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Teacher Interpretations of the Goals of Mathematics Professional Development and the Influence on Classroom Enactment

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Abstract

This multiple-case study is an investigation of how four high school teachers interpreted the goals of a professional development (PD) program and how these interpretations influenced their instructional practices during observed lessons. The teachers participated in PD that focused on using standards-based pedagogy and mathematical tasks with higher-level demands. Each teacher stated an interpretation of the goals that was consistent with the PD, but concentrated on one of the objectives for each of the goals. The teachers' interpretations of the goals influenced the lessons they taught and their use of ideas from the PD.

National standards such as the National Council of Teachers of Mathematics (NCTM) (1989, 2000), the National Governors Association Center for Best Practices and the Council of Chief State School Officers (NGACBP & CCSSO) (2010), and the National Research Council (NRC) (2001) have provided visions for mathematics teaching and learning in K-12 schools. These visions contain goals for students that include rigorous content, reasoning, modeling, communicating, connecting, constructing arguments, and supporting conclusions (NCTM, 2000; NGACBP & CCSSO, 2010). These standards-based visions include new ideas that can be challenging for teachers and schools to enact (Coburn, Hill, & Spillane, 2016; Munter, Stein, & Smith, 2015; NCTM, 2014). For example, teachers must learn new content, gain experience with different instructional techniques, and implement new assessment methods (Reys, Reys, Lapan, Holliday, & Wasman, 2003).

Research-based professional development (PD) can help mathematics teachers overcome the challenges of standards-based pedagogy (Lappan, 1997; Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2010), but studies indicate that the outcomes of PD programs are inconsistent. Some teachers are able to teach in a manner consistent with the goals of the PD that are focused on standards-based instruction, some teachers' instruction reflects portions of the goals, and other teachers struggle with using the reform concepts in their classroom (Coburn et al., 2016; Cook, Walker, Sorge, & Weaver, 2015; Munter et al., 2015). One explanation for the inconsistencies is that teachers attempting to use standards-based instructional strategies adapt to the new visions by interpreting and constructing understandings based on the way instruction is currently done (Coburn et al., 2016; Munter et al., 2015; Roth McDuffie, Choppin, Drake, Davis, & Brown, 2018). Research has reported examples of teachers who perceive that they are providing standards-based instructional practice, but observations by researchers reveal that what the teachers perceive is not consistent with this type of instruction (e.g. Cohen, 1990; Roth McDuffie et al., 2018; Spillane & Zeuli, 1999).

Due to the influence that teachers' interpretations have on the enactment of standards-based pedagogy in the classroom, it is important to learn more about the interpretations teachers develop as a result of PD experiences. In particular, understanding how teachers' interpretations of PD goals on standards-based instruction influence classroom practice could help PD providers and mathematics teacher supervisors understand inconsistencies concerning attempts to improve instructional practice. The question investigated in this research is: How do teachers interpret the goals of a standards-based mathematics PD program and how did their interpretations influence the enacted mathematics lessons?

Standards-Based Mathematical Practices

An important aspect of this research was PD aimed at helping teachers learn about and enact *standards-based visions for mathematics instruction*. Two sets of practice standards were used to define standards-based mathematics instruction. The first set of practice standards was the set of eight standards for mathematical practice (SMPs) identified by the NGACBP & CCSSO. The SMPs "describe varieties of expertise that mathematics educators at all levels should seek to develop in their students" (NGACBP & CCSSO, 2010, p. 6). The SMPs are (a) making sense of problems and persevering in solving them, (b) reasoning abstractly and quantitatively, (c) constructing viable arguments and critiquing the reasoning of others, (d) modelling with mathematics, (e) using appropriate tools strategically, (f) attending to precision, (g) looking for and making use of structure, and (h) looking for and expressing regularity in repeated reasoning. The second set of practice standards used in the PD was the set of eight mathematics teaching practices (MTPs) that "provide a framework for strengthening the teaching and learning of mathematics" (NCTM, 2014, p. 9). The MTPs include (a) establishing mathematics goals to focus learning, (b) implementing tasks that promote reasoning and problem solving, (c) using and connecting mathematical representations, (d) facilitating meaningful mathematical discourse, (e) posing purposeful questions, (f) building procedural fluency from conceptual knowledge, (g) supporting productive struggle in learning mathematics, and (h) eliciting and using evidence of student thinking.

Teachers who use the SMPs and MTPs to provide standards-based instruction in K-12 mathematics classrooms have specialized roles. One of the main responsibilities of a teacher in a standards-based mathematics classroom is to plan, establish, and sustain the mathematical learning environment. They are responsible for creating an environment where students can actively build mathematical understandings and share concepts (NCTM, 2000, 2014). Students have important roles in standards-based mathematics classrooms that are negotiated and developed through their participation over time (McClain & Cobb, 2001). For example, students are expected to make conjectures and share mathematical thinking, use reasoning to explain solutions to all members of the class, persevere in solving mathematical problems, and use mathematics to model experiences (Boaler, 2002; NGACBP & CCSSO, 2010).

Principles of effective PD for K-12 teachers of mathematics (e.g. Loucks-Horsley et al., 2010; Sztajn, 2011) were used to describe the PD program. These principles recommend establishing clear goals that incorporate school needs along with national, state, and local standards as a framework to support change (Sztajn, 2011). Goals and objectives provide benchmarks to monitor progress toward the vision of teaching and learning promoted by PD (Loucks-Horsley et al., 2010). Goals that reflect features that teachers find valuable are an important consideration because changes in instructional practice can be linked to perceptions about PD (Chapman, 2011; Martin & Gonzalez, 2017; Walker, 2018).

Methods

To address the research question, information was needed about teachers' interpretations of the goals of a PD program and how they enacted instruction in relationship to their interpretations of the goals. A multiple-case study design (Merriam, 2009) was used to analyze the influences on instructional practice in the complex social units of classrooms and schools. The role of the researcher was observer as participant (Merriam, 2009). The researcher was not involved in the design or execution of any parts of the PD or any mathematics lesson taught. Findings are reported as a case for each teacher. A summary of standards-based mathematical practices observed is provided with each lesson. These summaries provide additional information about the enacted lessons, but are not a focus of the research question. Findings and implications are presented following the cases.

Participant Selection

The PD program for this research was Teaching Algebra with Practice Standards (TAPS). TAPS was identified because it focused on helping teachers implement standards-based mathematical practices. It was a three-year program funded by a mathematics partnership grant. TAPS included partnerships between four Midwestern universities and four school districts, all from the same state. Faculty and graduate students specializing in mathematics education from all four university partners worked together to plan the PD activities. Each university was paired with a neighboring school district for the delivery of the activities.

This research focused on the Springfield School Corporation (SSC), which was one of the four partner school districts. There were fifteen SSC teachers who volunteered to participate in the first year of TAPS. Each of the fifteen teachers taught mathematics in grades six through twelve. All fifteen of the teachers received continuing education units and stipends for work done outside of school time.

The teachers participating in this multiple-case study were a subset of the fifteen teachers. At the beginning of the TAPS summer institute, the researcher presented the opportunity to participate in this research to all fifteen of the teachers. They were informed that participating in the research would require them to complete surveys, interviews, and allow the researcher to conduct classroom observations. As an incentive, teachers who participated in the research were credited with up to ten hours of independent work required by TAPS. Four teachers volunteered to participate. All of the research participants were high school teachers from Springfield High School (SHS) in SSC.

Data Collection

Data for this research were collected in 2015-2016 during the first year of TAPS. Data sources used to construct a description of the PD included the written PD proposal, field notes taken by the researcher during the PD sessions, and email interview responses from the PD facilitators. Data for instructional practices consisted of two enacted lessons for each teacher that were observed and videotaped. The researcher asked the teachers to self-select the lessons that were observed. The criteria for selection was that the observed lessons were developed during the PD workshops and consistent with the goals of the PD. The observations provided evidence of mathematics instruction that was intended to be consistent with the PD goals. The Wisconsin Longitudinal Study observation tool (Shafer, Wagner, & Davis, 1997) was used to organize data collection during the observations. The tool was adapted to identify evidence of the SMPs and MTPs during classroom instruction.

The four participating teachers were interviewed five times using protocols adapted from Shafer, Davis, and Wagner (1997) and Shafer, Davis, and Wagner (1998). The first interview took place in the summer after the PD was completed to learn about each teacher's interpretations of the PD goals and how they anticipated using the PD during the upcoming school year. The first interview included a question that asked each teacher to state the goals for TAPS in her or his own words. The next two interviews took place during the first half of the school year. These interviews were before and after the first observed lesson. Interview questions provided information about the planned lesson, the enacted lesson, and how ideas about standards-based instruction from the PD were included. The final two interviews took place during the second half of the school year. These interviews were before and after the second observed lesson.

Data Analysis

Classroom observation data were used to describe the alignment between each teacher's enacted instruction and the SMPs and MTPs. Lessons were classified as no evidence, sometimes, or yes for each of the SMPs and MTPs. No evidence was used when there were no classroom events or only one classroom event that aligned with a practice standard descriptor. Sometimes was used when there were two or three classroom events that aligned with a descriptor. Yes was used when there were more than three classroom events that aligned with a descriptor.

Each of the teacher interviews were transcribed. An inductive approach of comparative pattern analysis was used to create a category coding system for the transcripts (Merriam, 2009). The categories were further examined for sub-categories (Lincoln & Guba, 1985; Patton, 2002). For example, one of the coding categories for the transcribed interviews was "Teacher Role." Sub-categories for "Teacher Role" included: monitor or listener, source of mathematical knowledge, ensurer of correctness, and facilitator. Selected quotes in each of the teacher cases were representative of a coding category.

An independent education researcher checked the reliabilities of the observation classifications and the coding system. The researcher was trained on the classification and coding systems and completed independent coding. The reliability of the observation analysis was checked by calculating a Krippendorff (2004) alpha value of 0.8223. The reliability of the coding system was checked by calculating the percent of agreement, which was 90%.

The PD Program

TAPS was the PD program in this research (see Methods, Participant Selection). Goals for TAPS were developed jointly by the universities and school district partners from the analysis of a needs assessment. One area of need was teachers' knowledge and skills for teaching algebra. The assessment revealed that teachers needed to learn about research-based learning tasks, learn about research-based instructional strategies (including differentiated instruction), and have time to improve the mathematics programs based on these topics and student data (TAPS Proposal, pp. 6-7). A second area of need was students' algebraic knowledge and skills. The student passing rates for the four district partners was 25% below the state passing rate on state standardized mathematics tests. Each of the school districts also noted limited opportunities for students to engage in authentic learning tasks to enhance their algebraic understandings. Despite the limited opportunities, each district expressed a desire to learn more about authentic learning tasks and how to include them into the curriculum (TAPS Proposal, pp. 4-6).

Based on the needs assessment, TAPS identified two goals for the program (TAPS Proposal, p. 3). The first goal was to enrich teachers' knowledge and skills for teaching algebra. Objectives for the first goal included: (a) engaging in solving rich algebra tasks to enhance algebraic understanding and habits of mind (e.g., abstracting from computation, doing and undoing, and building rules to represent functions); (b) collaborating to locate and develop algebra activities, including modifying textbook tasks to increase cognitive demand, relate algebra to STEM and other real-world contexts, and address SMPs; (c) enacting research-based pedagogical strategies (e.g., productive discourse, multiple representations) within a system of structured reflection and feedback from critical friends; and (d) participating in a collaborative action-research project in which teachers identify their own focus for enhancing their classroom practice.

The second goal was to improve students' algebraic knowledge, algebraic skills, and disposition toward algebra. Objectives for the second goal included: (a) assessing and building upon students' prior knowledge of algebraic concepts; (b) engaging students in solving rich algebra tasks to enhance algebraic understanding and habits of mind; (c) providing opportunities for students to make meaning of algebra, including its conceptualization beyond symbolic manipulation and value as a tool for inquiry in STEM and other real-world contexts; and (d) improving students' performance on standardized and class-level assessments and motivation to engage with algebraic concepts.

Features of the PD

The PD was a year-round program that started with a ten-day summer institute in June 2015. Three follow-up sessions took place during the school year. In addition to the organized PD meeting times, each teacher was expected to teach lessons based on the standards-based mathematical practices, complete two observations of another teacher teaching a lesson from the PD, and provide data for research being conducted by the PD facilitators. Each teacher had an opportunity to participate in 86 hours of PD.

The standards-based mathematical practices were shared with the teachers at the beginning of TAPS as the vision for mathematics instruction for the PD. The SMPs were described to the teachers as descriptors of what students have an opportunity to do when learning mathematics (Field Notes, 2015-06-09). The MTPs were described to the teachers as descriptors for what teachers have an opportunity to do when teaching mathematics (Field Notes, 2015-06-09). In addition, the PD focused on the use of mathematical tasks with higher-level demands (Stein, Smith, Henningsen, & Silver, 2000). Teachers participated in mathematical tasks, discussed the characteristics of mathematical tasks, worked in small groups to create three tasks that would be used during the upcoming school year, and presented tasks to each other.

The PD facilitators used sample lessons and activities about patterns, relationships, and generalizations. Additionally, the PD facilitators provided active learning opportunities for teachers (Desimone, 2009; Loucks-Horsley et al., 2010) including journal responses, small and large group discussions, teacher peer observations, video study, student-like participation in mathematical tasks, and presentation of tasks with feedback from the group.

Summer Institute

The summer institute ran in conjunction with the SSC summer school program. This allowed the participating teachers to gain experience using the standards-based mathematical practices and mathematical tasks with the summer school students. It also provided an opportunity for the teachers to observe each other and to discuss the observations. The morning summer school sessions lasted three hours. The afternoon summer institute work-sessions also lasted three hours.

On the first day of TAPS, the facilitators discussed the goals of the PD with the teachers. The facilitators reviewed the standards-based mathematical practices with the teachers and shared that they would focus on developing and implementing activities aligned to these practices. The PD facilitators summarized the goals for the teachers as knowing more about algebra, teaching algebra, and ways to improve teaching algebra (Field Notes, 2015-06-08). These discussions were consistent with the program goals, but did not present the goals with the same detail as the TAPS proposal.

Most of the institute days included a reflection question that the teachers wrote about in reflection journals. The prompts included questions such as: "What do you see as the major challenges in teaching algebra?" (Field Notes, 2015-06-08) and "What connections are there between algebra topics, between algebra and other math, and between algebra and other non-math topics?" (Field Notes, 2015-06-12). After the personal writing, the teachers would discuss the questions in small groups and as a whole group.

In addition to the reflection questions, time was dedicated to understanding mathematical tasks. Teachers reviewed examples of mathematical tasks, sorted them as higher-level or lower-level (Stein et al., 2000), and developed characteristics of tasks that could be used as identifiers. For example, the teachers described higher-level mathematical tasks as having multiple steps, requiring justification, and allowing the opportunity for more than one correct answer. They described lower-level mathematical tasks as requiring only basic computation, having few steps, and being limited to the use of a formula or memorization (Field Notes, 2015-06-08).

An important feature of the PD was the time devoted to discussing and understanding the standards-based mathematical practices. For example, on the seventh workshop day the reflection question was: "Which MTPs do you feel most competent implementing in your classroom? Which do you wish you were better at?" Teachers responded during the whole group discussion:

> Teacher 1: I would like to be better with productive struggle and questioning.
>
> Teacher 2: I would like to get better with struggle without losing them, allow kids to struggle without stepping in.
>
> Teacher 3: I need to improve not jumping in to help.
>
> Teacher 4: It takes mistakes to learn. (Field Notes, 2015-06-16)

When teachers had time to work on the mathematical tasks for their classroom, the facilitators regularly asked the teachers to reflect on which of the standards-based mathematical practices were aligned with the task and to find ways to include more of the SMPs and MTPs.

Follow-Up Sessions

The three follow-up sessions took place after school in October, February, and April. Teachers met with facilitators for two hours. They shared the use of mathematical tasks in their classrooms and learned more about the standards-based mathematical practices. The follow-up sessions included reflection questions, readings from *Making Sense of Algebra* (Goldenberg et al., 2015), sample mathematical tasks led by the facilitators, and time for teachers to work on mathematical tasks for use in their classrooms.

The reflection questions, readings, and sample mathematical tasks provided opportunities for the teachers to learn more about the standards-based mathematical practices. For example, during the February meeting one of the PD facilitators shared how he selected and modified a presented mathematical task to align with the mathematical practices:

> Facilitator: Here is how I thought about the [standards-based mathematical practices] when I designed the task; the task included persevere because the scaling

> was not given to you; we had to reason abstractly because you had to go between context and numbers and solve the inequality; and you had to look for structure using shapes within shapes. (Field Notes, 2016-02-02)

Teacher Case Studies

Teacher 1: Doug Collins (DC)

Doug Collins was a male with thirteen years of teaching experience. This was his second year at SHS and he taught Algebra 1 and Geometry during the 2015-2016 school year. Mr. Collins had a bachelor's degree in mathematics and he was working on a master's degree in mathematics education. When asked to describe the goals of the PD in his own words, Mr. Collins stated that they were "to try to help improve the algebra one end-of-course exam scores at [SHS]" (DC Interview, 2015-09-15). His interpretation of the goals of the PD was to help students pass the state accountability and graduation test they took at the end of their algebra one course.

Doug Collins: Enacted lesson #1. Mr. Collins' first observed lesson was a task he developed during the PD Summer Institute on writing and solving multi-step equations. He described the academic standards that would be included in the lesson as following order of operations, solving equations, and checking solutions as reasonable (DC Interview, 2015-09-15). The task was done as review before an upcoming test. When asked about the purpose of the lesson, Mr. Collins replied, "I am hoping that the students get more experience with solving equations, showing all of their work, because I have students who don't like to do that, and hopefully to help build their confidence" (DC Interview, 2015-09-15).

Each student was given an algebraic expression on either a gold or a green piece of paper. Mr. Collins explained that a student with a gold sheet should find a student with a green sheet. They would set their algebraic expressions equal to each other and then find a value for the unknown that would make the equation true. Mr. Collins told the students that they should work together, show all of their work, and check to see if the solution made the equation true. He also stated that the students should complete at least five equations with five different partners.

The students worked in pairs on this task for thirty-five minutes. They checked answers with each other, explained methods used to find an answer, and used calculators to check answers. Students asked questions such as, "Can you do that?" and "Do you understand why I added seven?" (DC Observation, 2015-09-18). Mr. Collins moved around the room checking work done by students and helping students find new partners. He made comments to encourage the students to work together such as, "If you don't agree you will need to check with your partner" (DC Observation, 2015-09-18). At the end of the class, Mr. Collins asked the students to return to their seats and collected their work.

Mr. Collins's first lesson included some elements of standards-based mathematical practices emphasized by the PD. In comparison to the SMPs, evidence was seen of students *making sense of problems and persevering to solve them*. During the partner work, the students worked together to find and check solutions to algebraic equations. There was also evidence of students *constructing viable arguments and critiquing the reasoning of others*. This occurred as the students worked with different partners and explained how they found the solutions. When considering the MTPs, there was evidence of Mr. Collins *promoting reasoning and problem solving* and *facilitating meaningful mathematical discourse*. The teacher *promoted reasoning and problem solving* by providing challenging problems and having the students explain their work to each other and check the answers to see if they made the algebra equation true. *Facilitating meaningful mathematical discourse* was observed when he encouraged the students to talk with their partners and explain the steps for finding solutions.

Doug Collins: Enacted lesson #2. The second observed lesson took place at the end of a unit on quadratic equations. The topic for the lesson was using data to determine if relationships were linear or quadratic (DC Interview, 2016-04-25). Mr. Collins explained that the academic standards that would be addressed in this lesson were recognizing different types of equations, graphing ordered pairs, writing equations, and interpreting data and graphs (DC Interview, 2016-04-25). When asked where this lesson fit within the unit he was teaching, Mr. Collins stated:

> DC: It's at the tail end. We actually just got done. They are actually testing tomorrow on exponential equations, graphing them, solving word problems on

> them. So we've done all the math and now … here's an example of how [quadratic equations] can apply. (DC Interview, 2016-04-25)

For the lesson, Mr. Collins used a mathematical task presented by the PD facilitators, a modified version of "Bridge Strength" from *Thinking with Mathematical Models* (Lappan, 2005). At the beginning of the lesson, Mr. Collins asked the students to find a partner and to gather pennies, a cup, three strips of four different-length strips of paper, and books for suspending the strips to create a bridge (DC Observation, 2016-04-27). Mr. Collins passed out a work packet to each student and told them that they would need to read the packet so they would know how to do the activity for the day. Students were instructed to run through all of the experiments first and collect all of the resulting data (DC Observation, 2016-04-27). After all data were collected, the packet had fourteen questions for the students to answer about the experiment.

The students worked in pairs. They suspended the paper bridges, placed a cup on the bridge, placed pennies in the cup until the bridge collapsed, and recorded the number of pennies required to collapse the bridge in a data table. After a bridge collapsed, the students increased the thickness or the length of the bridge and repeated the process. Most of the teacher-to-student interactions involved clarifying how to set up the bridges or how to collect the data (DC Observation 2016-04-27). The student-to-student interactions included clarifying methods to collect and represent the data (DC Observation, 2016-04-27). When data collection was complete, the students made graphs and answered questions in the packet about the data to determine if relationships were linear or quadratic and to make predictions.

Some of the standards-based mathematical practices were observed during Mr. Collins's second lesson. For the SMPs, evidence was seen of students *making sense of problems and persevering to solve them*. This occurred during the small group work. Students were given a higher-level mathematical task and worked in small groups to make sense of the problem and answer related questions. The students also *modeled with mathematics* when they organized their data into tables and used the data to make graphs of the relationship between length or thickness and the weight of collapse. There was evidence of some of the MTPs. Mr. Collins was observed p*romoting reasoning and problem solving*. This occurred as the students made sense of the problem and made predictions using the collected data. There was also evidence of *facilitating meaningful mathematical discourse* as the students worked in small groups, clarified terminology with each other, and explained reasoning about the graphed relationships.

Doug Collins: Interpretation of goals and enacted lessons. Mr. Collins interpreted the goals of the PD to be a means to help students pass a state accountability and graduation test. His interpretation focused on TAPS' second goal: To improve students' algebraic knowledge, algebraic skills, and disposition toward algebra. His description of the goals in his own words was very close to the objective to improve students' performance on standardized and class-level assessments. When he identified the lessons to be observed that were consistent with the goals of the PD, both enacted lessons were a review or an extension to prepare students for an upcoming test.

Teacher 2: Kathy Gibson (KG)

Kathy Gibson was a female teacher with eleven years of teaching experience; ten of the years were at SHS. She taught Pre-Calculus and was the mathematics department chair. Her bachelor's degree was in mathematics education and she was working on a master's degree in mathematics education. After the PD summer institute, Ms. Gibson was asked to describe the goals of the PD in her own words.

> KG: Well, I don't know. I guess I would say the goals for me would have been to get more activities and more things that I could use in class that had a higher depth of knowledge questions and how I could improve in that area. I guess that was my main goal.
>
> Interviewer: What do you think the goals were for the presenters? What do you think [PD facilitator] was trying to accomplish or [other PD facilitator]? Do you think it was the same thing?
>
> KG: I don't know. (KG Interview, 2015-09-17)

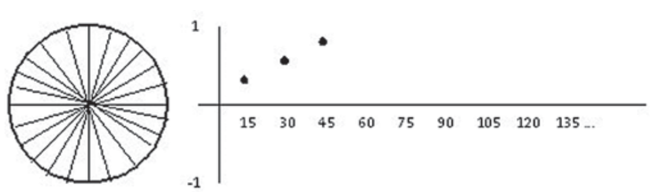
Initially, Ms. Gibson answered with her personal goals for the PD. She wanted to get more activities to use in her class with higher depth of knowledge questions. When she was asked what the goals were for the program, she replied that she did not know.

Kathy Gibson: Enacted lesson #1. Ms. Gibson's first observed lesson was an introduction to graphing sine and cosine functions. The lesson included a mathematical task that she developed during the PD sessions. When asked about the purpose of the lesson, Ms. Gibson replied, "It is discovering the graph of a sine function from the unit circle" (KG Interview, 2015-09-17).

The lesson started when students received a packet with instructions and questions and were told by Ms. Gibson that they would need to read the packet in order to know what to do (KG Observation, 2015-09-21). The students collected the needed materials for the lesson which included a large sheet of paper, a protractor, a compass, a meter stick, a piece of yarn about two meters long, and several pieces of uncooked spaghetti. The students worked in groups of three or four on the task.

Students used the compass to draw a unit circle with a radius equal to the length of one of the spaghetti noodles on one end of the large piece of paper. The students used the protractor to mark fifteen-degree increments around the circle and created a Cartesian plane next to the unit circle. The x-axis was labeled with the degrees of the circle and the y-axis was labeled with the vertical distances from the horizontal diameter of the circle to each of the given degrees (Figure 1). Students used additional spaghetti pieces to measure the perpendicular heights at the fifteen-degree increments and transferred the ordered pairs to their graph. The resulting graph was a sine curve. Once groups completed the sine curve, they followed similar steps to create a cosine curve.

FIGURE 1. *Task comparing unit circle to sine curve.*



The students worked together to make sense of the instructions, agree on terminology, use tools to construct a sine or cosine curve, and respond to questions in the packet. Student comments included, "If the spaghetti is the radius, then the circle is two-spaghetti wide," and "The curve follows the same pattern" (KG Observation, 2015-09-21).

Many groups noticed patterns with the different lengths. For example, students noticed that the perpendicular distance to the point on the circle at 45 degrees was the same as the distance at 135 degrees. Ms. Gibson walked around the room checking on the progress of the groups and asking questions to monitor student thinking. She asked the students questions about the activity like, "Do you see any patterns in the graph of the sine curve?" or "How are the graphs of sine and cosine the same and how are they different?" (KG Observation, 2015-09-21).

After completing the graphs, the students discussed their work. One group displayed a graph with a sine and cosine curve in the front of the class that was used as a reference during the discussion. The class discussed questions such as, "What is the period or the wavelength of the sine curve?" and "What are the zeros of the graph?" Students shared their thinking about the graphs such as, "It repeats after 360 because 0 and 360 are coterminal" (KG Observation, 2015-09-21). Ms. Gibson finished the whole class discussion by explaining that these were the parent graphs for the sine and cosine functions and the class would learn more about the properties of these functions.

Ms. Gibson's first observed lesson included many elements of standards-based mathematical practices; a few are highlighted here. One SMP observed during this lesson was *making sense of problems and persevering to solve them.* During the partner work, the students worked together to understand the instructions and work on the mathematical task, consider the relationship between the unit circle and the two trigonometric functions, and answer questions about the characteristics of the functions. A second observed SMP was *looking for and expressing regularity in repeated reasoning* during the lesson. This occurred when the students noticed patterns in the vertical distances at different degree measures around the circle (e.g., the sine values at 45 degrees and 135 degrees are equal). For the MTPs, evidence was seen of *using and connecting mathematical representations* and *posing purposeful questions*. Students had opportunities to *connect mathematical representations* by making the sine and cosine curves in proximity to a unit circle and using non-standard methods for measurement to find values of sine and cosine at different angles. Ms. Gibson *posed purposeful questions* during small group work and during the whole class discussion when she asked the students about patterns and asked them to compare the graphs.

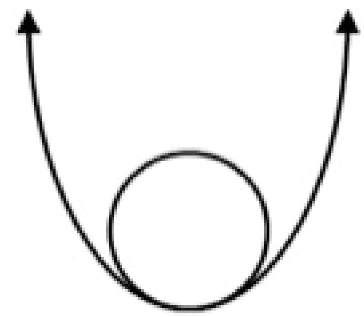
Kathy Gibson: Enacted lesson #2. The topic for the second observed lesson was solving non-linear systems of equations. Similar to the first observed lesson, she used a task that she developed during the PD. Ms. Gibson planned for the students to work in small groups and present solutions to the whole class so that they would "communicate and talk to each other about their ideas" (KG Interview, 2016-02-10).

At the beginning of the second lesson, Ms. Gibson told the students that they could work in small groups to answer two questions: (a) How many different possible intersection points are there if a line and a circle are graphed in the same coordinate plane? and (b) Write a set of equations for each of the possibilities you have and find the intersection points for each (KG Observation, 2016-02-16). The small groups had as many as four students and some students chose to work individually.

The students discussed mathematical ideas in relationship to the questions. For example, they discussed what it meant for a line to consist of an infinite number of points and the possibility of a circle and line intersecting at one point (KG Observation, 2016-02-16). Students made drawings to demonstrate the different intersection possibilities. Ms. Gibson moved around the room to monitor the different student groups. She discussed ideas in the lesson with the students such as how to find the points of intersection. She also challenged their current understandings by asking questions such as, "Can you do this in a different way?" (KG Observation 2016-02-16).

When groups finished the first questions, Ms. Gibson asked them to find the number of possible intersections between a parabola and a circle. Groups discussed the possibility of zero, one, two, three, and four intersections between the parabola and circle. Some debated the possibility of a circle and a parabola intersecting at an infinite number of points if the circle aligned "just right" with the vertex of a parabola (Figure 2).

FIGURE 2. *Image of parabola and circle debated as having infinite points of intersection.*



With about fifteen minutes remaining in the class, Ms. Gibson announced that the groups were going to share solutions (KG Observation, 2016-02-16). Different groups shared equations that were examples of a line and a circle intersecting or a parabola and a circle intersecting. The groups justified the points of intersection and explained how they selected the equations that they used. For example, one student explained:

> Student: We centered our circle around zero so it would be easier to work with. For one intersection we put our circle right underneath the parabola so it just hit at one point. From there we slowly started moving our circle up until it hit [the parabola] two, three, or four times. (KG Observation, 2016-02-16)

There was evidence of many of the standards-based mathematical practices in Ms. Gibson's second observed lesson. The lesson included opportunities for students to *construct viable arguments* as they argued the possibilities of different intersections and justified their reasoning during the small group work and during the whole class discussions. The students *used repeated reasoning* when they developed patterns for moving or changing properties (e.g. slope, radius, intercepts) of the lines, parabolas, or circles. MTPs evident during this lesson included *using mathematical tasks that promote reasoning and problem solving* and *posing purposeful questions.* Ms. Gibson started the task with two challenging questions that required the students to use reasoning and problem solving to conduct a mathematical investigation and apply many conceptual mathematical ideas. In addition, Ms. Gibson posed questions focused on exploring other solution methods and justifying solutions to other members of the class.

Kathy Gibson: Summary. When asked about the goals of the PD program, Ms. Gibson replied with her personal goal, to get more activities to use in her class with higher depth of knowledge questions. Her personal goal was similar to the first goal of the PD, to enrich teachers' knowledge and skills for teaching algebra. It centered on the objective to develop activities that would address the standards-based mathematical practices. Each of the lessons identified by Ms. Gibson for observation embodied her personal goal. In both lessons, Ms. Gibson was observed *posing purposeful questions*, which is consistent with her goal to include higher depth of knowledge questions.

Teacher 3: Laura Henderson (LH)

Laura Henderson was a female high school mathematics teacher with four years of teaching experience. This was her second year teaching at SHS and she taught Algebra 1. Her bachelor's degree was in mathematics education. Ms. Henderson described the goals of the PD in the following way, "I think the goals are to align the [SMPs and MTPs] and the content standards to make an algebra class more enriching to take it to that next level for the kids" (LH Interview, 2015-09-02).

Laura Henderson: Enacted lesson #1. The topic for Ms. Henderson's first observed lesson was a review of order of operations. She planned to have the students work in small groups "to thoroughly explain themselves and understand the content" (LH Interview, 2015-09-02).

At the beginning of the first lesson, Ms. Henderson gave a worksheet to each student that included a diagram with the numbers one through ten organized the way bowling pins are organized on a bowling lane. She explained to the students that they were going to play order of operations bowling (LH Observation, 2015-09-04). The students would roll four six-sided dice and use the four numbers with the order of operations to equal different values one through ten. For example, if 1, 2, 4, and 5 were rolled the students could do $5 \times 4 \div 2 - 1$ to get 9 and could do $5 + 4 - 2 - 1$ to get 6. If students could not find a way to get all of the values one through ten, then they could roll the dice a second time and use the new outcomes to get the remaining values.

Students were told to work with a partner and show all of their work on the worksheet. Students asked clarifying questions such as, "Can we use exponents?" or "Do we have to use all of the numbers?" (LH Observation, 2015-09-04). Most of the student discussions centered on checking answers and explaining how they used the four numbers to find values between one and ten. Ms. Henderson walked around the room checking the work done by the students. She reminded them to use grouping symbols and exponents (LH Observation, 2015-09-04). Some groups were not able to find a combination to get one of the values between one and ten. Ms. Henderson provided hints to groups when they only had a few numbers remaining. For example she visited one group and said, "You need an eight? What about six times five, divided by three, minus two?" (LH Observation, 2015-09-04).

The class discussed the task as a whole class during the last ten minutes. During the whole class review, Ms. Henderson asked the students to share the craziest equations they found. One student shared, "I had five to the power of one, minus two, minus two" (LH Observation, 2015-09-04). She asked if anyone used a square root, but none of the students shared an example. The students handed in their worksheets at the end of the class.

One standards-based mathematical practice was observed during Ms. Henderson's first lesson included. There was evidence of the SMP *attend to precision*. The main focus of the activity was applying the rules for order of operations to find different integer answers. Students manipulated numbers, performed calculations, and checked their work to calculate the different integer answers.

Laura Henderson: Enacted lesson #2. For the second observed lesson, Ms. Henderson planned for the students to work on data analysis and creating approximate best-fit lines. She felt that the lesson was aligned to the PD because she was using a mathematical task that was shared by the facilitators (LH Interview, 2016-04-29). The lesson involved students comparing data about the length and the width of different bird eggs (Mathematics Assessment Resource Service, 2011).

The lesson started when Ms. Henderson introduced the task.

> LH: You are going to work on a task involving bird eggs. It involves bivariate data, which we have talked about. You will need to read the questions and answer them the best that you can. If you get confused, I will clarify the question for you. After a while, you can work with a partner and compare what you have with what they have. There isn't just one right answer for these. Just because someone has a different answer doesn't mean that you are completely wrong. (LH Observation, 2016-05-04)

Ms. Henderson handed out a packet with a scatter plot graph of data comparing the length and the width of different bird eggs. Students were instructed to use the data to answer a series of questions about the relationship.

The students worked independently on the task at the beginning. After about twelve minutes, Ms. Henderson asked the students to work with a partner. The students compared answers and explained their solution methods to each other (LH Observation, 2016-05-04). Ms. Henderson walked around the class checking work done by the students and listening to the small group discussions. She asked some groups clarifying questions such as, "What can you do to check the equation?" (LH Observation, 2016-05-04).

The students returned to their seats after fifteen minutes. At this point, Ms. Henderson led a whole class review of answers. She asked questions such as, "How did you start this?" and "What do we do next?" (LH Observation, 2016-05-04). The students responded and explained steps in their solutions. For example, Ms. Henderson asked one student how he added a point to the graph given an egg with a length of 57 millimeters and width of 41 millimeters. The student explained how he graphed the point with length on the x-axis and width on the y-axis (LH Observation, 2016-05-04). In other cases Ms. Henderson re-read the questions from the packet and explained how to do the problems. For example, one question asked which egg had the greatest ratio of length to width. She explained that the students needed to create ratios for five different eggs and see which ratio was the largest (LH Observation, 2016-05-04). After the whole class review of the answers, the students passed in their work from this task.

Ms. Henderson's second lesson included some of the s tandards-based mathematical practices. In comparison to the SMPs, there was evidence of students *making sense of problems and persevering to solve them.* Ms. Henderson facilitated instruction by requiring the students to read and understand the mathematical task, limiting the amount of direction and answer-giving to students, and having the students work independently and in small groups. There was evidence of *attending to precision* when the students compared and checked solutions and estimated the length or width of eggs not on the provided scatter plot using an estimated best fit line. For the MTPs, Ms. Henderson implemented a task that included opportunities for the students to *use and connect mathematical representations*. This was evident when the students represented the data graphically and algebraically.

Laura Henderson: Summary. Ms. Henderson described the goals of the PD in her own words as aligning the SMPs and MTPs to mathematics content standards to improve her algebra class. Her goal was aligned with objectives from the first and the second PD goals: developing algebra activities that would address the SMPs and MTPs and engaging students in solving rich algebra tasks to enhance understanding. Each of the observed lessons reflected her interpretation of the PD goals because each of the lessons was an attempt to include practices like *attending to precision* and *making sense of problems and persevering to solve them* as students worked on tasks and explained the mathematical content.

Teacher 4: Ruth Lawrence (RL)

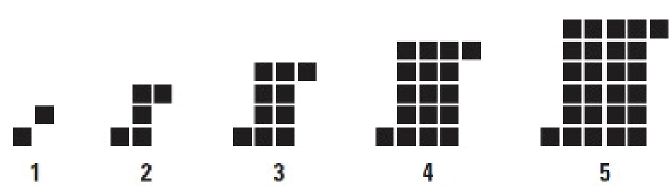
Ruth Lawrence was a female teacher with nine years of teaching experience, eight of them at SHS. At the time of this research, she was teaching Algebra 1 and Geometry. Ms. Lawrence had a bachelor's degree in mathematics. After the summer PD, she was asked to describe the goals for the PD program.

> RL: I think the goals were to expose us to mathematically rich tasks. And I think also we were supposed to learn about the SMPs and the MTPs and maybe how to keep those in our focus while we are teaching throughout the school year. (RL Interview, 2015-09-14)

Ms. Lawrence identified two key aspects or the PD, to help teachers learn about mathematical tasks and to help teachers learn how to use the standards-based mathematical practices in the classroom.

Ruth Lawrence: Enacted lesson #1. The first lesson involved creating algebraic representations of patterns using the S-Pattern task (Institute for Learning: Learning Research and Development Center, 2015) that was shared during the summer PD (Figure 3). Ms. Lawrence stated that the purpose of the lesson was "to explore this activity and represent a pattern with a quadratic equation" (RL Interview, 2015-01-07). The lesson took place over two days.

FIGURE 3. *Sequence of figures following the S-Pattern.*



At the beginning of the lesson, Ms. Lawrence asked the students to get into pairs, explained that the worksheet included a sequence of figures with patterns, and asked the students to answer questions about the patterns (RL Observation, 2016-01-08). The students described different patterns in the picture with their partners. Students asked their partners questions such as, "How do you explain that?" and justified statements such as, "It is F – 1 because it is the fifth figure, but there are only four squares in the row" (RL Observation, 2016-01-08). When the students shared their patterns and solutions, they frequently asked each other if they understood. When students did not understand, they would ask for an explanation (RL Observation, 2016-01-08).

Ms. Lawrence walked around the room checking on the student work. She encouraged the students to consider other patterns by asking questions such as, "You are doing different patterns. Are you noticing anything else?" (RL Observation, 2016-01-08). Ms. Lawrence also probed for student understanding with questions such as, "Does the height change too? Can you relate it to that?" (RL Observation, 2016-01-08). The groups answered questions about representing the total number of squares algebraically until the end of the class on the first day.

When the students returned the next day, Ms. Lawrence asked different students to come up to the front of the class and share the equations they created. They explained how they developed the parts of the equations based on the figures. For example, one student shared:

> Student: [Writes T = (F – 1) (F + 1) + 2 on the board]. F is figure number and T is total squares. If we talk about figure five, five would go where the F's are. [Draws a circle around the 4 × 6 rectangle in the middle of the picture, excluding the two individual squares on the top right and bottom left]. This is F + 1 [points to the side with length 6] and this is four [points to the top with length 4] and five minus one is four. And you add these two [points to the two excluded individual squares] at the end. So it is 26 squares. Six times four is twenty-four and add two. (RL Observation, 2016-01-09)

Many of the standards-based mathematical practices were observed throughout Ms. Lawrence's first lesson. The alignment with the practices was largely due to her use of a high-level mathematical task, providing opportunities for students to make sense of the problem, working in small groups, encouraging students to explain their thinking, and encouraging students to compare their solutions to other solutions. One of the most common SMPs was *reasoning abstractly and quantitatively* when the students developed equations to represent the relationship between the figure number and the number of squares in each figure and described patterns to answer the questions for the mathematical tasks. Students *constructed viable arguments* during the small group and whole class discussions as they explained their thinking and clarified statements if ideas were unclear. Two of the MTPs observed during this lesson were *connecting mathematical representations* and *facilitating meaningful mathematical discourse*. The students *connected mathematical representations* by creating algebraic equations to represent the relationship between figure numbers and number of squares. In some cases, students created data tables for the figures. Ms. Lawrence *facilitated meaningful mathematical discourse* by having the students work in pairs and asking them to explain patterns or solutions to their partners.

Ruth Lawrence: Enacted lesson #2. The topic for the second observed lesson was writing linear, quadratic, and cubic algebraic expressions. M.s Lawrence felt that the lesson was aligned to the PD because the activity included algebraic habits of mind and productive struggle (RL Interview, 2016-01-12). The mathematical task for the second lesson was the painted cube problem (Lappan, Fitzgerald, & Fey, 2006), which was shared by the PD facilitators during the summer. This lesson also took place over two days.

Ms. Lawrence started the lesson by passing out a worksheet with a data collection table and asking the students to get into small groups. Using a model of a 3x3x3 cube, she explained that the outside of a cube with edge length three could be painted and the unit cubes would have paint on zero, one, two, or three sides (RL Observation, 2016-01-15). Ms. Lawrence showed the class how to fill in their data table for the cube with edge length three such that eight unit cubes had paint on three faces, twelve unit cubes had paint on two faces, six unit cubes had paint on one face, and one unit cube had paint on no faces (RL Observation, 2016-01-15). The students were told to complete the worksheet about painted cubes with edge length two, three, four, five, six, and any length "n".

The students used snap-cubes to build models and worked in groups of two or three to complete the data table and the worksheet. Students asked questions such as, “Are there twenty-four cubes with one face painted?” and “Is it always times six?” (RL Observation, 2016-01-15). They explained answers to their partners while pointing to models and counting cubes located in the correct positions for three, two, one, or zero faces painted.

Ms. Lawrence walked around asking questions about different ways to work on the problem such as, “Is there a faster way to find the number of cubes in the middle instead of adding nine and nine and nine?” or “Can you think about the cubes in groups?” (RL Observation, 2016-01-15). She also shared ideas to help the students find patterns for a cube with any unit length. For example, she told a student, “I want you to go back and see how you found these [the number of cubes with paint on one side]. That will help you see the pattern” (RL Observation, 2016-01-15).

During the second class period, Ms. Lawrence told the class that they needed to explain patterns and write algebraic expressions for a cube with an edge length of “n” and be prepared to discuss their expressions with the whole class (RL Observation, 2016-01-16). During the whole class discussion, one student wrote the number of unit cubes with paint on one face for the different lengths of the large cube as 0, 6, 24, 54, and 96. She explained that each of the numbers is a multiple of six. Zero is 6×0, six is 6×1, twenty-four is 6×4, fifty-four is 6×9, and ninety-six is 6×16 (RL Observation, 2016-01-16).

Again, many standards-based mathematical practices were observed during Ms. Lawrence’s second lesson. The students *modeled with mathematics* by creating data tables, algebraic equations, and graphs from the cube models. They also *looked for and made use of structure* by using patterns and algebraic expressions to represent mathematical relationships they found with the cubes. With respect to the MTPs, Ms. Lawrence *promoted reasoning and problem solving* by using a higher-level mathematical task and allowing the students to work in small groups to make sense of the problem. She also provided opportunities for the students to *connect mathematical representations* as they created models, completed data tables, wrote algebraic equations, and graphed the relationships. The students negotiated an understanding of these relationships during the small group and whole class discussions.

Ruth Lawrence: Summary. Ms. Lawrence interpreted the goals of the PD as exposing the teachers to mathematical tasks and using the standards-based mathematical practices as a focus throughout the school year. Her interpretation of the goals aligned with the first goal: To enrich teachers’ knowledge and skills for teaching algebra. The description she provided was similar to the objective to develop activities that would address the SMPs. Both of the lessons Ms. Lawrence identified for observation that were consistent with the goals of the PD included mathematical tasks shared during the PD and contained many standards-based mathematical practices due to instructional strategies used like questioning and small group expectations.

Findings

There were two goals for TAPS. The first goal was to enrich teachers’ knowledge and skills for teaching algebra. To help accomplish this goal, the PD providers engaged the participating teachers in solving algebra tasks to enhance algebraic understanding, asked each teacher to develop tasks or lessons that would address the standards-based mathematical practices, and provided a system for structured reflection and feedback. The second goal was to improve students’ algebraic knowledge, algebraic skills, and disposition toward algebra. The PD providers addressed this goal by helping teachers engage students in solving rich algebra tasks and provide opportunities for students to make meaning of algebra.

When the teachers were asked to describe the goals of the PD in their own words, each teacher stated a goal that was consistent with the PD goals, but concentrated on one of the objectives. The teachers’ interpretations of the goals influenced which lessons were selected for observation and the enactment of the lessons (see Table 1). Doug Collins described the goal of the PD as helping him improve student scores on the state accountability exams. Both observed lessons were review or extension activities where students applied learned content before an upcoming unit test. Kathy Gibson’s goal, to include activities with higher depth of knowledge questions, was an influence on the lessons that were observed. During the observations there was evidence of Ms. Gibson *posing purposeful questions* and the students *constructing viable arguments.* The observed standards-based mathematical practices were consistent with using higher depth of knowledge questions. Laura Henderson’s interpretation of the goal of the

PD was to align the standards-based mathematical practices with content standards and improve her algebra class. Her lessons included a few of the practices, but she included strategies like small group work and questioning that encouraged students to explain their work about order of operations and bivariate data. Ruth Lawrence's interpretation of the goal was to include mathematical tasks and the standards-based mathematical practices from the PD. The influence of her interpretation of the goals started with her choice to use mathematical tasks from the PD for each of her lessons. In addition, her enacted instructional strategies such as providing opportunities for students to make sense of the problem, working in small groups, and encouraging students to explain their thinking provided opportunities for the practices to be a part of the lesson.

Table 1: Comparison of teachers' interpretations of PD goals and observed lessons.

Teacher	Interpreted goal	Observed lessons
Doug Collins	Help improve the algebra one end-of-course exam scores	Review lessons for unit tests
Kathy Gibson	Get more activities with higher depth of knowledge questions	Mathematical tasks with instances of *posing purposeful questions*
Laura Henderson	Align the standards-based mathematical practices and the content standards to make an algebra class more enriching	Mathematical tasks with partner work, students were observed *attending to precision*
Ruth Lawrence	Expose the teachers to mathematical tasks and use the standards-based mathematical practices	Lessons involved mathematical tasks shared during the PD and many practices were observed due to enacted instructional strategies

Similar to the research demonstrating that teachers adapt to new visions for mathematics instruction by interpreting and constructing understandings (Coburn et al., 2016; Munter et al., 2015; Roth McDuffie et al., 2018), each teacher in this research interpreted the goals for the PD. The teachers interpreted the goals in a way that was consistent with the goals for TAPS, but tended to concentrate on an objective related to one of the two goals. Their interpretations of the goals are in line with findings described by Ball (1996) and Loucks-Horsley et al. (2010) where teachers focused on elements of PD that addressed curricular, pedagogical, and/or outcome needs for her or his class.

Discussion and Implications

This research describes how four teachers interpreted the goals of a mathematics PD program and how those interpretations influenced enacted lessons. The enacted lessons were lessons that the teachers selected to have observed because they felt the lessons were consistent with the goals of a PD program. The observed lessons were similar to the teachers' interpretations of the goals. A multiple-case study cannot suggest that the interpretations and aligned instructional patterns found are comprehensive for all teachers, but these patterns illuminate how teachers' interpretations of the original goals of the PD program can influence enacted lessons.

An implication of this research is that PD providers need to spend time learning about teachers' interpretations of the goals of PD. Similarly, when mathematics supervisors are working with teachers to enact a new vision for mathematics instruction, they should spend time learning about teachers' interpretations of the vision. Learning about teachers' interpretations of goals or a vision can help PD providers and mathematics supervisors in two ways. First, learning about the interpretations can help providers and supervisors identify misunderstandings or misalignment with the desired outcomes. This offers an opportunity to develop learning experiences that will enrich understandings about the goals or vision. If PD providers and mathematics supervisors feel that the teachers' interpretations are too narrow or too broad, they can work with teachers adjust their interpretations. This would help the providers/supervisors and teachers build a consistent vision and move in a similar direction towards achieving desired instructional practices. Second, because the teachers' interpretations influence enacted instruction, providers and supervisors should learn about the interpretations of goals or visions before performing classroom observations. Knowing a teacher's interpretation could provide insight about enacted instructional strategies for lessons, especially when the lessons are selected for observation by the teachers. PD providers and mathematics supervisors can use the interpretations of the goals and evidence from observed lessons to discuss the goals for mathematics instruction and work with teachers to promote strategies that are consistent with the goals.

This research focused on teachers' interpretations of goals while participating in PD and how those interpretations influenced enacted lessons. Many additional questions about the relationship between the interpretations and standards-based mathematical practices are not understood with this data, but would be helpful if investigated. For example, what influence could the interpretation of goals have on the enactment of specific standards-based mathematical practices? In addition, it would also help to know if the teachers' interpretations of the goals were associated with the development of the goals based on the needs of the school and the teachers, each teachers' prior experience with classroom situations (especially standards-based mathematical practices), or other influencing factors. Additional research on how teachers interpret the goals of a mathematics PD program would add to the understanding about impact on classroom practices. ✪

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