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Characterizing How Expert Algebra Teachers Promote Productive Struggle

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Abstract

While frameworks for analyzing teacher actions have been developed, little research describes how expert teachers promote productive struggle in their classrooms. In this paper, we report findings from using a productive struggle framework and a cognitive demand framework to characterize how nine National Board Certified algebra teachers promoted productive struggle in a lesson. After analyzing videos of their lessons, we found these teachers, whom we label as expert teachers, used a high-cognitive demand task and gave responses that promoted students' productive struggle, often maintaining the high-cognitive demand of the task. This analysis provides mathematics teacher educators and leaders concrete evidence of how expert algebra teachers promote productive struggle in the classroom. We also discuss implications useful for educators and administrators who decide on algebra curricula and professional development, including tailored information for how teachers can respond to student struggle in ways that promote the high cognitive demand of algebra tasks.

Introduction

Mathematics education research has described students actively struggling to learn mathematical concepts as essential in fostering conceptual understanding of the material. This process is often called productive struggle, and research has identified the important role mathematics teachers play in promoting students' productive struggle (Warshauer, 2014, 2015). For example, the way a teacher responds to a student when they are struggling can impact the cognitive demand of the task. Specifically, the cognitive demand is lowered if a teacher responds by telling the student the answer as opposed to raising the cognitive demand if a teacher responds by providing the student a reason-provoking statement.

Two key frameworks have been developed and used to describe teacher actions influencing productive struggle in the classroom. First, Stein and Smith (1998) developed a cognitive demand framework to characterize the tasks teachers choose to incorporate in the classroom and the resulting cognitive demand the tasks invoke from students. Second, Warshauer (2014) developed a productive struggle framework measuring teacher interactions with students during episodes of struggle. While these two frameworks have been used in a variety of mathematics education settings, they have not been used jointly to quantitatively measure teachers' selection of a task and the interactions with students during the task, and determine correlations in the change in cognitive demand. Using both frameworks within a single study could reveal important findings about how a teacher might select a low-level cognitive demand task but implement it in a way that increases the cognitive demand by promoting student struggle from his/her responses to episodes of students struggle. Alternatively, a high-level cognitive demand task might be implemented in a way that reduces the cognitive demand through teacher responses to student struggle that include telling or directed guidance.

Furthermore, little is known about the ways specific types of teachers incorporate tasks and respond to student struggle. For example, how do expert teachers enact tasks and promote student struggle? Are the tasks and ways expert teachers respond to student struggle different than novice teachers? Another example is examining how teachers of a specific mathematical subject differ in task implementation and response to student struggle. What kinds of tasks do algebra teachers enact, and how do they tend to respond to student struggle? Are there differences between algebra teachers and teachers of other mathematical subjects (geometry, trigonometry, etc.)? Answers to these questions are currently unknown, and knowing more about the patterns of specific groups of teachers could help (a) mathematics teacher educators provide tailored professional development to improve the level of cognitive demand within these teachers' classrooms, (b) designers of algebra curricula incorporate tailored advice on how teachers can respond to student struggle in ways promoting the high cognitive demand of the task, and (c) education researchers routinely incorporate these two frameworks to understand teacher actions.

This study contributes to the literature by reporting findings from using the productive struggle framework and cognitive demand framework to characterize how nine national board certified algebra teachers, whom we call expert mathematics teachers, promoted productive struggle in a lesson. We investigate the following three research questions: (1) What types of student struggle are typical during an algebra task implemented by expert mathematics teachers? (2) How do expert mathematics teachers respond when students struggle within an algebra task? (3) What relation, if any, exists between how expert algebra teachers respond to students' struggle and the associated change in the cognitive demand of the task? We incorporated qualitative methods to answer these questions by using Warshauer's (2014) productive struggle framework and Stein and Smith's (1998) cognitive demand framework to code videos of student-teacher interactions within algebra classrooms. From these codes, we characterized the episodes of struggle and types of teacher responses. Then we used quantitative analysis to determine correlations between characterizations of student struggle and the associated cognitive demand of the task.

Productive Struggle Framework

Similar to Vygotsky's (1978) Zone of Proximal Development, Towsend (2018) offered the idea of students' *zone of productive struggle*. The idea was to encourage students to dig deeper into algebraic relationships and experience productive struggle. While students were offered the opportunity, the teacher needs to ensure students are not working outside their zone which may make them feel overwhelmed. Teachers sometimes have difficulty encouraging productive struggle, as this aim might seem at first counterintuitive towards the goals of the lesson. But research suggests that when properly structured and implemented, productive struggle can lead to success for both the teacher and the students (Edwards, 2018; Freeburn & Arbaugh, 2017; Lobato, Clarke, & Ellis 2005). For example, when teachers went through and experienced productive struggle as students, they saw the importance of the process, group discussions, enjoyed the process, and developed confidence that productive struggle supports mathematical goals such as conceptual understanding and problem solving (Murawska, 2018).

A productive struggle framework was developed by Warshauer (2014) using three principal areas of mathematics education research to build her productive struggle framework. First, the framework draws upon literature surrounding the important role struggle plays in students learning and understanding mathematics (Hiebert & Grouws, 2007). Second, the framework incorporates literature documenting how characteristics of mathematical tasks impact students' struggle (Smith & Stein, 1998). Third, the framework relies upon "the ways teachers respond to students' struggles in classroom interactions to capture episodes of struggle, episodes within the stages initiation, interaction and resolution" (Warshauer, 2014, p. 377) and the impact of teacher responses on the cognitive demand (Henningsen & Stein, 1997; Herbel-Eisenmann & Breyfogle, 2005). Using these areas of literature, Warshauer (2014) used an embedded case study to identify and describe the student struggle, teacher actions in response to the struggle, and the resulting impact on cognitive demand.

Table 1: Types of Struggle Experienced by the Student

Kind of Struggle	Descriptors
Get started	• Confusion regarding what task is asking • Forgetting how to solve a type of problem • Gesturing uncertainty and resignation • No work written down
Carry out a process	• Unable to progress on a problem due to inability to use or process a formulated representation, carry out an algorithm, or recall needed facts or formula
Uncertainty in explaining and sense-making	• Difficulty in explaining or making sense of their work • Express uncertainty • Unclear reasons given for their choice of strategy
Express misconceptions and errors	• Misconception related to mathematical content in problem • Performing an arithmetic or technological error

Adapted from Warshaur, 2014.

Table 2: Types of Teacher Responses

Teacher response	Descriptors	Dimensions
Telling	• Supplying information • Directing students towards a strategy • Correcting an error • Referring or referencing simpler problem	• Cognitive demand lowered • Removed struggle efficiently • Suggested an explicit idea
Directed Guidance	• Redirect student thinking • Narrow down possibilities for action • Direct an action • Break down problem into smaller parts • Alter problem to an analogy	• Cognitive demand lowered or maintained • Teacher builds on student thinking
Probing Guidance	• Ask for reasons and justification • Offer ideas based on students' thinking • Seek explanation that could get at an error or misconception	• Cognitive demand maintained • Encouraged student's self-reflection • Questioned and built on student thinking • Used as basis for guiding student
Affordance	• Ask for detailed explanation • Build on student thinking • Press for justification and sense-making with group or individually • Afford time for students to work	• Cognitive demand maintained or raised • Acknowledged, questioned, and allowed student time • Built on student thinking, perhaps by clarifying and highlighting student ideas

Adapted from Warshaur, 2014.

The result of her work was the productive struggle framework which provides the means to classify students' struggles, teacher responses, outcomes of the struggle, and changes in cognitive demand. While literature about the cognitive demand of the activity was incorporated into the literature, specific focus on the task is not included in the Productive Struggle Framework. Table 1 summarizes Warshaur's (2014) four characterizations of student struggle: *(1) getting started, (2) carrying out a process, (3) uncertainty in sense making and explaining,* or *(4) expressing misconceptions and errors.* Table 2 summarizes Warshaur's four characterizations of teacher response: *(1) telling, (2) directed guidance, (3) probing guidance,* or *(4) affordance*. Table 3 summarizes Warshaur's outcomes: *(1) productive, (2) productive at a lower level,* or *(3) unproductive.* The methodology section details how we interpreted and applied these categories to analyze video data.

Table 3: Outcome of Struggle

Outcome Type	Descriptors
Productive	• Maintained the intended goals and cognitive demand of the task. • Supported students' thinking by acknowledging effort and mathematical understanding. • Enabled students to move forward in the task execution through student actions.
Productive at a lower level	• Lowered somewhat in the cognitive demand of the intended task. • The teacher rather than the students actively guided the students through the struggle. • The students passively following a directed guidance.
Unproductive	• Students continued to struggle without showing signs of making progress toward the goals of the task. • Reached a solution but to a task that had been transformed to a procedural one that significantly reduced the task's intended cognitive demand. • Students simply stopped trying.

Adapted from Warshaur, 2014.

Table 4: Changes in Struggle

Changes	Descriptors
Factors Associated with the Maintenance of High-Level Cognitive Demands	• Teacher uses scaffolding, questioning, comments, and feedback to press for student reasoning, explanation, justification, and conceptual connections. • Teachers supports students in monitoring their own progress and the modeling of high-level performance. • Teacher allows sufficient time for task.
Factors Associated with the Decline of High-Level Cognitive Demands	• Teacher emphasizes complete and correct answers rather than the meanings and understanding of the concepts. • Teacher provides their own thinking and reasoning at the expense of student reasoning. • Teacher reduces the complexity of the task by providing explicit procedures or proscribed routines. • Teacher accepts unclear or incorrect student explanations. • Teacher expectations are not clear or appropriate for high-level cognitive activities or does not maintain classroom environment suitable for high-level cognitive activities. • Teacher does not allow sufficient time for task or too much time is allowed, resulting in off-task behavior. • Teacher selects a task that is inappropriate for the group of students (e.g., students do not have prior knowledge needed or task expectations are not clear enough to put students in the right cognitive space).

Adapted from Stein and Smith, 1998.

Cognitive Demand Framework

The learning processes in mathematics are quite sensitive to the selection of the task by the teacher. Lappan and Briars (1995) state "there is no decision that teachers make that has a greater impact on students' opportunities to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages students in studying mathematics." (p. 138). Tasks that represent higher levels of thinking are especially important because they provide students the opportunity to think and reason in complex and meaningful ways (Stein and Smith, 1998).

Stein and Smith (1998) identified four ways a task can be approached on the same topic, each with a different kind of cognitive demand on students. The first category of tasks is called *memorization tasks*, which are lower-level cognitive demand tasks requiring students to use previously learned factors, rules, formulas, or definitions. These tasks have explicit and clear direction too short to use procedures and have no connection to underlying facts, rules, or formulas. The second category is *procedures without connections*, another lower-level cognitive demand task that include algorithmic use of procedures with little ambiguity or connections to underlying concepts. These tasks focus on producing correct answers rather than developing mathematical understanding through explanations. The third category is *procedures with connections*, which are higher-level cognitive demand tasks that have students use procedures to develop understanding of mathematical concepts through multiple representations and engagement with conceptual ideas. The fourth category is *doing mathematics*, which is a higher-level cognitive demand task that has students use complex and non-algorithmic thinking to explore and understand the nature of mathematical concepts, processes, and relationships. These tasks necessitate students to self-monitor and self-regulate, to access prior knowledge and apply it to the task, and to examine constraints of the task (Smith & Stein, 1998).

In addition to the selection of a task, teachers' implementation of the task also impacts the cognitive demand. The seminal Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUSAR) project found that "having the opportunity to work on challenging tasks in a supportive classroom environment translated into substantial learning gains on an instrument specially designed to measure exactly the kind of student learning outcomes advocated by NCTM's professional teaching standards" (Stein and Smith, 1998, p. 16). Based on this work, the researchers created three ways teacher-student interactions can impact the cognitive demand of a task: *(1) factors associated with the decline of the task, (2) factors associated with maintenance of the task,* or *(3) factors associated with the increase of the increase of the task*. Table 4 details the characteristics of each categorization and the associated teacher-student interaction descriptors. These descriptors align with Warshauer's (2015) productive struggle framework. For example, when teachers address students' struggles by supplying information they are essentially removing the demand. Teachers can direct student actions or use probing guidance to address students' struggles in ways that maintain the intended cognitive demand. Teachers can also have an opportunity to increase the intended level of cognitive demand when they can take a procedure with connections task and utilize opportunities to incorporate doing mathematics (Warshauer, 2015).

Methodology

Our participants included nine National Board Certified teachers who taught algebra during the time of the study. The designation of National Board Certification (NBC) in the United States was created to reward the most accomplished teachers and is proclaimed to be the most respected professional certification available in K-12 Education. The NBC standards contain five propositions based on research of what accomplished teachers should emulate in the classroom, such as being committed to students and their learning (proposition 1) and managing and monitoring student learning (proposition 3) (NBC, 2017). To gain certification, teachers must also show evidence of participating in professional learning communities and of ongoing reflection on teaching.

Before applying for NBC, an individual must have a bachelor's degree, a valid teaching license, and three years of teaching experience. The certification process for Mathematics – Adolescence and Young Adulthood involves (a) taking a computer-based mathematical content knowledge assessment, (b) submitting instructional materials and work samples with commentary, (c) submitting two videos showing evidence of how the teacher's classroom practices and learning environment contributed to student engagement and to meet the mathematical goals of the lesson, and (d) submitting a portfolio demonstrating evidence of how the teacher develops and applies knowledge of students to plan and impact student learning.

In this study, we examined nine algebra videos from nine teachers that were submitted as part of part (c) of the NBC process. NBC instructions were that these videos should focus on student engagement and the teaching practices and format used to help students meet the mathematical learning goals for the lesson (National Board for Professional Teaching Standards, 2016). Thus, these videos provided valuable data on task selection and the teacher-student interactions of implementing the lesson. Since the teachers used the videos to become National Board certified, we thus call our participants expert mathematics teachers. Therefore, the videos were appropriate for answering our research questions about how students struggle on an algebra task selected by expert mathematics teachers, how mathematics teachers respond to student struggle, and how teachers' responses impact the associated changes in the cognitive demand of the task.

The researchers of this study used publicly available CBS videos of nine expert algebra teachers implementing a lesson with their students; four were male and five were female. We selected these nine teachers because they were the only teachers in the available data set who submitted a video using an algebra lesson. The videos were between ten and twenty minutes, and the teacher decided which lesson and segment were recorded and shared. We also collected the associated tasks that were implemented in the video. The video and associated task documents of the nine teachers comprised the data collection for this study.

Data Analysis

We used Hiebert and Grouws' (2007) definition of productive struggle as students' "effort to make sense of mathematics, to figure something out that is not immediately apparent," (p. 287). To analyze the written documents for student struggle, we used Stein and Smith's (1998) four levels of cognitive demand to code the type of enacted algebra task (1). The lesson was coded as one of the cognitive demands: *memorization* (1a), *procedures without connections* (1b), *procedures with connections* (1c), and *doing mathematics* (1d). The first two levels (*memorization, procedures without connections*) are considered lower levels of demand, and the second two levels (*procedures with connections, doing mathematics*) are considered higher levels of demand (Stein and Smith, 1998).

We analyzed the video data using Warshauer's (2014) productive struggle framework for identifying and coding the four elements of each struggle episode: (2) the struggle experienced by the student, (3) the teacher response, (4) the outcome resulting from the response, and (5) the subsequent change in cognitive demand demonstrated by the student. To code elements (2) through (4), we identified the unit of analysis as an episode of struggle using Warshauer's (2014) definition: (a) the time, beginning with an indication of student uncertainty, confusion, or teacher-directed question; (b) the corresponding teacher response and teacher-student interactions; and (c) the outcome, either productive or unproductive struggle.

In each episode, we coded the type of struggle experienced by the student (2) using one of Warshauer's characterizations: confusion about an approach or what the task was asking, which was coded as *get started* (2a), an inability to carry out an algorithm, implement a process that is generally algebraic in nature (2b), which was coded as *carry out a process*, difficulty explaining their work or making sense of their work, which was coded as *uncertainty in explaining and sense-making* (2c), and *an expression of a misconception or error* (2d).

In each episode, we coded the teacher's response (3) using Warshauer's categorizations: when the teacher supplies information, directly corrects an error, or suggests a strategy, coded as *telling* (3a), when the teacher redirects student thinking, directs an action, or narrows down the possibilities for action, which was coded as *directed guidance* (3b), when the teacher asks for reasons and justification or seeks an explanation that could get at an error or misconception, which was coded as *probing guidance* (3c), and when the teacher asks for a detailed explanation, presses for justification and sense making, or builds on student thinking, which was coded as *affordance* (3d).

In each episode, we coded the outcome of the struggle (4), using Warshauer's three categorizations: when the student or group of students work through the struggle while maintaining the intended level of cognitive demand or are at least able to continue engagement, which was coded as *productive* (4a), when the struggle is addressed by reducing or removing the struggle or making the task easier, which was coded as *productive at a lower level* (4b), and when the students are unable to proceed past the struggle or the teacher completely removes the struggle and fundamentally changes the original intentions of the task, which was coded as *unproductive* (4c). Finally, in each episode, we coded the level of cognitive demand following the outcome (5) using Warshauer's three categorizations: *lowered* (5a), *maintained* (5b) or *increased* (5c) adapted from Warshauer (2014).

In an effort to answer the third research question regarding what relation, if any, exists between how expert algebra teachers respond to students' struggle and the associated change in the cognitive demand of the task, we performed statistical tests in Statistical Package for the Social Sciences (SPSS). The first analysis we performed was a Spearman Rank Correlation. The Spearman Rank Correlation was done because the categories to be analyzed consisted of ordinal variables. We then ran a Kruskal-Wallis H Test with the teacher as the factor to see if the individual teacher was a significant factor in relation to the types of responses given to students.

Results

After coding each episode of struggle based on the type of struggle experienced by the student, we found 29 of the 58 (50%) episodes of student struggle were *uncertainty in explaining* and sense making, 20 of the 58 (34%) episodes were *carrying out a process*, 5 (9%) were *getting started*, and 4 (7%)

FIGURE 1.
Results from the nine videos of expert teachers enacting an algebra lesson.

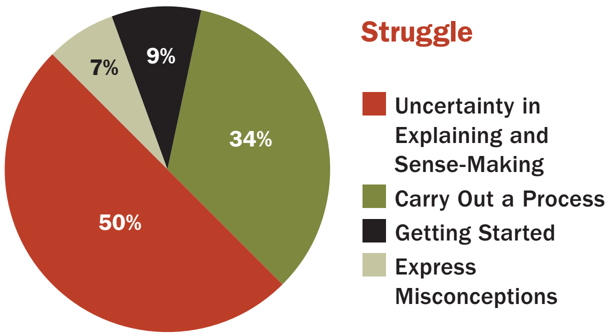


FIGURE 2.
Results from the nine videos of expert teachers enacting an algebra lesson.

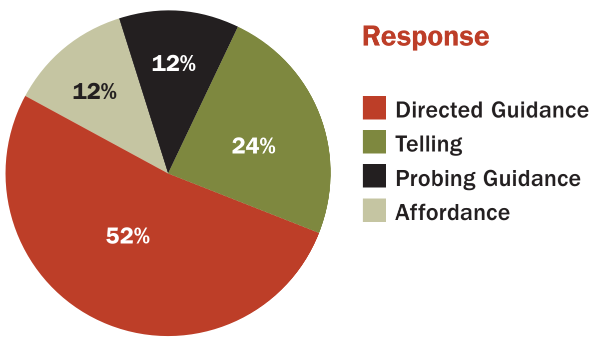
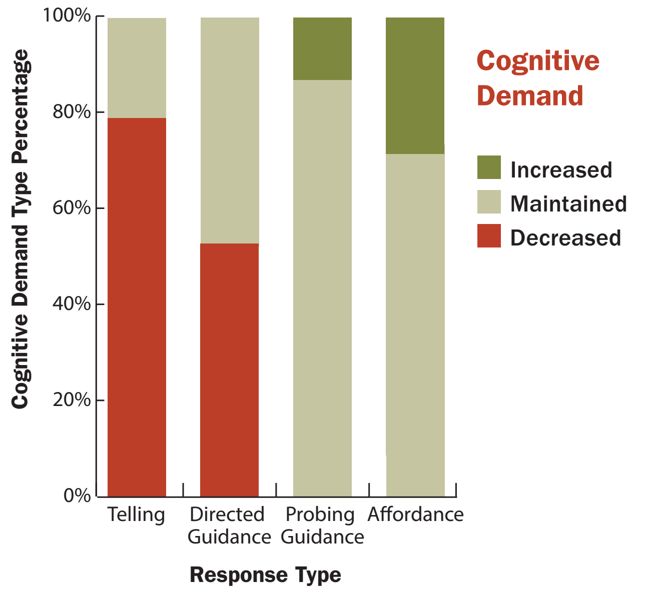


FIGURE 3.
A breakdown of how each type of teachers' response impacted the subsequent cognitive demand of the task.



were *misconceptions*. Figure 1 gives a visual representation of the findings, showing the high prevalence of *uncertainty in explaining and sense making* experienced by students.

After coding each episode of struggle based on teachers' responses, we found 30 out of 58 (52%) episodes were responded to with *directed guidance*, 14 of the 58 (24%) episodes were *telling*, 7 (12%) were *probing guidance*, and 7 (12%) were *affordance*. Figure 2 shows a visual representation of how the majority of teachers' responses were directed guidance.

We found a pattern between how expert algebra teachers respond to students' struggle and the associated change in the cognitive demand of the task (Figure 3). Of the 14 *telling* responses, 11 (79%) decreased the level of cognitive demand, 3 (21%) maintained the level of cognitive demand, and 0 increased the level of cognitive demand. Of the 30 *directed guidance* responses, 16 (53%) decreased the level of cognitive demand, 14 (47%) maintained the level of cognitive demand, and 0 increased the level cognitive demand. Of the 7 *probing guidance* responses, 0 decreased the level of cognitive demand, 6 (86%) maintained the level of cognitive demand, and 1 (14%) increased the level of cognitive demand. Of the 7 *affordance* responses, 0 decreased the level of cognitive demand, 5 (71%) maintained the level of cognitive demand, and 2 (29%) increased the level of cognitive demand.

When the *telling* and d*irected guidance* responses are combined and the *probing guidance* and *affordance* responses are combined, a clearer pattern emerges. Of the 44 *telling* and *directed guidance* responses, 27 (61%) decreased the level of cognitive demand, 17 (39%) maintained the level of cognitive demand, and 0 increased the level cognitive demand. Of the 14 *probing guidance* and *affordance* responses, 0 decreased the level of cognitive demand, 11 (76%) maintained the level of cognitive demand, and 3 (24%) increased the level cognitive demand.

A Spearman Rank Correlation analysis verified this pattern, showing the relationship between student struggle and teacher responses to be statistically significant (Table 5). This provided evidence that lower level struggles (*getting started, carrying out a process*) were addressed by teachers *telling* or *giving directed guidance*.

Table 5: Correlations Using Spearman Rank Across the Variables Task, Struggle, Response, Outcome, and Cognitive Demand.

		CORRELATIONS				
Spearman's rho		**Task**	**Struggle**	**Response**	**Outcome**	**Cognitive Demand**
Task	Correlation Coefficient	1.000	.017	-.026	-.034	.086
	Sig. (2-tailed)	.	.899	.847	.799	.519
	N	58	58	58	58	58
Struggle	Correlation Coefficient		1.000	.296*	.300*	.192
	Sig. (2-tailed)		.	.024	.022	.149
	N		58	58	58	58
Response	Correlation Coefficient			1.000	.652**	.584**
	Sig. (2-tailed)			.	.000	.000
	N			58	58	58
Outcome	Correlation Coefficient				1.000	.774**
	Sig. (2-tailed)				.	.000
	N				58	58
Cognitive Demand	Correlation Coefficient					1.000
	Sig. (2-tailed)					.
	N					58

** Correlation is significant at the 0.05 level (2-tailed).*

*** Correlation is significant at the 0.01 level (2-tailed).*

The Spearman Rank Correlation test also revealed a significant correlation between the way teachers responded to students' struggles and the impact on cognitive demand (Table 5). The correlation of .584 and significance p<.01 indicates teachers' response to student struggle can greatly affect the potential changes in cognitive demand. Thus, the *telling* and *directed guidance* responses were less likely to maintain or raise the cognitive demand of the task in comparison to *probing guidance* and *affordance* responses.

A strong and statistically significant correlation was found between the teachers' response to a struggle and the associated outcome of the episode (r=.65, n=58, p<.001). This was unsurprising because the outcome of an episode (productive, productive at a lower level, or unproductive) was a direct result of the interactions from the response. We found a statistically significant correlation between the outcome of an episode and the resulting cognitive demand of the task (r=. 77, n=58, p<.001). This is again unsurprising

FIGURE 4.

Kruskal-Wallis H test across the variables response, task, struggle, outcome, and cognitive demand.

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of Task is the same across categories of Teacher.	Independent- Samples Kruskal-Wallis Test	.000	**Reject the null hypothesis.**
2	The distribution of Struggle is the same across categories of Teacher.	Independent- Samples Kruskal-Wallis Test	.003	**Reject the null hypothesis.**
3	The distribution of Response is the same across categories of Teacher.	Independent- Samples Kruskal-Wallis Test	.209	Retain the null hypothesis.
4	The distribution of Outcome is the same across categories of Teacher.	Independent- Samples Kruskal-Wallis Test	.234	Retain the null hypothesis.
5	The distribution of Cognitive Demand is the same across categories of Teacher.	Independent- Samples Kruskal-Wallis Test	.004	**Reject the null hypothesis.**

because if an episode resulted in a productive outcome, the productive aspect influenced the cognitive demand of the task.

A Kruskal-Wallis H test was conducted using the nine expert teachers as a grouping variable to determine if the kinds of student struggle differed across the teachers, the way teachers responded to the struggle differed, differences in the outcome of the episode across the teachers, or differences in the cognitive demand across the teachers. Since each teacher used a different task that had an original cognitive demand rating associated with it, we did not compute how the original task differed across teachers.

The Kruskal-Wallis H test showed that there was a statistically significant difference in struggle score between the different teachers (test statistic = 8.520, p = 0.003). When specific teachers were compared, the test revealed teachers eight and nine were significantly different in the types of struggles students encountered (p=.001). The test also showed that there was a statistically significant difference in cognitive demand score between the different teachers (test statistic = 22.378, p = 0.004). When specific teachers were compared, the test revealed teachers two and seven were significantly different in how they responded to student struggle (p <.001). Figure 4 details these findings as well as the lack of significance between the types of responses across the same category of teachers and the outcome across categories of teachers.

Discussion

By examining an algebra lesson from nine expert mathematics teachers, we found student struggle was evident in all lessons. The first research question asked what types of student struggle are typical during an algebra task implemented by expert mathematics teachers. We found students struggled in ways that aligned with Warshauer's (2014) framework, most often by *struggling to carry out a process*, or *getting started*. These types of struggle are perhaps unsurprising because algebra tasks tend to incorporate many processes that students must perform, or about which students must reason. We note students struggled to carry out algebraic processes within expert mathematics teachers' classrooms and that a teachers' response to this struggle is the determining factor for the cognitive demand of the task.

The second research question asked how expert mathematics teachers responded when students struggled within an algebra task. We found teachers responded in ways that aligned with Warshauer's (2014) framework, most often by offering *directed guidance* or *telling*. These responses can perhaps be explained by the nature of the tasks and the types of student struggle to which the teachers were responding. The algebra tasks primarily caused students to struggle to carry out a process, specifically an algebraic process. The teachers' response to give them the solution method is perhaps a natural resolution to the struggle, as this is a quick way to have students overcome the problem and move forward within the task.

We found an interesting relationship when answering our third research question regarding the relation between teachers' responses and changes in the cognitive demand of the task. We founded *directed guidance* or *telling* responses lowered the cognitive demand of the task 61% of the time and maintained the cognitive demand of the task 39% of the time. *Directed guidance* or *telling* responses were never observed to raise the cognitive demand of the task. While the *probing guidance* and *affordance* responses were less commonly provided by the expert teachers, when these responses were given, they always maintained the cognitive demand of the task (76%) or raised the cognitive demand of the task (24%). This suggests that even though providing probing guidance and affordance responses to students' struggle during algebra tasks might be more time consuming, it is an important part of the learning process, as suggested by other researchers (Lewis, & Özgün-Koca, 2016; Stephan, Pugalee, Cline, & Cline, 2016; Townsend, Slavit, & McDuffie, 2018; Zeybek, 2016).

This study has a limitation of the small sample size (n=9) having been selected as the only algebra teachers available to us. Having collected data from more expert algebra teachers or multiple lessons from the nine expert algebra teachers would have increased the number or episodes of student struggle, ensuring saturation had been achieved, and likely have had a positive impact on the validity of the findings (Fusch & Ness, 2015).

Implications

One implication of this research for educational leaders is that Warshauer's (2014) framework can be useful in describing student struggle and teacher responses. We propose that educational leaders categorize exemplars of how teachers provide *probing guidance* and *affordance* responses to students *struggling to carry out a process or* *getting started*. This would provide constructive and practical content for professional development sessions to help algebra teachers to foster productive struggle in their classroom while maintaining the cognitive demand of tasks.

We also made note of a few teaching characteristics that were not captured in Warshauer's (2014) framework that may be lurking variables useful for educational leaders and educators to consider. First, we saw some teachers used wait time more effectively than others to have students resolve the struggle. Examining the wait time duration and context in comparison to the resolution of the uncertainty might help researchers understand the role this plays in promoting students' productive struggle. Second, the use of groups during the task allows students to provide each other responses to their struggle, thus avoiding an episode or response from the teacher. This type of environment seemed to provide students opportunities to resolve their struggle in comparison to settings where the teacher was the sole person attending to students' struggle. Teachers should be encouraged to consider going through a productive struggle experience to see first-hand the dynamics of the process, group work, and how the entire experience can impact problem solving (Murawska, 2018). Like other researchers (Warshauer, 2014), we suggest teachers incorporate group settings to better promote productive struggle while maintaining the cognitive demand of the task.

A second implication of this work is that mathematics teacher educators can use this data to provide evidence and examples for pre-service and novice teachers on what productive struggle looks like in the classroom and ways to best respond to student struggle to maintain or increase the cognitive demand of the task. Encouraging struggle can be a difficult thing to do for teachers often trained to remove impasses for students. Finding the appropriate strategies to implement opportunities for struggle are an essential aspect of teaching (Freeburn & Arbaugh, 2017; Lobato et al., 2005). New teachers could be influenced by seeing expert teachers incorporating student struggle as a regular, positive, and necessary occurrence in the classroom. Practicing *probing guidance* and *affordance* responses to students rather than *directed guidance* or *telling* responses could be beneficial for teachers in all stages of their career.

A third implication of this work is for educational leaders who select or have input on deciding algebra curriculum materials. The need for high cognitive demand tasks is well established in the literature. Creating appropriate opportunities for productive struggle can be an issue of equity. Task selection for productive struggle opportunities can be important when considering differentiation for diverse learners (Lynch, Hunt, & Lewis, 2018). We contend that in addition to providing high cognitive demand tasks within curricula, educational leaders should also provide suggested *probing guidance* and *affordance* responses that algebra teachers can use with students. Having these examples in the teacher editions of algebra textbooks might give teachers ideas on how to avoid *directed guidance* or *telling* responses, thus maintaining or raising the cognitive demand of the task.

In conclusion, we recommend three kinds of future studies. First, we suggest similar research examining expert teachers within other mathematical topics, such as geometry, trigonometry, and calculus, to see if comparable tasks and responses to student struggle occur. Second, this study had only high cognitive demand tasks used by the expert teachers; we recommend investigating other expert algebra teachers who incorporate low cognitive demand tasks and determining the types of responses given to episodes of student struggle. Third, comparing novice and expert teachers' responses to student struggle within mathematics classrooms would provide useful information about how to tailor teacher education, professional development, and curricular materials to teachers at various stages in their careers. ✪

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