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Design and Impact of Flexible, Asynchronous Online Video-based Mathematics Professional Development

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Abstract

In this article, we share the experimental research design and preliminary impact results from the Video in the Middle project, which is adapting existing face-to-face video-based mathematics professional development materials to online two-hour modules that can be used in flexible asynchronous formats: independent, locally facilitated, or developer facilitated. Preliminary research results indicate that teachers appreciated the variety of formats, found the modules useful and engaging, and learned to appreciate and use visual methods for solving problems, including using color to distinguish and highlight the relationship between numeric, algebraic, and geometric models. The benefits of this asynchronous PD became pronounced as the pandemic emerged during the research study and teachers found themselves shifting to remote instruction with little time to prepare.

Incorporating video within the professional development (PD) environment provides an opportunity for teachers to unpack the relationships among pedagogical decisions and practices, students' work, and the disciplinary content (e.g., Borko et al., 2011; Brophy, 2004; Harford & MacRuairc, 2008; Rich & Hannafin, 2009; Rosaen et al., 2008; Sherin, 2007). Collectively viewing and discussing video clips allows for the complexities of classroom practice to be stopped in time, unpacked, and thoughtfully analyzed, helping to bridge the ever-present theory-to-practice divide and support instructional reflection and improvement. In the classroom, teachers must constantly make individual in-the-moment decisions, while viewing video during PD allows them the opportunity to collectively deconstruct and discuss familiar experiences and to actively generate new understandings about content, pedagogy, and student thinking (Cullen, 1991; Korthagen et al., 2001). With video, teachers have the opportunity to observe and study the complexity of classroom life, to reflect on their own instructional decisions, and to integrate multiple domains of knowledge to solve problems of practice (Blomberg et al., 2013). Recent comprehensive reviews of the literature on video in PD point to the value of video as a learning tool that can promote improvements

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in instructional practice (Gaudin & Chaliès, 2015; Major & Watson, 2018). In addition, cognitive science research suggests that strongly connected learning and transfer situations improve knowledge transfer (Novick, 1988).

Classroom video clips, by themselves, are unlikely to foster teacher learning without being intentionally integrated into a PD program or course (Blomberg et al., 2014). Along with the purposeful selection of video clips, a central component of designing effective PD materials is determining how to embed the video within the broader curriculum to accomplish identified learning goals. It is essential to situate the video in a framework that supports detailed analysis and interpretation, thereby providing access and opportunities for teacher learning across the totality of the PD experience. Both the video and the activities surrounding the video should be designed to target predetermined learning goals for both the PD curriculum as a whole as well as each individual session (Blomberg et al., 2013).

Many, but not all, video-based mathematics PD programs have teachers engage in specific activities before and after watching the focal video (Borko, et al, 2015; LeFevre, 2004; Santagata, 2009). For example, prior to watching a clip, PD facilitators may ask the teachers to solve and discuss the math problem shown in the video in order to develop content knowledge, motivate teachers to notice particular elements of the content contained within the clip, and attend to specified activities such as a unique solution method or teacher questions that prompt extended student reasoning. After viewing the video, there may be a guided discussion and, perhaps, follow-up activities in which the teachers relate what they have seen on the video to their own classroom practice. The discussion and follow-up activities extend teachers' thinking and analysis by probing more deeply into topics or issues presented within the video.

We label this intentional sequencing of video viewing such that it occurs between designated activities with specified learning goals a "video in the middle" design (Seago et al., 2018). In video-based mathematics PD that incorporates this design feature, video is located in the middle of the learning experience, sandwiched between activities such as mathematical problem-solving and pedagogical reflection. We will describe how we use this sequence in more detail when we discuss the Video in the Middle project that is the focus of this paper and will provide an illustrative vignette to depict how the specific sequence looks in action. Our goal is not to argue that this design feature is new to the field of professional development, but rather to highlight and label it, consider how the design is likely to support teachers' learning, and help inform leaders who facilitate mathematics PD.

Teacher Noticing as a Conceptual Frame

Mathematics teachers come to professional learning situations with varying levels of knowledge, much like the K-12 students who come to their mathematics classrooms. One unique aspect of teachers' knowledge is their "professional vision", which refers to their ability to notice and analyze features of classroom interactions, make connections to broader principles of teaching and learning, and reason about classroom events (Sherin, 2007; van Es & Sherin, 2002). Over the years, diverse conceptions of noticing have emerged in the literature, but in general most discussions of mathematics teacher noticing involve two main processes: (1) Attending to particular events in an instructional setting (i.e., teachers choose where to focus their attention and for how long) and (2) Making sense of events in an instructional setting (i.e., teachers draw on their existing knowledge to interpret what they notice in classrooms) (Sherin et al., 2011). Sherin et al. (2011) argue that these two aspects of noticing are not discrete, but rather interrelated. Teachers attend to events based on their sense-making, and how they interpret classroom interactions influences where they choose to focus their attention.

Teacher education programs that incorporate video foster the development of teachers' noticing skills (Koellner & Jacobs, 2014; Roller 2016; Santagata & Yeh 2013; van Es & Sherin, 2002). As they attend to and make sense of PD focused on cases of instruction, teachers are also likely to consider the implications for their own practice (Koh, 2015). In other words, what teachers notice appears directly relevant to how they elect to carry their learning into their classrooms (Sherin & van Es, 2009). Participants in PD do not all make sense of their experiences in the same way; rather, individuals bring differing knowledge and beliefs about teaching and learning, students, content, and curriculum to bear on what they notice (Erickson, 2011; VanEs, 2011). This individual diversity impacts what they notice, how they engage in the professional development and what they take and use in their own practice. It also has implications for the purposeful design of video-based PD and teacher education (Hatch et al., 2016).

Online Teacher Professional Development

As video technology and online video sharing have become more accessible and widespread, video-based professional learning is well-positioned to leverage the benefits of digital platforms especially during the pandemic (Teräs & Kartoglu, 2017). Online teacher professional development (oTPD) allows mathematics teachers access to professional development resources that may not be available to them locally and can also support those who are reluctant to share ideas in face-to-face settings in becoming more comfortable doing so in digitally mediated interaction (Dede et al., 2009). Online teacher PD is considerably more scalable than comparable face-to-face PD, and in many cases is subject to fewer monetary and logistical constraints for teachers (Killion, 2013). Research to date on online professional development has shown some positive effects for teachers, even compared to face-to-face formats (Chauvot et al., 2020; Nite & Bicer, 2020; O'Dwyer et al., 2010; Telese & Chamblee, 2020). Most research comparing online, and face-to-face versions of PD has found that well-designed online courses utilizing high-quality learning materials intended for individual use can produce learning outcomes that are similar to or better than face-to-face options (Fisher et al., 2010; Fishman et al., 2013).

Fishman (2016) reminds us that oTPD is PD. The professional learning opportunities in an online environment and a face-to-face setting are both determined by the learning design of the program, as different approaches can lead to different learning experiences (Fishman, 2016; Prosser & Trigwell, 1999). Herrington et al. (2010) propose principles of authentic e-learning within a framework based on the theory of situated learning (Lave & Wenger, 1991). Situated learning theory assumes that learning takes place in an authentic context that preserves the complexity of practice —a context that can occur in differing settings such as live, virtual, or video representations of practice (McLellan, 1994). The authentic e-learning principles proposed by Herrington and colleagues focus on authentic context, authentic tasks, access to experts, multiple perspectives, collaboration, reflection, articulation, scaffolding, and assessment. This paper reports on the design and preliminary findings from a project that is adapting face-to-face mathematics PD materials to an asynchronous digital format that utilizes these authentic e-learning principles.

Affordances of Asynchronous Teacher Professional Development

By asynchronous PD, we mean learning activities that happen at different times for different participants; that is, participants are not required to be available at the same time (Dash et al., 2012). Asynchronous PD environments can include social networks, discussion boards, self-paced online courses, resource-sharing sites, and are often transformed or defined by technology (Bates et al., 2016). In recent years, asynchronous, remote opportunities have provided teachers with more and more opportunities to engage in professional learning when high-quality in-person PD is not available or practical (Appana, 2008; Kleiman, 2004; Laferriere et al., 2006; Walker et al., 2008; Wells et al., 2006). While face-to-face professional learning provides many benefits, teachers may struggle to participate due to a number of possible factors such as: the costs of substitute teachers, travel time, scheduling conflicts or a national pandemic (Abbott et al., 2006; Archibald & Gallagher, 2002; Elges et al., 2006; Wentling et al., 2000). Teachers who do not have a school or district peer teaching the same subject or grade level may also struggle to find meaningful, in-person PD opportunities, and for those working in rural or other remote or isolated settings, high-quality in-person PD opportunities may not exist at all (Kleiman, 2004). Even when teachers are able to participate in some face-to-face opportunities, research shows that consistency and coherence is key (Darling-Hammond et al., 2017); asynchronous PD experiences may also be used in conjunction with less frequent face-to-face or synchronous opportunities in a way that provides teachers with a more impactful experience.

While asynchronous teacher PD can pose some challenges for collaboration and interactivity due to their focus on self-directed learning (Alterman & Harsch, 2017), it also offers a unique set of affordances that make it a genuinely attractive option and not merely a fallback alternative when in-person PD is not possible (Meritt, 2016; Pletola et al., 2017. During the Video in the Middle project research study in March 2020, the benefits of this asynchronous PD became more pronounced as the pandemic emerged and teachers found themselves shifting to remote instruction with little time to prepare. In addition to providing meaningful professional learning during times that are convenient or that may not otherwise be available to teachers locally, asynchronous experiences may offer teachers the ability to choose offerings that address their immediate classroom needs, suit their individual learning styles, or allow them to interact with material in a variety of multi-media formats (Docherty & Sandhu, 2006; Garrison & Cleveland-Innes, 2005; National Staff Development Council, 2001; Richardson, 2002; Spicer, 2002; Treacy et al., 2002). For teachers working in remote environments, asynchronous PD can also connect teachers to networks of other professionals and reduce feelings of isolation (DuFour, 2002; National Staff Development Council, 2001).

Asynchronous teacher PD may also foster higher-quality, more reflective dialogue. Text-based discussions in online PD tend to be more exact and organized (Garrison et al., 2001; McCreary, 1990), involve more formal and complex sentences (Sotillo, 2000; Warschauer, 1995) and incorporate critical thinking, reflection, and complex ideas (Davidson-Shivers et al., 2001; Marra et al., 2004). There is also evidence that asynchronous professional learning experiences can support more open and uninhibited dialogue about sensitive subjects since teachers are able to share ideas and questions when they feel ready rather than feeling "on the spot" in a face-to-face environment (Spicer, 2002; Treacy et al., 2002). The ability to work at their own pace and has also been shown in some cases to increase the amount of PD in which teachers are willing to participate (Paskevicius & Bortolin, 2015; Russell et al., 2009).

While asynchronous professional development has grown in popularity in recent years, instructional leaders and PD providers are finding that, in the current pandemic, conducting high-quality, asynchronous teacher PD is not only possible, but more critical than ever. During the pandemic, teachers, coaches, and other PD providers continue to work from home or hybrid settings and juggle a variety of competing priorities while attempting to learn an entirely new way of teaching, flexible, easy-to-access professional learning experiences that teachers can engage with at their convenience are greatly needed (Boaler et al., 2020; Darling-Hammond et al., 2020; Reimers & Schleicher, 2020). The benefits of this asynchronous PD became pronounced as the pandemic emerged during the research study and teachers found themselves shifting to remote instruction with little time to prepare.

The Video in the Middle Project

The goal of the Video in the Middle (VIM): Flexible digital experiences for mathematics teacher education (NSF Award #1720507) project is to design, develop, and research an asynchronous, video-based form of mathematics professional development/teacher education. The VIM project draws upon the *Learning and Teaching Linear Functions: Videocases for Mathematics Professional Development* (NSF; ESI-9731339) video and ancillary resources (e.g., lesson graphs, transcripts, mathematics and video commentaries) to develop a bank of 40 individual two-hour VIM modules grounded in teachers' mathematical knowledge for teaching linear functions, expressions, and equations. These modules will serve as the component ingredients for creating suggested sequences and pathways of multiple VIM modules based on mathematical and pedagogically focused professional learning opportunities. Mathematical learning goals focus on content-related ideas such as conceptualizing and representing slope, distinguishing between and connecting recursive and closed methods and presentations, and exploring the impact of shifting the starting point (y-intercept). Pedagogical goals provide opportunities for managing meaningful mathematical discourse, examining purposeful questions, using and connecting mathematical representations, and establishing goals to focus student learning (NCTM/NCSM, 2020).

The VIM modules are designed to be offered in three asynchronous digital delivery formats: (1) independent, (2) locally facilitated groups, and (3) VIM project-facilitated groups. Each of these formats offers unique affordances for teachers and provides users with both flexibility and choice in their professional learning, as we believe that teachers will appreciate constrained but flexible options. Some teachers may prefer to work independently at their own pace and on their own time schedule; others may prefer to work with colleagues at their school with local facilitation from a coach. Or districts may want to offer their teachers the opportunity to participate with other teachers nationally in a facilitated experience. VIM's final design will offer a variety of suggested pathways depending upon goals, grade levels, and mathematics content, with options to personalize a professional learning plan (depending on one's goals) or swap a particular module with another from the bank of VIM modules.

Video in the Middle Module Design

Each two-hour module places a video clip at the center, or "in the middle," of professional learning as teachers take part in an online experience of mathematical problem solving, video analysis of classroom practice, and pedagogical reflection (Figure 1). The overall structure of this design is consistent across all VIM modules and is intended to support teachers professional learning opportunities around mathematical knowledge for teaching (Ball & Bass, 2002) and teacher noticing of student thinking and teacher-student interactions (VanEs & Sherin, 2002). Each VIM module contains the same set of activities embedded in the Video in the Middle design as described in Figure 1.

FIGURE 1. *Video in the Middle PD activities*

Pre-Video Activities:
1. Introduction: Module Goals (mathematical, pedagogical, instructional)
2. Explore Math Task and Reflect in Journal
3. Padlet Wall: Share Your Work on the Math Task (on a community wall)
4. Consider Other Solutions and Perspectives
5. Explore Math Task and Reflect in Journal
Video Activities:
1. Review the Context of the Lesson (examine where the video clip is situated within the lesson)
2. Watch Video and Reflect in Journal
3. Reflect on the Lesson Graph and Solution Methods Documents
4. Examine Video Transcript and Share Your Thoughts
5. Watch Video Again with Math Educator Annotations
6. Watch Video and Reflect in Journal
7. Reflect on the Lesson Graph and Solution Methods Document
Post-Video Activities:
1. Padlet Wall: Reflect on Your Learning (e.g. "I used to think…. Now I think…") on a community wall.
2. Bridge to Practice: Connecting Your Learning to Classroom Practice
3. Reflect in Your Journal

The underlying VIM design principles are consistent with. the nine principles of authentic e-learning as defined by Herrington et al. (2010). Figure 2 illustrates the nine principles and how each is exemplified within the learning design of the VIM modules.

Research Study

The first two years of the VIM project concentrated on the iterative testing and design of the video-based asynchronous modules and accompanying resources. Year three of the project focused on conducting an experimental randomly controlled trial pilot to study the potential for teacher and student impact. During Spring 2020, the pilot efficacy study was conducted with 67 teachers across the three delivery formats (Independent: 25, Locally facilitated: 25, VIM project-facilitated: 17) to address the following research questions:

1. *What is the impact of teachers' participation in the three delivery formats on teachers' mathematical knowledge for teaching and their teaching practice?*
2. *What is the impact on their students' performance?*

Method

Intervention

All teachers experienced the same sequenced four, two-hour modules for a total of eight hours of professional development. Figure 3 (pgs. 34 - 35) displays the mathematical tasks, video clip description and learning goals for each of the four VIM modules used for the research study.

Participants

Middle and high school mathematics teachers were recruited from across the state of California. For the locally facilitated condition, math coaches/leaders from two school districts with which researchers had existing relationships were recruited. The coaches and leaders then recruited teachers of grade 6-8 math as well as Algebra 1/ Math 1. The math coaches/leaders in each district served as the local facilitators for groups in their districts. For the self-paced/non-facilitated condition and the VIM project-facilitated condition, teachers were recruited from districts across California and randomized into two groups. Where multiple teachers were recruited from the same district, teachers were split between the two groups. For districts where only one teacher was recruited, participants were matched using demographic characteristics of the

FIGURE 2. *The Elements of Authentic e-Learning (Herrington et al., 2010) and Their Application in the VIM Module*

Principles of Authentic e-Learning	Exemplified in the Design of Each VIM Module
Authentic context reflects the way knowledge is used in real life, preserving the complexity.	Unedited video clips of un-staged mathematics classroom interactions, highlighting the relationship between content, teacher, and students.
Authentic tasks have situationally relevant content and offer opportunities for practical implementation.	Teachers examine mathematics tasks within the context of a lesson, view and analyze a video clip of the lesson, and consider implications for their own practice.
Access to expert performances by having the opportunity to observe how experts solve problems as well as learn with and from their colleagues.	Teachers have opportunities to consider a mathematician's perspective on the mathematics task and a mathematics educator's perspective on the video clip, as well as the perspectives of their peers.
Promoting multiple perspectives by sharing different viewpoints and experiences.	Teachers share their work on the mathematics task with others, consider other solution methods, comment on their peers' work, and receive feedback on their own solution methods.
Collaborative construction of knowledge is characterized by collegial sharing, interaction and collaboration between participants.	Teachers explore a mathematics task and post their work on a community wall for their colleagues to view and comment on.
Reflection offers the opportunity to compare one's thoughts to the ideas of other learners, experts and mentors.	Teachers compare their mathematical work and module reflections to that of their peers, their instructional strategies to those of videotaped teachers, and their analysis to that of mathematicians and mathematics educators.
Articulation is encouraged when participants discuss their growing understanding and publicly present and defend arguments.	Teachers present their solution methods to the mathematical task on a community wall for public presentation to their peers and respond to questions or comments.
Scaffolding and coaching are available when needed.	Each module is scaffolded according to mathematical and pedagogical learning goals. Facilitators in local and project formats read and respond to teachers' journals and community wall posts.
Authentic assessment provides learners with the opportunity to be effective performers with the skills and knowledge they have acquired.	At the end of each module, teachers engage in a "Bridge to Practice" activity designed to provide them with the opportunity to use what they have learned in their own practice.

district (race, free/reduced lunch, and EL status). Of the 68 teachers who began the study, 56 (82%) completed all or nearly all study activities, including all four VIM modules. Table 1 shows the completion percentage for each condition.

Across all conditions, grade levels that participants taught ranged from 6 to 12 (some teachers taught multiple grades). Table 2 shows the breakdown of grade levels taught (some teachers taught multiple grades) and Table 3 shows years of teaching experience.

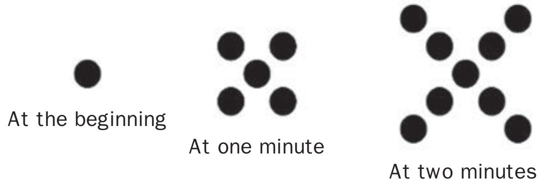
Table 1: Participants' Completion of the Study by Condition

Condition	Began study	Completed study	
		n	%
Self-paced	29	24	83%
Locally facilitated	19	16	84%
VIM project-facilitated	20	16	80%

FIGURE 3. *Four Selected VIM Modules*

VIM 1: James & Danielle: Representing Recursive and Explicit Approaches

Growing Dots

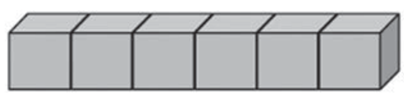


Describe the pattern. Assuming the sequence continues in the same way, how many dots are there in 3 minutes? 100 minutes? *t* minutes?

Learning Goals	Video Clip Description
• Examine, represent, and compare recursive and explicit approaches to solving linear tasks. • Listen to, interpret, and understand differing student approaches to solving the dots task. • Think about goals and instructional decision-making in launching a task.	The teacher asks his 9th grade students to share their solutions and methods for solving growing dots 1. Danielle shares the equation x4+1 and shows the one as the center with a circular growth of 4 dots at each minute. James shares his equation as x + 4 and points to the dot sequence as he shows that 4 is added each time to the previous picture. James says that he didn't count the center because then center is not growing.

VIM 2: Breanna & Cody: Representing Mathematical Thinking

Cubes in a Line



How many faces (face units) are there when two cubes are put together sharing a face? 10 cubes? 100 cubes? How many faces for any number of cubes?

Learning Goals	Video Clip Description
• Examine, represent, and compare the mathematics behind various solution methods. • Listen to, interpret, and understand differing student's mathematical thinking in solving the cubes task. • Think about posing questions in orchestrating a classroom discussion.	This 3rd grade class was given the task of predicting the number of faces for 10 cubes. This segment is a whole class discussion of their predictions based on 2 students' methods. Breanna says you just count down and add 4 more so it is 42. Cody says that you multiply the cubes by 4 and add 2.

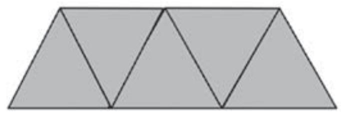
Table continues on next page

FIGURE 3. *Four Selected VIM Modules (continued)*

VIM 3: Lindsey's Question: Connecting Geometry to a Rule

Polygons

If I line up (sharing one side) 100 regular triangles in a row, whay will the perimeter be?



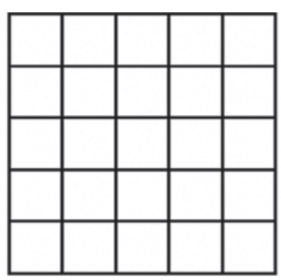
Can you create a rule for finding the perimeter for any number of triangles?

Learning Goals	Video Clip Description
• Make sense of how two different approaches to the general rule for the task connect to its geometry. • Examine how a teacher responds to students' ideas and questions. • Consider how you might purposefully plan your questioning in order to elicit student thinking.	During the 7/8th grade whole class discussion of the triangle problem, Kristen says that the perimeter for any number of triangles would be the number of triangles plus 2. The teacher writes t + 2 = p on the overhead. She asks the class why the rule says we're only adding t + 2 when every time we add a triangle, we are adding 3 edges. Nick responds that two sides get closed off. Chris says that you have the top and bottom and you add two for the ends. Lindsey asks, "Why isn't it plus 4?"

VIM 4: Siri & Tiffany: Using and Connecting Mathematical Representations

Pool Border

Find the number of 1 by 1 tiles required to surround a 5 by 5 pool.



Pool

Find a rule to predict the number of tiles required to surround a square pool of any size. See if you can express that rule as an equation. Be prepared to explain how your equation relates to the pool and border.

Learning Goals	Video Clip Description
• Connect the structure of a visual representation to a mathematical equation • Discuss the role of the teacher in enabling students to communicate and represent their mathematical ideas • Use and connect mathematical representations	After the 8th grade students work in groups on the Pool Border task, the teacher asks Siri and Tiffany's group to share their equation, $n = s4 + 4$, with the class. They explain that if you decompose the border into sides and corners, and group one side and one corner together, you have 4 of them. They share that this is the same thing as adding s + 1 four times, which they share as their second equation.

Table 2: Grade Levels Taught by Study Participants

Grade Level	n
Grade 6	12
Grade 7	25
Grade 8	31
Grade 9	21
Grade 10-12	10

Note: Most teachers taught more than one grade level

Table 3: Study Participants' Years of Teaching Experience

Teaching Experience	n
0-1 years	5
2-5 years	22
6-10 years	11
More than 10 years	30

Measures

In order to answer our research questions, a variety of measures were used to gather impact data on teachers and students. Teacher measures included an online pre-post video and student work analysis, weekly online teacher logs, teacher interviews, and PD embedded pre-post community wall posts and comments. The pre-post student measure was an online quiz aimed at analyzing student's conceptualization of linear functions. Each measure is described in more detail below.

ARTIFACT ANALYSIS

Teachers were given a pre-post online Artifact Analysis measure designed to examine teachers' mathematical knowledge for teaching. The Artifact Analysis is a three-part instrument in which teachers:

1. Solve a mathematical task, predict student solution methods, analyze different representations, and predict student misconceptions.
2. View and answer a series of increasingly specific questions about several short videos of a class discussion centering on students' presentation of their various solution methods.
3. Comment on three pieces of written student work for the same task.

WEEKLY ONLINE LOGS

Online logs, designed to gather information on teachers' instructional practice, were completed weekly by participating teachers. Specifically, the logs documented how teachers reported implementing key content and instructional strategies highlighted in the module learning goals in their classrooms. In addition, fourteen teachers were interviewed individually about their experiences with the VIM modules.

EMBEDDED COMMUNITY WALL RESPONSES

Within the VIM RCT four module experience, two types of community wall pre-post responses were analyzed: (1) VIM 1 and VIM 4 posted mathematical work and teacher comments/questions regarding each other's methods, and (2) VIM 1 and VIM 4 posted reflections from module experience.

STUDENT ONLINE QUIZ

A short, targeted online student quiz was created to assess students' conceptual understanding of linear functions and their ability to use them to solve problems and communicate their reasoning. The pre quiz was completed by 5,070 students and took no longer than half an hour to complete. It was delivered via a Google Form and included three questions with two parts each—short written explanations as well as multiple choice answers.

Due to the COVID-19 pandemic and schools moving to remote instruction after week 6 of the study, we were unable to administer the post student quiz measure or to conduct teacher observations as planned. All other data, the artifact analysis teacher pre-post measure, weekly teacher logs, and community wall responses and reactions, were completed due to the fact that they were collected online. Post teacher interviews were conducted via telephone. For this paper, we will share our early analysis of the teacher log data results, teacher interviews, and community wall mathematics task responses. We are currently in the process of analyzing the pre-post artifact analysis measure and pre-post community wall reflections responses and anticipate having results by spring 2021.

Table 4: Completion Rates of Online Teacher Logs

Condition	Week 1 *n* = 53	Week 2 *n* = 47	Week 3 *n* = 46	Week 4 *n* = 54	Week 5 *n* = 51	Week 6 *n* = 46	Week 7 *n* = 52	Week 8 *n* = 52
Self-paced	86%	83%	59%	79%	76%	69%	72%	72%
Locally facilitated	74%	58%	74%	84%	74%	68%	84%	84%
VIM Project-facilitated	70%	60%	75%	75%	75%	60%	75%	75%

Results

Weekly Online Teacher Logs

Each week, teachers were asked to complete an online teacher log consisting of eight questions focused on mathematical content taught, student interaction structures, and instructional strategies used during that week. Of the 68 participants who completed the study, Table 4 shows the percentage of the 68 participants across the three conditions who completed each log.

Although the response rates remained high for weeks 7 and 8, nearly all teachers responded that they did not teach mathematics for those weeks due to school closures, so responses for those weeks were not included in the analysis. Of the 68 teachers who began the study, 52 completed all four VIMs and indicated in at least four of the first six logs that they taught math that week. (Teachers sometimes missed logs or indicated that they did not teach math that week due to school breaks or other reasons.) These responses were analyzed for similarities and differences in completion rates within and across the three conditions. With a few exceptions, the completion rates across all conditions were fairly consistent. During the first six weeks of the study, the percentage of teachers reporting they taught topics related to linear functions and linearity gradually decreased somewhat (Table 5).

Neither teachers' reported use of VIM instructional strategies (Table 6) nor students' use of related solution strategies (Table 7) changed dramatically over the course of the six weeks. This may be due to several factors:

- Teachers reported teaching less linearity content as the six weeks went on, and some may have felt unsure how (or if) these techniques applied to content not addressed by the VIM modules;
- If teachers' adopted materials were substantially different from the tasks used in the VIM modules, they may have been unsure how to apply these strategies with their materials;
- The study period may have been too short a time for some teachers to become comfortable using new instructional strategies in their classrooms;
- Teachers who did not complete all four VIMs and four of the first six logs were excluded from analysis, so the remaining teachers may represent a group more enthusiastic about reform teaching strategies or trying new methods; it is possible that more teachers in average in this group had encountered these strategies before and were already using them in class. Analysis of teachers' pre/post Artifact Analysis measures will shed light on whether these teachers had higher-than-average MKT before the study.

Table 5: Types of Mathematics Participants Reported Teaching

Type of Mathematics	Week 1 *n* = 47	Week 2 *n* = 42	Week 3 *n* = 38	Week 4 *n* = 50	Week 5 *n* = 46	Week 6 *n* = 44
Linear Functions	55%	48%	45%	50%	41%	32%
Other Linearity Topics	45%	36%	29%	24%	20%	23%
Other Algebra Topics	43%	43%	39%	42%	43%	52%
Other Math Topics	28%	36%	42%	40%	37%	41%

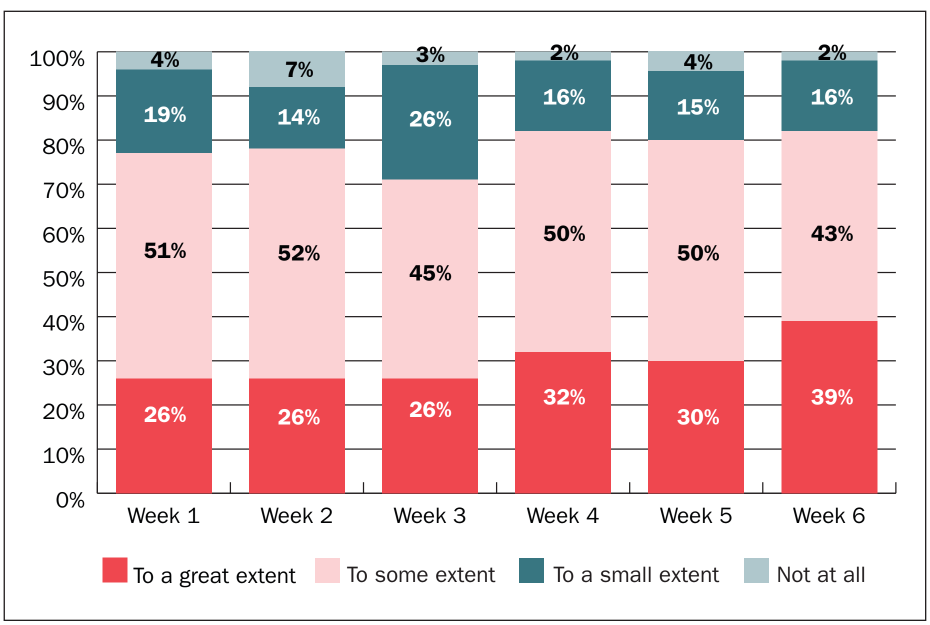
Table 6: VIM Strategies Participants Reported Using

VIM Teacher Strategies	Week 1 *n* = 47	Week 2 *n* = 42	Week 3 *n* = 38	Week 4 *n* = 50	Week 5 *n* = 46	Week 6 *n* = 44
Linking algebra and geometry	74%	48%	58%	52%	48%	64%
Use of color to connect representations	38%	33%	29%	34%	30%	36%
Highlighting multiple solutions	68%	60%	61%	62%	59%	68%
Probing to elicit math ideas	66%	79%	82%	68%	72%	64%
Connecting representations	53%	55%	58%	46%	48%	48%
Other	9%	14%	11%	6%	15%	2%

Table 7: VIM Strategies Teachers Reported Students Using

VIM Student Strategies	Week 1 *n* = 47	Week 2 *n* = 42	Week 3 *n* = 38	Week 4 *n* = 50	Week 5 *n* = 46	Week 6 *n* = 44
Linking algebra and geometry	64%	50%	55%	52%	52%	52%
Use of color to connect representations	28%	29%	32%	28%	22%	23%
Listening to/critiquing others' solutions	53%	71%	55%	62%	70%	59%
Connecting representations	64%	57%	58%	70%	59%	73%
Other	11%	14%	11%	6%	7%	5%

FIGURE 4. *Participants' Responses to the Question, "I can apply VIM ideas with my adopted materials."*



Three Likert-style questions asked teachers to reflect on their teaching experience each week and select one of four answers (not at all, to a small extent, to some extent, to a great extent). Two questions showed increases from week one to week six.

With the question "I am able to apply VIM ideas when working with my district's adopted materials," there was a 13% increase from week one to six in teachers answering, "To a great extent" (Figure 4). There was not much change among teachers who initially answered "To a small extent" or "Not at all" (23% to 18%); this may be because some teachers simply felt that their materials were too different from the tasks presented in the modules to support the use of VIM strategies.

With the second question ("I am able to understand student solution methods that are different than my own"), teachers' responses did not change substantially from week 1 to week 6 (see Figure 5). This may be due to the possible selection bias discussed above, where teachers with higher than average MKT may be overrepresented among those who completed all four VIM modules and at least four out of six weekly logs (those whose weekly logs were included in the analysis). As a result, we may be seeing a ceiling effect in responses to this question.

FIGURE 5. *Participants' Responses to the Question, "I am able to understand student methods different from my own."*

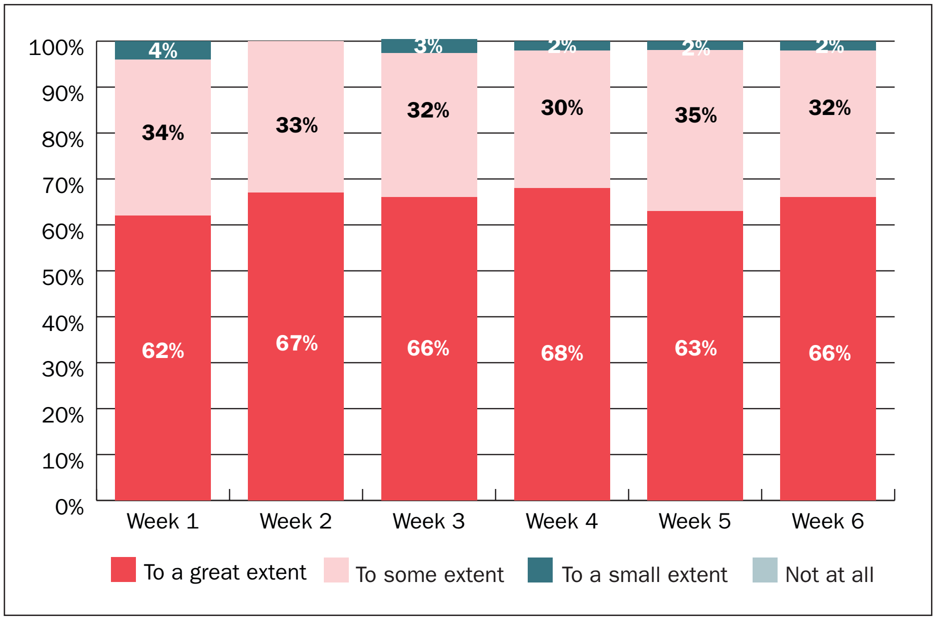
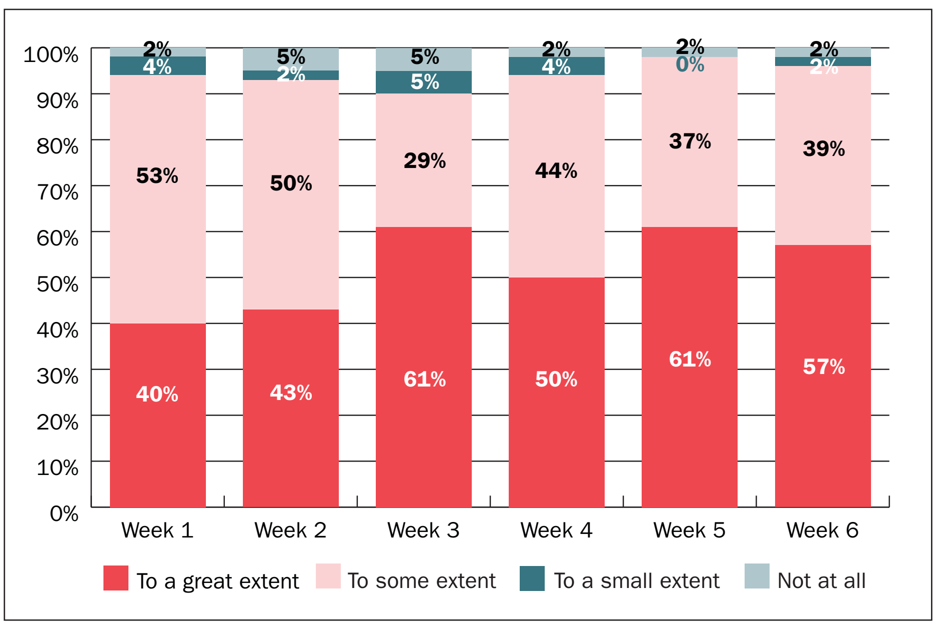


FIGURE 6. *Participants' Responses to the Question, "VIM strategies deepen conceptual knowledge"*



Question three of the teacher log asked teachers if VIM activities helped deepen students' conceptual knowledge of algebraic ideas (Figure 6). There was an 17% increase in teachers responding "To a great extent", with a 21% increase from week 1 to week 5. While teachers reported teaching less linearity content in the later weeks, virtually all were still working on completing VIM modules, so we predict this increase was due mainly to what teachers were experiencing in the PD modules.

Teacher Interviews

Of the 56 teachers who completed the study, nine were randomly selected for interviews in June and July 2020, three from each condition. We also requested interviews with the ten teachers who did not complete the study and received five positive responses.

All teachers interviewed who completed the study expressed that they found the VIM PD modules engaging and useful. Interestingly, interview data did not align with the weekly log data where teachers did not report using VIM strategies more frequently over the course of the study; when asked to highlight specific ways in which the PD had impacted their thinking or practice around teaching linear functions topics, interview subjects mentioned the following more than once:

- Changing questioning strategies and patterns to "focus" student thinking on the learning goal, rather than "funneling" towards a particular strategy or conclusion.
  - The language of focusing and funneling was included in VIM 3 Bridge to Practice activity in which participants examined a chart comparing how two different teachers use the Triangles task in their classroom and facilitated the class discussion by asking questions of students by focusing or funneling (Herbel-Eisenmann & Breyfogle, 2005).
- A greater focus on students' mathematical thinking and reasoning (vs. finding answers)
- Adjusting participation structures and lesson formats to give students more time to work collaboratively
- Desire to use more "open" or visual math tasks, usually thanks either to seeing these tasks used in real classrooms or to feeling more confident in their ability to use them effectively
- Renewed commitment to supporting productive struggle (e.g., letting students struggle with a problem for longer, and asking probing questions rather than giving answers when students are stuck)
- A greater emphasis on multiple representations, including connecting representations through color and probing questions
- Increased openness to multiple ways of seeing and describing linear growth and mathematical structure
- Openness to using manipulatives with older students

When asked to comment on features or elements of the VIM modules that they found most beneficial, the videos, lesson graphs, and community walls were all mentioned by a majority of teachers. Many commented that watching a video of a real classroom helped them better understand what teacher moves described in the PD would look like and how real students might respond. In particular, seeing a video of elementary students working on one of the tasks gave some teachers confidence that their middle school students could approach and benefit from it. Many also expressed that it was helpful to see a variety of ways tasks could be approached or solved, whether in the videos, the solution methods document, or in other participants' work posted on the community walls.

As we hypothesized, teachers in different conditions described different affordances of each. For example, most teachers in the facilitated groups appreciated receiving feedback from a coach in their district or a VIM facilitator, while those in the self-paced group enjoyed the flexibility of being able to complete the modules at their own pace. As one self-paced participant said, "I like this particular experience because I can go at my own pace, and it was still almost like it was facilitated because there were questions that you had to answer."

The benefits of asynchronous, online PD became even more pronounced as the pandemic worsened in March and teachers found themselves shifting to remote instruction with little time to prepare, while also juggling family health concerns and supporting their own children's remote learning. Many expressed gratitude both for the opportunity to complete the PD experience even under shelter-in-place orders as well as the ability to fit their module work around other work and family obligations.

Teacher Community Walls

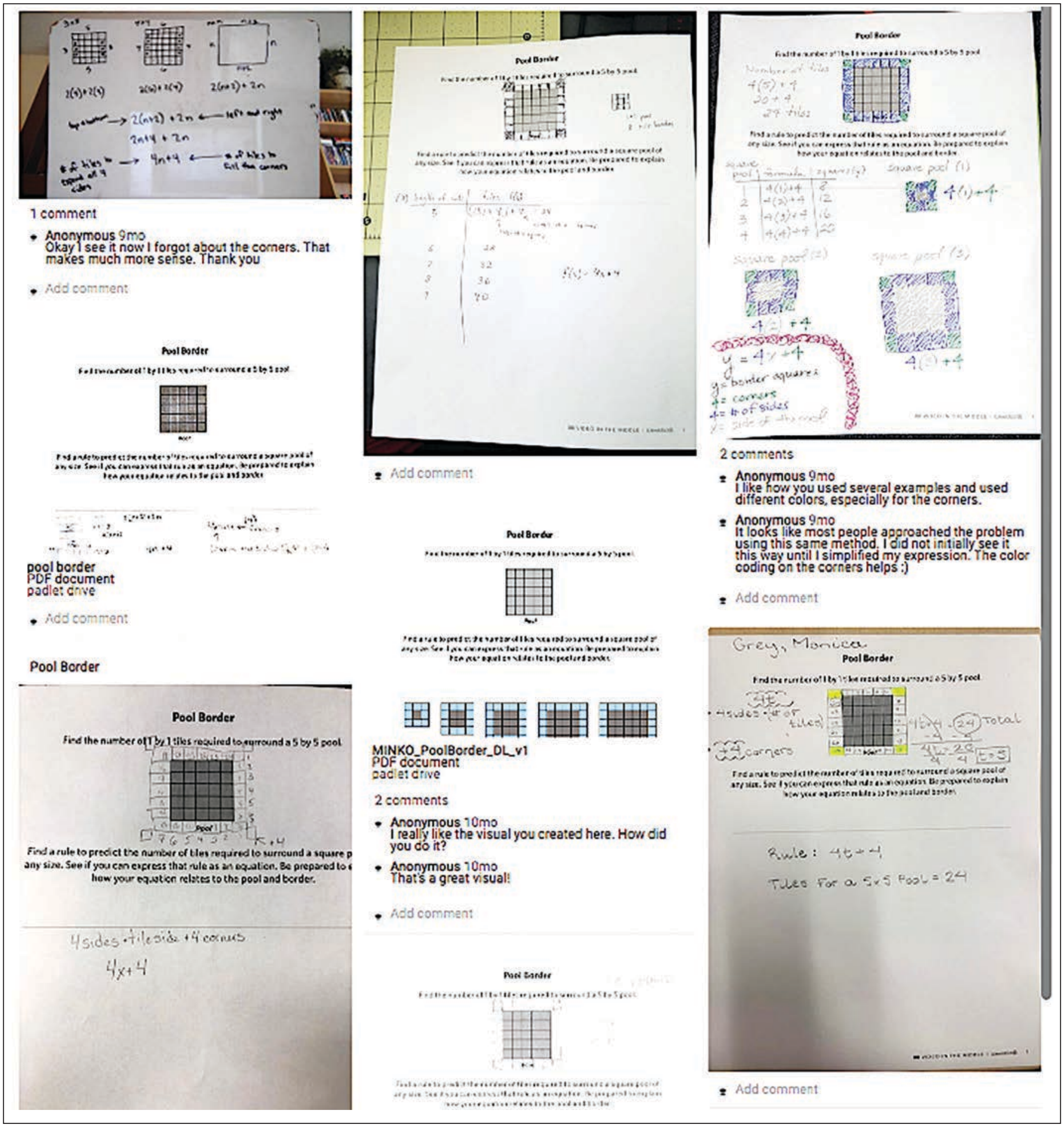
Within each of the four VIM modules, teachers worked on the mathematical task that the students in the video clip engaged with. After solving the problem, they uploaded an image of their work and other teachers (and facilitators in the facilitated conditions) commented or asked questions (Figure 7).

The community mathematics wall participation was high among all three conditions. In the locally facilitated condition, 80% of participants posted their mathematical work in the first VIM module and 95% posted their work in the final VIM module. In the self-paced group, 88% of the participants posted their mathematical work for the first module and 100% posted in the final module. In the VIM project facilitated group, 100% of the participants posted their work in both the first module and last modules. The VIM project facilitated group had the least amount of pre non-facilitator comments, but a similar number of comments to the other two conditions (Table 8).

The most notable pre-post results emerged in the analysis of the visual versus numerical methods used by teachers. Specifically, by condition:

- *Locally facilitated:* Visual methods from 3% of the total methods posted in module 1 to 89% in module 4; numerical methods from 70% of the total methods posted in module 1 to 11% in module 4

FIGURE 7. *VIM Community Mathematics Task Wall*



- *VIM project facilitated:* Visual methods from 6% of the total methods posted in module 1 to 94% in module 4; numerical methods from 82% of the total methods posted in module 1 to 6% in module 4
- *Self-paced:* visual methods from 18% of the total methods posted in module 1 to 85% in module 4; numerical methods from 82% of the total methods posted in module 1 to 6% in module 4

Table 8: Participants' Posts, Comments, and Methods in Module 1 and 4

Condition	Module 1				Module 4			
	Posts & Comments		Methods		Posts & Comments		Methods	
	Posts	Comments	Visual	Numerical	Posts	Comments	Visual	Numerical
Locally Facilitated *Mod 1: 25* *Mod 4: 21*	20	18	6	14	20	16	17	2
VIM Project-Facilitated *Mod 1: 17* *Mod 4: 16*	17	Part: 12 Fac: 14	1	16	16	Part: 16 Fac: 7	14	2
Self-paced *Mod 1: 25* *Mod 4: 20*	22	26	4	18	20	14	17	1

The large majority of methods across all conditions in the VIM 1 community mathematics task wall responses were numerical, while the large majority of methods across all conditions in VIM 4 were visual. We hypothesize that this result could be related to a number of things:

- The VIM learning goals highlighted multiple methods with an emphasis on visual methods.
- The solution methods resource for each VIM highlights visual methods and the links between numeric and visual representations.
- The participants had repeated exposure to the various student visual methods in the four VIM module video clips.
- The participants had repeated exposure to each other's methods with each of the four VIMs.

In addition, these results map onto the **teacher log** results showing an increase of 14% from week 1 to week 6: "I am able to understand student solution methods that are different than my own". These results also correspond to the **interview data**, in which teachers indicated:

- A greater emphasis on multiple representations, including connecting representations through color and probing questions
- Increased openness to multiple ways of seeing and describing linear growth and mathematical structure

Analysis of Community Wall Comments

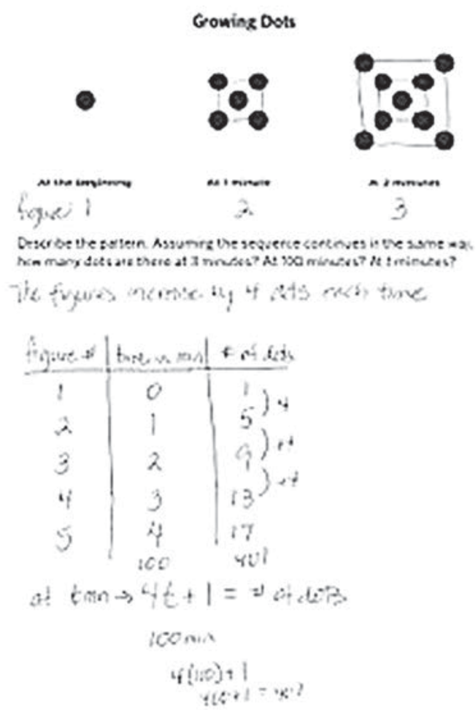
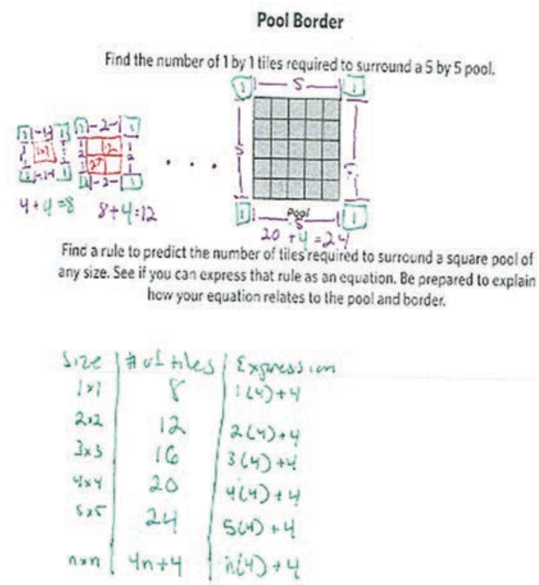
In addition to analyzing the comments quantitatively, we examined the comments qualitatively. In general, comments in VIM 1 were focused on recognizing, agreeing with, and appreciating the tabular approaches. A couple of comments were focused on providing advice/teaching tips. Comments in VIM 4 included more appreciation for a variety of approaches, recognizing the value of using color, and connecting to/learning from other participants' work. Table 9 shows differences in each condition from Module 1 to Module 4.

Discussion

The preliminary results on teacher impact show some consistent findings across different data sources—weekly logs, post PD interviews and pre-post community mathematics task walls. Teachers appeared to have learned to appreciate and use visual methods for solving problems, including using color to distinguish and highlight the relationship between numeric, algebraic, and geometric models. In addition, teachers engaged with and interacted with each other by examining, commenting on, and questioning each other's mathematical work.

A surprising preliminary result was the fact that there were no substantial differences across the three conditions regarding teacher engagement and interaction on the community mathematics task wall. We hypothesized that the

Table 9: Differences in Community Wall Comments from Module 1 to Module 4 by Condition

Group	Module 1	Module 4
Locally Facilitated	• Mostly appreciated table and chart solution methods. • The work that showed an error generated the most comments (4). • One comment was different in nature: *"I like how you connected your dots. It makes it easier to see the pattern of adding four."* 	• Identified with/appreciated others' approaches. • Appreciated others' explanations. • Appreciated visuals and use of color: "Your rule matches the pattern I found in one of my tables. Things like that make me go "Ah hah!". 

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facilitated group would be more engaged and post more comments in response to their colleagues' methods. This did not turn out to be the case, as teachers across all three conditions commented in similar numbers and shifted from numeric to visual methods from pre to post.

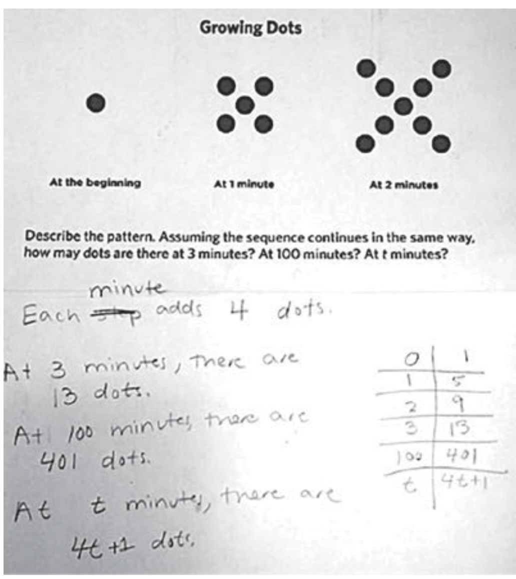
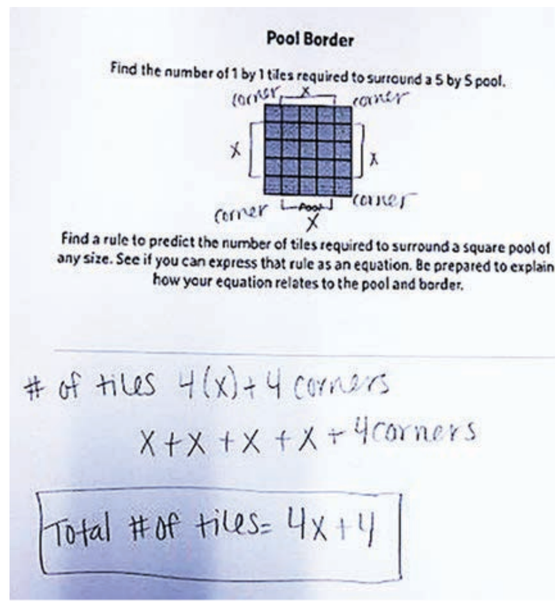
Implications for Mathematics Education Leaders

Mathematics leaders are often placed in the position of creating their own PD materials more or less from scratch or pulled together from many different sources. Because of time and resource limitations, this often results in all teachers receiving "one-size-fits-all" PD experiences that are not necessarily responsive to their needs and interests.

The VIM project aims to support mathematics education leaders by disseminating the VIM modules and resources as open education resources to mathematics leaders in a variety of flexible formats and bundlings, beginning Spring/Summer 2021. We plan to advertise the release of these modules on WestEd's website (www.wested.org) as well as through mathematics education and mathematics leadership professional organizations.

Using the VIM modules, leaders can provide the teachers they support with high-quality PD experiences that can be completed asynchronously, allowing teachers to schedule their PD work around their other responsibilities. As we saw in our research study, this flexibility may be particularly

Table 9: Differences in Community Wall Comments from Module 1 to Module 4 by Condition (continued)

Group	Module 1	Module 4
VIM Project-Facilitated	• Facilitators made 14 comments, Participants made 12 comments • A couple of teachers focused on learning from looking at someone's example: "I like seeing others do it so I can relate my answer to theirs." "Your demonstration was very neat and clear. I had to go back and fix mine after observing yours." • There were a few "seeking to understand" types of comments: "It's unclear what n or t mean since it's not stated on the paper, but the solution is valid." • There was one comment that talked about having learned from looking at someone else's example: "Your response was the first one I saw. When I saw your (0,1) ordered pair, I realized I did it wrong…" 	• Several focused on the value of the visuals—marking to show the growth: "I like how you separate the sides and the corners." "I like the x's to help students see why there is a plus 4." • Appreciating different approaches: "I love it when someone does it different than me. Such a good learning experience." • Connecting to and learning from others: "I notice we had the same equations; however, we interpreted the x and corners are different in our drawings." 

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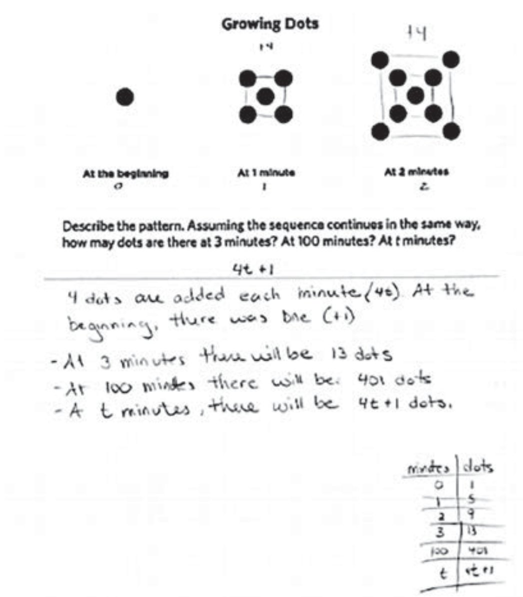
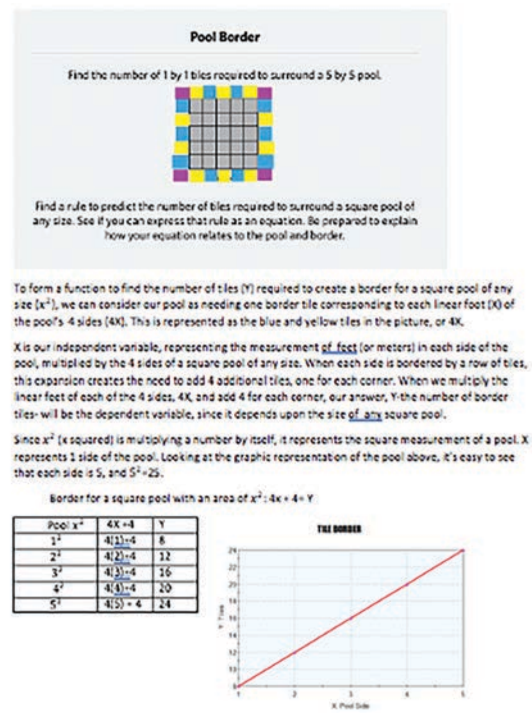
welcome as staff continue to work from home and juggle many competing priorities. In addition, sequence recommendations ("pathways") and sample facilitation guides will be shared in an attempt to support math leaders in meeting teachers' needs. Teachers have different professional development needs and interests due to a variety of factors (Chval et al., 2008; Desimone, 2009; Bautista & Ortega-Ruíz, 2015; Matteson et al., 2013); because the modules address a range of mathematical and pedagogical topics, leaders will be able to select modules that align with district or department priorities, or let teachers choose the modules or bundles that most interest them. The option to deliver the modules in either a self-paced or facilitated format will provide leaders with additional flexibility; depending on district goals, resources, and teacher needs and preferences, they may choose to offer a

Table 9: Differences in Community Wall Comments from Module 1 to Module 4 by Condition (continued)

Group	Module 1	Module 4
Self-Paced	• Most of the comments (9) focused on appreciating tables: "I like your table—easy to follow, showing a pattern." • Others provided advice/teaching tips: "How would you connect your "+4" pattern to your "4t" for your students?" • A couple of teachers commented on how someone saw it visually: "It's interesting how you saw the +4 as progressively larger squares." 	• Many teachers appreciated visuals and use of color: "The color coding on the corners helps." "I like the visual you provided and the color-coded keys." • One piece of work generated comments reflecting two different perspectives: "I like the way you used colors to identify the parts of your rule. Visually clarifying." "I find the different colors distracting. I can see the four corners being a different color." 

facilitated experience where teachers engage in the same modules or pathways, or to implement a non-facilitated option, either where a group completes the same modules or pathways or individual teachers select which modules to work on.

Finally, the asynchronous, online nature of the VIM modules makes them highly scalable; unlike many face-to-face and synchronous online PD options, math leaders will not need to limit participation due to space or cost concerns, a welcome feature as many LEAs must now balance shrinking budgets. At the same time, community walls still allow for interaction and collaboration among teachers working on the same modules or pathways. PD looks differently during the pandemic and the flexibility of the VIM modules may be a good fit for PD leaders at this moment in time. ✪

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