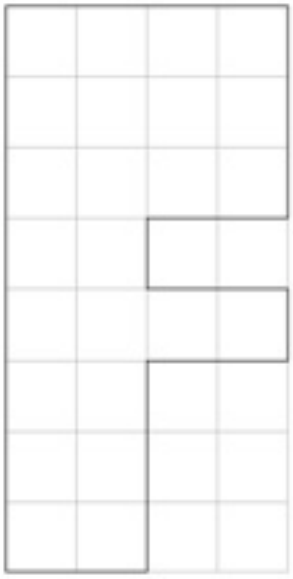


FIGURE 6b.
The F-Shape with an inch-grid superimposed on it.



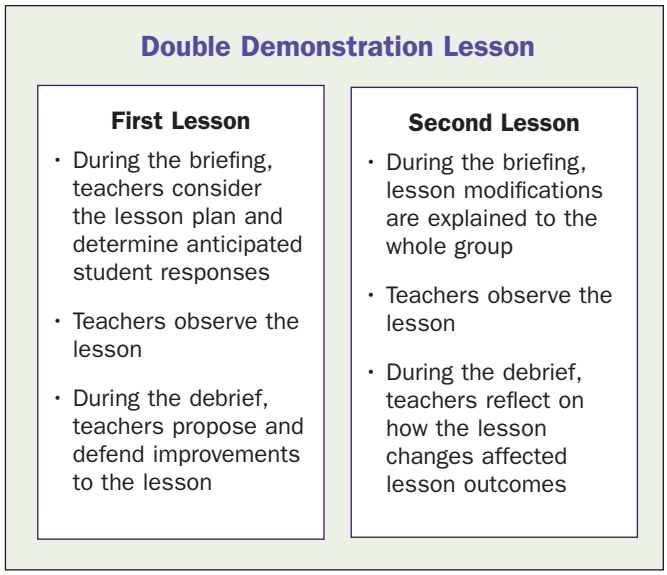
During the day two demonstration lesson, the participants observed how the lesson built from the previous day's work, as students counted the centimeter squares to determine the area of the L-shape and compared this solution and process to their ideas from the previous day's lesson. Once students realized that the subareas should be added rather than multiplied, they were better prepared to successfully find the area of the F-shape. In doing so, students either replicated the F-shape onto graph paper and counted the squares or decomposed the F-shape into rectangles and added the areas of these subregions. After the lesson, participants' discussions during the debriefing centered on the notion of using students' mathematical reasoning to build from one lesson to the next.

Double Demonstration Lessons

As the IMPACT team planned the work for year four of the project, we considered the professional growth of the participants. During the day two demonstration lessons, we noticed that some of the participants were still operating with a practitioner's stance toward professional development (Farmer et al., 2003). That is, they were focusing on specific ideas that could be taken from the demonstration lessons and used with little modification in their classrooms. However, we also observed other participants adopting an inquiry stance during the debriefing. These participants focused on how they could use the demonstration lesson to investigate the teaching process, suggested changes to the lesson, and hypothesized how those changes might influence the lesson outcomes. In fact, one participant, who was clearly demonstrating an inquiry stance, stated, "I wish we could teach this lesson again to see how our suggestions will impact the lesson." In response, we introduced *double demonstration lessons* during the fourth year with the goal of supporting all participants in adopting an inquiry stance. A general description of this model is provided in the next section, followed by an example taken from Project IMPACT.

Description. Double demonstration lessons (see Figure 7) incorporate two rounds of the briefing, observation, and debriefing cycle in a single day. In the first briefing, teachers review the lesson plan for the demonstration lesson. As teachers reflect on the lesson plan, they discuss: what they hope to observe during the lesson with regard to teacher and student actions; how they will know it if they see it; how they will record their observations; what student misconceptions they might observe; and what portions of the lesson plan currently concern them. This section of the briefing is intended to help teachers see the demonstration lesson as an inquiry process in which they can learn about the lesson in order to improve the lesson for student learning.

FIGURE 7.
Double demonstration lessons.



After the briefing, an instructor teaches the lesson to a class of students. During the first debriefing, teachers reflect on areas for improvement in the lesson with regard to student engagement in the task, content, and the mathematical practices. Teachers decide on recommended revisions for the lesson and present these with justifications. Then, depending on the size of the group, a subset of teachers determines the final revisions for the lesson. With these revisions in hand, the same instructor teaches the modified lesson to a second class of students. Finally, during the debriefing of the second demonstration lesson, teachers reflect on how the changes from the first to second lesson affected lesson outcomes.

Example. To describe how the double demonstration lesson process unfolded in Project IMPACT, we share an example that is based on the Sharing Chocolate Task (Enns, 2014). This problem features a group of four students sharing three chocolate bars equally and a group of eight students sharing six chocolate bars equally. The problem asks students to consider how much chocolate students in each group receive as well as which group of students will receive more chocolate. Although there are several potential mathematical goals for which this problem could be used, the IMPACT instructor chose to use the problem as a means to help students understand that the fraction of chocolate received by each person can be represented by the number of chocolate bars divided by the number of people. The IMPACT instructor wrote the lesson plan to match the description of its enactment found in the article by Enns (2014).

Time in the initial briefing was spent acquainting the participants with the problem and its accompanying lesson plan. IMPACT staff also made participants aware of the double demonstration goal: to watch the first lesson with an eye on modifications that could be made to the lesson that would influence student learning. After observing the demonstration lesson, the first debriefing involved participants in small groups discussing their observations and developing suggested modifications for the lessons with justifications. Then, each group presented their ideas to the

Table 1: Summary of Demonstration Lessons Used in Project IMPACT

Model	Motivation	Intended Impact
Exemplar Demonstration Lesson[s]	• Participants did not scaffold students' engagement in problem solving. • Participants demonstrated procedures to be duplicated by students.	• To provide participants with instructional strategies for scaffolding towards problem solving. • To set a new vision for what effective mathematics instruction might look like.
Day Two Demonstration Lesson	• Participants expressed uncertainty regarding how to follow up a lesson that exposed students' mathematical misunderstandings/ shortcomings.	• To support participants' understandings of designing lessons that build on students' mathematical reasoning.
Double Demonstration Lesson	• Participants wondered how their suggestions for lesson modifications would influence the learning outcomes.	• To provide all participants' with the opportunity to engage in practices associated with an inquiry stance towards teaching.

[a] Although Project IMPACT chose not to use replicated demonstration lessons, many participants elected to utilize the exemplary demonstration lessons as if they were replicated demonstration lessons.

whole group. Because our group was large (60 participants), a subset of participants (i.e., those that had been with the project since its inception) were tasked with making the final decisions regarding lesson modifications. Then, the IMPACT instructor taught the modified lesson, which was followed by a second debriefing that focused on evaluating the impact of the lesson modifications.

As participants reflected during the second debrief, several noted that small changes led to significant influences on students' mathematical understandings. Other participants stated that seeing the enactment of the lesson modifications caused them to rethink some of the instructional assumptions that led to the suggested modifications. In this way, double demonstration lessons provided participants with an opportunity to adopt an inquiry stance and to recognize that the act of teaching is an opportunity for their own personal professional learning to occur.

Summary of Demonstration Lesson Models

As we reflect on the models of demonstration lessons, both from the literature and from our own work, we recognize that the use/development of each model was motivated by different circumstances and with different intentions based on project goals and participants' needs. Table 1 summarizes the models as they were utilized in Project IMPACT.

Conclusion

With increased expectations regarding the mathematics that students are to learn (e.g., CCSSI, 2010), there exists the need to support mathematics teachers in understanding the instructional decisions that will lead to deep mathematical learning (NCSM, 2014). Although Project IMPACT utilizes a variety of professional development activities, feedback from participants has indicated that demonstration lessons hold the most potential for supporting teachers' reflection on instructional practices. In this paper, we have not only described two models for demonstration lessons that have emerged from our work but also the circumstances that led to their development. In doing so, it is our hope that other mathematics education leaders might utilize these new models, should they fit within the circumstances of their work, and contemplate other models developed in response to the needs of their teachers.

This work was conducted under the Mathematics and Science Partnership program administered by the Tennessee Department of Education. Any opinions expressed herein are those of the authors and do not necessarily represent the views of the Tennessee Department of Education.

References

Arizona Board of Regents. (2002). *Reformed teaching observation protocol.* Retrieved from http://www.ecept.net/rtop/

Burns, M. (1996). *50 Problem-solving lessons grades 1-6: The best from 10 years of Math Solutions Newsletters.* Sausalito, CA: Math Solutions Publications.

Casey, K. (2011). Modeling lessons. *Educational Leadership, 69*(2), 24–29.

Conference Board of the Mathematical Sciences. (2012). *The mathematical education of teachers II* (Vol. 17). Providence, RI: American Mathematical Society.

Clarke, D., Roche, A., Wilkie, K., Wright, V., Brown, J., Downton, A., . . . Worrall, C. (2013). Demonstration lessons in mathematics education: Teachers' observation foci and intended changes in practice. *Mathematics Education Research Journal, 25*, 207–230.

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics.* Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

Enns, Ed. (Editor). (2014). Problem solvers: Problem (sharing chocolate). *Teaching Children Mathematics, 21*, 136–139.

Ernest, P. (1989). The knowledge, beliefs and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching: International Research and Pedagogy, 15*(1), 13–33.

Farmer, J. D., Gerretson, H., & Lassak, M. (2003). What teachers take from professional development: Cases and implications. *Journal of Mathematics Teacher Education, 6*, 331–360.

Herbert, S., Vale, C., Bragg, L. A., Loong, E., & Widjaja, W. (2015). A framework for primary teachers' perceptions of mathematical reasoning. *International Journal of Educational Research, 74*, 26–37.

Loucks-Horsley, S., Stiles, K. E., Mundry, S., Love, N., & Hewson, P. W. (2010). *Designing professional development for teachers of science and mathematics* (3rd ed.). Thousand Oaks, CA: Corwin Press.

National Council of Supervisors of Mathematics. (2014). *It's TIME – themes and imperatives for mathematics education: A leadership framework for Common Core Mathematics.* Bloomington, IN: Solution Tree Press.

National Council of Teachers of Mathematics. (2010). *Research brief. Goals of professional development.* Reston, VA: Author. Retrieved from http://www.nctm.org/Research-and-Advocacy/research-brief-and-clips/Goals-of-Professional-Development/

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: Author.

Watanabe, T. (2008). *Can 5th graders derive area formulae? Implications from Japanese curriculum materials.* Presentation at the Georgia Mathematics Conference in Rock Eagle, GA.