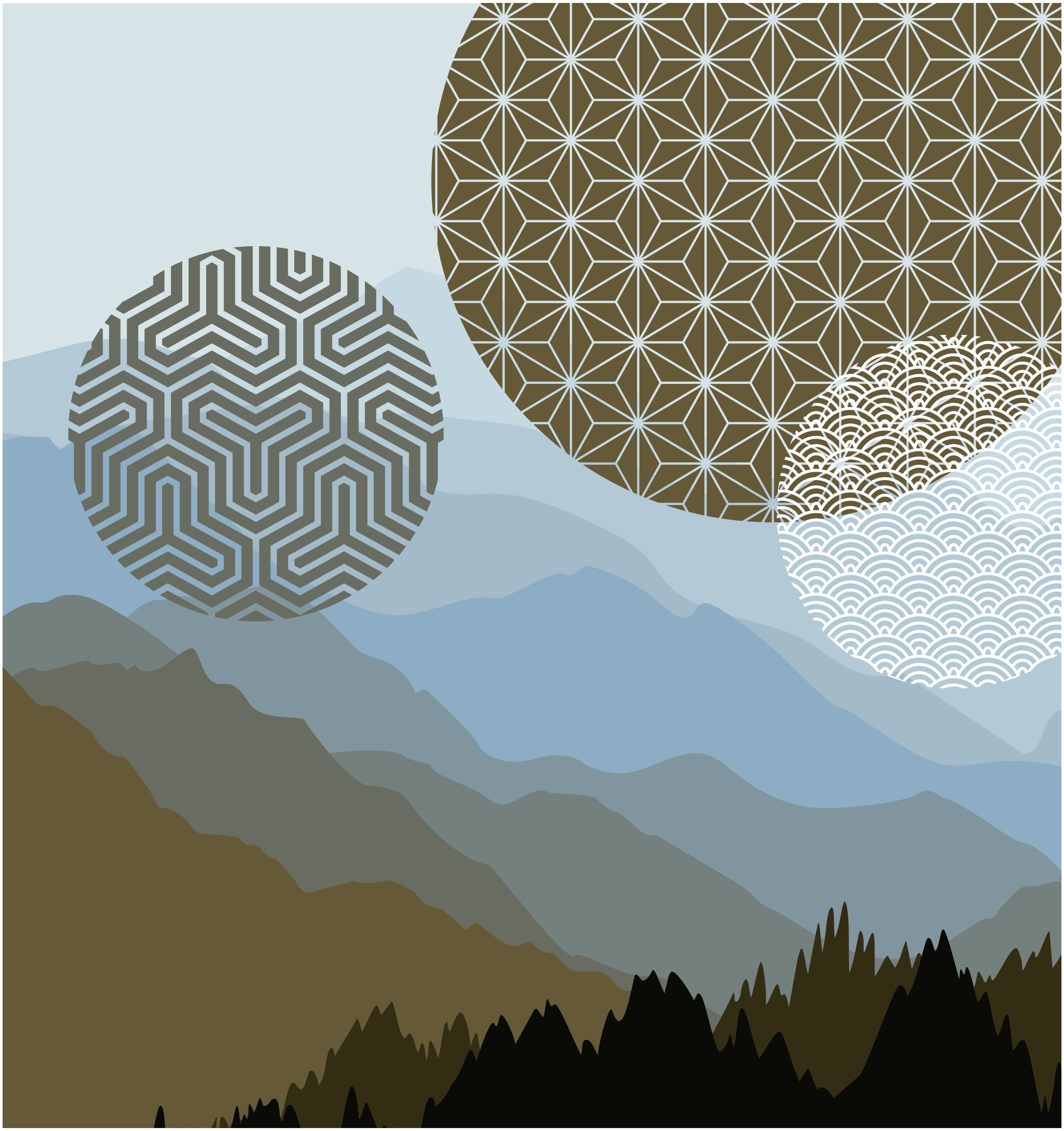
NCSM Journal
of Mathematics Education Leadership

FALL/WINTER 2021-2022
VOL. 22, NO. 2



NCSM–Leadership in Mathematics Education
www.mathedleadership.org

Table of Contents

Inclusion and Intervention: Understanding "Disability" in the Mathematics Classroom

Lara Jasien and John Hayes
CPM Educational Program

Abstract

All students' learning—including students with learning and intellectual disabilities—is deepened when students with multiple ability levels engage in teamwork on high cognitive demand tasks. Yet, we know little about supporting teachers in inclusive mathematics classrooms. This knowledge void presents challenges for mathematics education leaders who wish to foster inclusion. Synthesizing a small but growing body of mathematics education research, this manuscript is a resource for leaders supporting teachers in inclusive standards-based classrooms. In particular, this manuscript articulates (1) why productive struggle is essential for students with disabilities, (2) progressive definitions of disability and inclusion, and (3) conceptual descriptions of pedagogy in inclusive mathematics classrooms. It is followed by an appendix filled with tangible strategies that mathematics education leaders can adopt and adapt in their own contexts.

Introduction

Mathematics education leaders work tirelessly to help teachers learn how to best support their students. Yet, little is known about how to support students with learning and intellectual disabilities to be successful in inclusive general education classrooms that feature productive struggle on high cognitive-demand tasks. This manuscript synthesizes what is known about supporting students with learning and intellectual disabilities to meaningfully participate and learn in classrooms based on the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School, 2010). Most research on this topic is qualitative and conducted from a sociocultural perspective. Qualitative research is not designed to make claims about causality. Thus, this paper does not make claims about "what works" at scale. Instead, it informs how and why particular methods support students' learning. This manuscript is not an exhaustive review of existing research. It draws on literature purposefully to answer the following question:

> *How can mathematics educators support students with learning and intellectual disabilities to experience productive struggle during collaborative problem-solving on cognitively-demanding tasks in inclusive classrooms?*

Thus, this manuscript will help mathematics education leaders better support teachers to practice equitable ambitious instruction for all students in inclusive classroom settings.

The first section describes why productive struggle is essential for doing mathematics, including for students diagnosed with disabilities. The second section draws on multiple research paradigms to discuss definitions of disability and the differential impact of these definitions on students. Mathematics education leaders can use these sections to help expand views on who inquiry learning is appropriate for and what it means to teach students with disabilities. After providing this conceptual underpinning, the third section explores what research says about supporting students with disabilities to learn mathematics through instruction involving authentic problem-solving. Mathematics education leaders can use this section's synthesis of research to help teachers expand their teaching practice to create inclusive learning environments. This manuscript's appendix contains six tables summarizing the evidence-informed strategies from the third section so that mathematics education leaders can try them out with teachers in their unique contexts. Overall, this manuscript provides conceptual and practical resources for reframing disability and providing quality instruction that gives all students access to collaboration with their peers, productive struggle, and cognitively demanding tasks.

A Moral Imperative: Fostering Productive Struggle for All Students

Much research in mathematics education is grounded in the idea that learning mathematics requires problem-solving. Mathematical problem-solving is not simply completing a task, it is doing something non-routine, perplexing, and difficult for the doer (Schoenfeld, 1988). In other words, mathematical problems are only problems insomuch as they create a sense of struggle for the problem solver. Struggle on mathematical tasks happens when students put effort into making sense of mathematics. This struggle is productive because it produces deep learning. Warshauer (2014) describes tasks' affordances for fostering productive struggle as related to cognitive demand, with the following types of tasks moving from having the least cognitive demand to the most cognitive demand: tasks that require primarily memorization, tasks that require using procedures without requiring conceptual understanding, tasks that require using procedures with conceptual understanding, and tasks that require doing math, engaging in high-level tasks that are worthwhile and not-straightforward. Thus, tasks requiring (a) conceptual understanding and (b) doing math most reliably foster productive struggle.

Mathematical tasks should be designed to foster productive struggle for all students, including students with learning disabilities and most intellectual disabilities. For example, the tasks in CPM Educational Program's materials support problem-based learning; they have a low floor and high ceiling. Low floor tasks allow students who have not mastered all relevant prior mathematical knowledge to get started on tasks. High ceiling tasks have task extensions that challenge students with rich relevant prior knowledge.

During work on such tasks, equitable mathematics learning is made possible through collaboration in heterogeneous student teams (Cohen & Lotan, 2014). When students of multiple ability levels work together on tasks whose solution strategy is not readily apparent, learning is deepened for typically achieving students, gifted students, and students who have been diagnosed with learning and intellectual disabilities, to name a few. Denying any student the inherent struggle of mathematical problem-solving simultaneously denies them much more: It denies them meaningful mathematics learning, opportunities to develop critical thinking, and the joy of aha moments. In the words of educational researchers Akyuz and Stephan (2020):

> Critical thinking, problem-solving, and modeling is necessary for twenty-first-century employment… and to withhold inquiry mathematics instruction to students with disabilities is immoral. … [And while] it is clear that direct instruction increases achievement [for students with learning disabilities] … there is no evidence that this is the only type of effective instruction for students with disabilities (pp. 2-3).

It is the moral responsibility of mathematics education leaders to establish inclusive teaching environments and support teachers with mathematics-specific teaching practices that make inquiry instruction through standards-based curricula available and accessible to students with disabilities.

Appropriateness of standards-based curricula

Students with identified exceptionalities can learn through standards-based mathematics curricula (Lambert & Sugita, 2016). In other words, students with identified exceptionalities can meaningfully learn from curricula in which:

1. mathematics is encountered through problem-solving,
2. mathematics is embedded in contexts such that mathematical strategies and topics are connected to real-world applications,
3. mathematics emerges through collaborative teamwork and with mathematical tools (e.g., algebra tiles, calculators), and
4. mathematics begins with student-invented strategies rather than standard algorithms (Jitendra, 2013).

Because standards-based mathematics curricula can be made accessible to students with identified exceptionalities, they also should be made accessible to these students. While some argue that working on grade-level content in standards-based curricula steals time from learning skills useful for life after school, Courtade et al. (2012) provide seven substantive reasons for why standards-based curricula are appropriate for students with intellectual disabilities:

1. Standards-based curricula are part of all students' right to a full educational opportunity.
2. Standards-based curricula are relevant for students with disabilities.
3. Students with disabilities seem to learn standards-based content and use it in their lives in unique ways.
4. Functional skills are not a prerequisite to academic skills.
5. Standards-based curricula are not a replacement for functional curriculum.
6. Individualized curricula are limited when they are the only curricula.
7. Students with disabilities show that they want to learn with their peers and succeed.

Beyond these practical justifications for inclusion, the *Individuals With Disabilities Education Improvement Act* (IDEA regulations, 2016) states that students with identified exceptionalities should be "To the maximum extent appropriate … educated with children who are nondisabled" except if "education in regular classes with the use of supplementary aids and services cannot be achieved satisfactorily" (IDEA Sec. 300.114). In other words, all students have the right to inclusive education in the "least restrictive environment" with their peers. Unfortunately, even though IDEA has essentially existed by other names since 1975, almost 50% of students receiving special education services still spend over 60% of their day in segregated (or "dedicated") special education interventions (Wehmeyer et al., 2021). More recently, the Supreme Court ruling Endrew F. v. Douglas County School District created the precedent that the word "appropriate" in IDEA means appropriately ambitious, in that students with identified exceptionalities have the right to meet challenging objectives and fulfill their potential for growth (Wehmeyer, 2019; Wehmeyer et al., 2021). Unfortunately, students are given increasingly fewer opportunities for participation in general education classrooms as they progress through the system (Cook & Cook, 2020).

The overrepresentation of Black and Brown bodies in special education

The problem of funneling students with identified exceptionalities out of general education classrooms is exacerbated for Black and Brown students. These students are disproportionately excluded from inclusive settings (Cook & Cook, 2020, p. 137, citing Skiba et al., 2006; Sullivan, 2011). The achievement gap, or, more appropriately, the opportunity gap and overrepresentation in special education is due to systemic racism through structures such as educational resource allocation, inappropriate curriculum and pedagogy, and inadequate teacher preparation (Annamma et al., 2013; Blanchett, 2006). In other words, critical scholars argue that the overrepresentation of Black and Brown bodies labeled as disabled is the result of (a) carceral pedagogies that emphasize compliance and memorization over critical thinking and problem-solving and (b) lack of access to standards-based curriculum (Annamma, 2017, 2018). Studies that counter these claims self-admittingly do not account for critically important moderators such as:

1. geographic location (e.g., that U.S. states with histories of slavery, de jure or de facto housing segregation, and discrimination are more likely to overidentify otherwise similar minority children than states without such histories),
2. disability type (e.g., that minorities are overidentified for more stigmatizing disability conditions, including intellectual disabilities and behavioral disorders), and
3. how the disproportionality may fluctuate in regard to context and setting (e.g., that minority children with disabilities are more likely to be placed in segregated or restrictive settings than otherwise similar White children with disabilities) (Morgan et al., 2017; Morgan et al., 2018, p. 12).

In general, educators need increased awareness that white children receive better educational healthcare when educators are more responsive to white parents' concerns. They also need increased awareness that minoritized students' parents may be more hesitant to seek out or accept a disability diagnosis due to the historical and experienced marginalization and criminalization of Black and Brown bodies in school buildings (Gregory et al., 2010; Guerrero et al., 2011; Morgan et al., 2013).

Mitigating the consequences of these potential tendencies will require that mathematics education leaders provide teachers with meaningful professional learning opportunities to examine implicit bias and critique institutional procedures that may need revising. The following section describes disability and related concepts in ways meant to help mathematics education leaders and the teachers they support to examine their implicit biases towards students diagnosed as disabled. Examining educators' implicit biases is essential in shifting students' educational experiences.

What is disability?

What is disability? The answer to this question is ideological and thus contested, with implications for appropriate designs for learning. There are three primary answers to this question (Thurber et al., 2018):

1. ***Medical model:*** A disability is a deficiency or abnormality.
2. ***Social model:*** A disability is a difference.
3. ***Cultural model:*** A disability is a valuable form of human diversity.

These different perspectives on disability are rooted in distinct models of disability. These latter two models of disability are closely related to each other and build on the work of disabled disability activists (Union of Physically Impaired Against Segregation & The Disability Alliance, 1975). These models argue that associating negative labels with neurological differences is problematic. The cultural model of disability goes further than the social model: it argues that neurological differences are valuable contributions to human diversity (i.e., neurodiversity; Armstrong, 2012) rather than a neutral difference. Thus, the cultural model flips the medical model's difference-as-deficiency framing to difference-as-exceptionality.

The majority of research on learning disabilities has been conducted under a behaviorist learning theory, which aligns with the medical model of disability (Lambert & Tan, 2016). Like the medical model of illness, this perspective of disability locates the cause of disabilities within individuals and aims to diagnose (i.e., identify error patterns in mathematical problem-solving), diminish, and correct perceived deficits through remediation (Lambert & Tan, 2016; Thurber et al., 2018).

Unsurprisingly, this has resulted in a plethora of research investigating the effectiveness of interventions that break down tasks into small chunks in hopes of remediating through simplifying. This approach can effectively support students to perform better on discrete tasks. However, it largely fails to support conceptual understanding because it tracks students away from high cognitive demand tasks (Bannister, 2016; Tan et al., 2019; Woodward & Montague, 2002). This segregation is unfortunate, as students with neurological differences can learn conceptually complex mathematics well enough to major in mathematics, even when those neurological differences cause them to struggle with early mathematics such as number sense (e.g., dyscalculic tendencies with compensatory aspects; Lewis & Lynn, 2018).

Following the cultural model, this paper locates the source of disability within social institutions and processes, including the physical and social environment, rather than within individuals. Ability and disability are not inherent nor static; they are socially constructed in particular teaching contexts (Lambert, 2015). Thus, instead of remediating children, mathematics education leaders must remediate learning environments to increase accessibility.

Reframing neurological differences as valuable diversity

Here, learning and intellectual disabilities are defined as neurological differences resulting in different needs than typical neurological development. Neurodiversity encapsulates a wide range of neurological differences, including dyscalculia, ADHD, and autism, to name a few. These diagnosed differences all occur on a spectrum. Acknowledging that there is a great deal of diversity within disability, speaking broadly, in this paper the term exceptionality encompasses learning and intellectual disabilities. Some papers cited herein focus on specific disabilities, yet, the instructional practices described are beneficial for all students, those with and without disabilities. When studies focus on specific diagnoses, those terms (e.g., autism, dyslexia) are used.

In regards to labels, individuals have preferences about person-first (i.e., a person with a disability) or identity-first (i.e., a disabled person) language, and these preferences should be honored in personal interactions. The remainder of this paper uses the term "students with identified exceptionalities" for its qualities of putting the person rather than the difference first ("students … with"), for qualifying that identity-labels are socially constructed and not inherently important ("identified …"), and for pointing to difference rather than deficit ("exceptionalities"). This new label does not romanticize the real challenges faced by students with identified exceptionalities. It reframes and relocates the source of these challenges. Of note, some disabled disability activists and others dislike terms other than the medical diagnosis terms such as learning disability and intellectual disability; they find terms like "students with identified exceptionalities" to be infantilizing. To challenge historical and commonplace views of disability which can constrain students' access to meaningful learning opportunities, we use this phrase without intent to infantilize and hope that this explicit acknowledgment makes our intent match impact.

In this paper, the phrase "students with identified exceptionalities" encapsulates diagnoses of learning disabilities (e.g., dyslexia), mathematical disabilities (specifically mathematics learning disabilities such as dyscalculia), and mathematical difficulties, and intellectual disabilities (e.g., autism). Non-sensory physical disabilities are not addressed in this paper because we take it as common sense that students with physical disabilities should receive adequate supports to engage in productive struggle on cognitively demanding tasks, including work with materials required to engage in such tasks.

Sensory disabilities, such as visual or hearing impairments, are included in this paper. Equitable classrooms are not classrooms where students learn simply through reading a textbook (inequitable for those with visual impairments) or through listening to lectures (inequitable for those with hearing impairments) but are places of multifaceted pedagogy and opportunities to learn. It follows that simply including students with identified exceptionalities in the classroom is not sufficient. Inclusion is not only about keeping student bodies in the classroom; it is not about place. Inclusion requires the general education classroom to be a place where all students experience effective instruction. This is a matter of equity in education; it requires transformational instruction that creates equitable learning opportunities for learners with a wide variety of needs.

Rehumanizing mathematics education for students with identified exceptionalities

Students with identified exceptionalities have experiences of being disabled due to the design of society to favor the needs of the average or neurotypical person. Although the education system is designed to highlight these children's and youths' differences, they are children and youth first. It is the job of mathematics education leaders to rehumanize the educational experience of students with identified exceptionalities. Many of these students have similar learning preferences as their neurotypical peers. For example, Klingner and Vaughn (1999) found that students labeled with learning disabilities tend to prefer the same activities, homework, books, grading, and grouping as their peers without similar labels (Rexroat-Frazier & Chamberlin, 2019). They also found that these same students valued clear explanations, experiencing content in multiple ways, and responsive lesson pacing. Arguably, these are features of teaching and learning that all students might value, with or without identified exceptionalities. Listening to the voices of students with identified exceptionalities is a start to rehumanizing their experiences in mathematics classrooms.

To have conversations where students with identified exceptionalities can reflect on and share their experiences of meaningful learning, we must lift the veil of secrecy in diagnosis. Too often students often experience secrecy around their diagnosis (Lambert et al., 2019; Rexroat-Frazier & Chamberlin, 2019; Vaughn & Klingner, 1998). This secrecy is detrimental because it creates fixed, shame-ridden mindsets and obfuscates students' ability to advocate for themselves around their specific learning needs and goals. In a study of individuals' self-perceptions of the nature of their learning disability diagnoses, Lambert et al. (2019) found many who echoed the sentiment poignantly shared by Lynn Pelkey in her essay in *Learning Disabilities and Life Stories*:

> I do not know when I was labeled as learning disabled. It was not until junior high and maybe into high school that the term LD started to surface with frequency. For years, my fellow LDers and I wondered what LD meant. No one ever told us. We did know that it set us apart from others and that we were different. Being LD was not something that we received awards for. It was secretive and suspicious. It was something talked about in hushed tones. It was discussed at secret parent/teacher meetings. It was the reason that I had to go to summer school. Is it any surprise then, before I knew what LD meant, I felt ashamed about being LD? (Rodis et al., 2001 p.19, as cited in Lambert et al., 2019, p. 7).

If secrecy fosters such shame, perhaps it is time for mathematics education leaders to create policies and practices that open communication with students about their diagnoses. Such communication empowers students and gives them opportunities to advocate for themselves.

Of course, these conversations should include not only students' deficits, but also a meaningful conversation about what disability, or exceptionality, is, as well as clear highlighting of individuals' strengths. In the Lambert et al. (2019) study, participants described what they called gifts of their exceptionalities, namely, creativity and conceptual thinking, multimodal thinking, persistence, and motivation. Indeed, individuals with identified exceptionalities lament the endemic educational emphasis on their deficits, arguing that attempts to remediate them to the average learner happened at the cost of fostering their strengths (e.g., Lewis & Lynn, 2018; Robinson, 2016; Roy, 2015). Education can foster these strengths. The same Lynn who described feelings of shame about her LDness described her experiences with conceptual learning in mathematics general education as "magical":

> As I sat in that class, something magical happened to me. I could understand what he was teaching. I was learning. I even started participating in the class, raising my hand, and answering questions. I was LD. But then again I wasn't. I still couldn't multiply or divide very well, and I had to use elaborate ways to come up with the answer. But I wasn't memorizing, I was thinking, and I was figuring out the answer. I was learning. This was one of the experiences that shot a pinhole in the bubble that trapped me in my LDness. (Rodis et al., 2001 p. 21, as cited in Lambert et al., 2019, pp. 14-15).

Thus, conceptual learning can be empowering for students with identified exceptionalities. With her statement "I wasn't memorizing, I was thinking … [and that] shot a pinhole in the bubble that trapped me in my LDness," Lynn articulated that opportunities to engage in conceptual learning opened up possibilities for her and helped her see herself as capable despite the challenges her neurological differences led her to encounter with memorization.

Stories such as Lynn's are not typically elevated in special education research although they are in the budding area of mathematics education research on disability. The lack of overlap between special education research and mathematics education research (Garderen et al., 2009; Lambert & Tan, 2016, 2019) reflects these fields' distinct differences in terms of subscribing to medical, social, or cultural models of disability. Special education's leading theory of learning aligns with the medical model of disability. This leading theory is behaviorism, a theory that generally defines learning as a change in behavior. Behaviorism leads scholars to design for learning by focusing on individual instruction with instructional sequences that move from simple to more complex actions, with many opportunities for practice. In contrast, mathematics education's theories of learning align with the social and cultural models of disability. These theories are sociocultural (goals of enculturation) and sociopolitical (goals of emancipation, Gutierrez, 2013). These theories generally define learning as a change in participation, which is different from a change in behavior because participation includes a different orientation towards activity as a social and cultural endeavor. Sociocultural and sociopolitical studies take the stance that all students are sensible, competent mathematical doers and thinkers and thus offer an alternative narrative from the hegemony of behaviorist, quantitative, special education studies that make up the bulk of research on exceptionalities (Connor et al., 2011).

Research paradigms and their implications for educational design

Mathematics education leaders need to be aware of important differences between special education and mathematics education research so that they can use discernment when making research-based decisions. For example, special education research tends towards domain neutral interventions such as using mnemonic techniques while mathematics education research attends to the structure of mathematics as fundamental to intervention designs (Garderen et al., 2009), thus making instructional practices rooted in mathematics education research more likely to support students' learning of mathematics. Special education and mathematics education research also differ in their research settings, with the former occurring in one-to-one, team, or specialized settings and the latter occurring in inclusive whole-class settings (Garderen et al., 2009). If mathematics education leaders want to foster inclusive mathematics classrooms, then drawing on research that occurs in such classrooms has tremendous value. In addition, most research on MLDs in special education, about 75% of it, is conducted in elementary settings, perhaps due to flawed perceptions of mathematics as a hierarchical subject in which basic skills must be mastered before conceptual learning is possible (Garderen et al., 2009; Lewis & Fisher, 2017).

If mathematics education leaders want to create environments where teachers can support students with identified exceptionalities in inclusive classrooms then they will need the underused tool of qualitative research to understand how these students do learn. While the behaviorist quantitative research of special education helps identify the persistent errors students make, it falls short of explaining why students make particular errors and why errors persist despite instruction (Lewis, 2016, p. 100). Unfortunately, and as an indictment to mathematics education research, very little of such research on exceptionalities has yet been published in mathematics education journals (Lambert & Tan, 2019). Still, there is a small but growing body of qualitative research that is beginning to fill this gap.

When examining student thinking, qualitative research conducted from sociocultural and sociopolitical theories focuses on the ways that students do understand mathematical concepts and representations instead of only the ways that they do not understand them (i.e., their error patterns). For example, Lewis (2016) identified the persistent understandings of two girls with identified exceptionalities during fraction comparison tasks (e.g., Which is bigger, 2/8 or 5/8?). Lewis found that the errors made by the two girls were the result of three persistent understandings in which the girls (a) used fraction complements[1] instead of the fraction itself, (b) had a single factor understanding[2] of fractions, and (c) understood 1/2 as an action of halving rather than as a quantity. By analyzing what sense the students were making rather than only what errors they made, Lewis' analysis not only portrays the students as sensible doers and thinkers of mathematics but also provides a foundation to build on to move students beyond their current understanding of fractions.

Of course, this has implications for assessment. There is a pressing need for research-based assessments that capture the conceptual understandings of students with identified exceptionalities in content areas such as algebra and geometry (Garderen et al., 2009). Existing research focuses on how to support and assess learning of basic facts and procedural skills, an outcome of the lack of mathematics education research on the learning of students with identified exceptionalities. Assessments designed to capture conceptual understanding may reveal mathematical competence even when students have trouble with symbolic and non-symbolic processing. For example, as noted earlier, Lewis and Lynn (2018) documented how a student successfully majored in statistics despite mathematics learning disabilities with number sense and automaticity (dyscalculia). Qualitative research has shown that standards-based mathematics curricula can be made accessible to students with identified exceptionalities (Lambert & Sugita, 2016), and so the development of research-based assessments for middle and high school mathematics concepts is urgently needed. Without new forms of assessment, efforts to rehumanize the mathematics education experiences of students with identified exceptionalities will be constrained. Mathematics education leaders can begin this work and seek relationships with scholars who can support them and their teams.

The findings of qualitative research are an important balance to quantitative studies of efficacy. In a review of the 50 highest impact research reports on inclusive classrooms, Cook and Cook (2020) found that quantitative research on the efficacy of inclusive education was highly inconclusive, with both positive and negative effect sizes. For this reason, they suggest that educators engage in evidence-informed practice by taking both research and practical matters (such as families' values, etc.) into account when making decisions for and with students with identified exceptionalities. The next section overviews qualitative research on students with identified exceptionalities so that mathematics education leaders can make such evidence-informed practice possible for the teachers and staff they support.

How can we support all students in inclusive classrooms?

To understand how to create inclusive environments where students with identified exceptionalities can thrive in their mathematics learning, a definition is needed of what it means to do mathematics. Going through an a priori set of steps to complete a mathematical task is not doing mathematics. Doing mathematics requires struggle. Just ask a mathematician. For this reason, all students need to engage in struggle in mathematics classrooms. According to Warshauer (2014), common sources of struggle in mathematics learning for typically achieving students include:

1. getting started, for example, because of confusion about what the task is asking, forgetting the type of a problem, resigning due to uncertainty, or not putting any work onto paper;

---

[1] A fraction complement is the unshaded portion of a fraction. So, for example, the girls compared three pieces for 5/8 to six pieces for 2/8.

[2] A single factor understanding of fractions refers to focusing either on the size of pieces (e.g., fifths are smaller than halves, so 3/5 is smaller than 1/2) or on the number of pieces in the whole (e.g., 5 is more than 2, so fifths are larger than halves).

2. carrying out a process, for example, due to being unable to implement a process from a representation or due to its algebraic nature, or being unable to remember a fact or formula;
3. experiencing uncertainty in explaining and sense-making, for example, because of uncertainty in the reasons for their strategy choices or being unable to make sense of their work; and
4. expressing misconception and errors related to content (p. 385).

These struggles are also experienced by students with identified exceptionalities, and they become unproductive if students struggle without making progress towards task goals or give up.

Thus, finding instructional strategies to support all students to make progress, without lowering the cognitive demand of tasks, is essential in standards-based classrooms, especially inclusive ones. Students with identified exceptionalities are systematically denied a sense of intellectual authority, or "the belief that one has the responsibility for making sense of problematic situations rather than relying on someone else" (Akyuz & Stephan, 2020, citing Kamii, 1982). In her book *Culturally Responsive Teaching and the Brain*, Dr. Zaretta Hammond describes such authority as essential for independent learning, as contrasted to dependent learning in which students heavily rely on the teacher when they experience even small moments of uncertainty (Hammond, 2014).

Lynch and colleagues (2018) warn about authority-reducing instructional pitfalls that teachers commonly make when trying to support students with identified exceptionalities to engage in standards-based curricula.

Pitfall 1: Hinting. Hinting typically reduces the cognitive demand of the task and thereby removes the struggle and the learning. Hinting reduces the cognitive demand of tasks by narrowing students' focus. One way that teachers may hint is through funneling, where the teacher leads the student towards a correct answer by asking a series of questions that require short, fill-in-the-blank type answers from students (Wood, 1999). This prevents students from making connections and often redirects student thinking altogether. In this way, hinting leads students towards solutions built on the teacher's thinking rather than on the students' thinking, which in turn diminishes students' mathematical authority.

Pitfall 2: Backgrounding Problem Context. Backgrounding a problem's context removes sensemaking resources for students to draw on as they enter mathematical tasks. For example, consider a task that involves characterizing the rate of a redwood tree's growth (see Dietiker et al., 2015). Launching this task by focusing on how to represent growth or plot points on a coordinate plane, rather than focusing on students' informal expectations for how the rate of tree growth might be measured, backgrounds the problem's context, resulting in an overemphasis on procedures.

Backgrounding a problem's context also reduces the need to have a collaborative discussion. For example, pre-teaching mathematical concepts or skills relevant to a lesson can short circuit students' opportunities to learn from their peers. Pre-teaching requires making assumptions about which supports students will need, but allowing students to explore the problem context allows for more responsive scaffolds for student learning through just-in-time instruction. Backgrounding problem context and pre-teaching strip problems of meaning and learners of engagement and curiosity.

Pitfall 3: Providing Formulas. Providing formulas removes students' opportunity to engage in authentic mathematics. Because these pitfalls remove productive struggle, they are (unfortunately) some of the core strategies in special education's behaviorist interventions. In other words, because special education's theory of learning focuses on how individuals can come to successfully perform (not understand) a task and veils cognition, the roles of social interaction, and culture in learning, special education's designs for learning typically reduce tasks to a series of steps. Each of the pitfalls described above is key in transforming a cognitively demanding task into a series of steps. Instead of helping students with identified exceptionalities meaningfully engage in content, it steals their aha moments and their collaboration with their peers.

Adherents of the behaviorist theoretical perspective have two assumptions about standards-based curricula that lead them to believe that hinting, backgrounding problem context, and providing formulas are supportive of learning (Lambert & Sugita, 2016). The first assumption is that students with identified exceptionalities need expert help to construct problem-solving strategies. However, research has shown that students with identified exceptionalities can construct effective strategies without intensive scaffolding (see Lambert & Sugita, 2016, p. 352 for a list of such studies). The second assumption is that teachers in standards-based classrooms never make specific content or strategies explicit. However, a quick look through the renowned *5 Practices* book (Smith & Stein, 2018) indicates quite the opposite. Teaching a standards-based curriculum is complex and requires multiple modes of instruction. For example, mathematical discussions make mathematics explicit as students verbalize connections to prior content and collectively work to formalize mathematical concepts with canonical vocabulary. This is especially important during lesson launch and closure as students with identified exceptionalities can struggle to get started and put it all together.

Many argue that students with identified exceptionalities cannot engage in the productive struggle that is part and parcel of cognitively-demanding problem-solving tasks and teamwork. This ableism is so entrenched in education culture that it can seem like common sense. Yet, the common-sense appeal of such ableism is flawed and contributes to a lack of opportunities for students with identified exceptionalities to engage in the productive struggle that supports meaningful mathematics learning.

In the remainder of this section, describe how to increase opportunities for students with identified exceptionalities to engage in the productive struggle that supports meaningful mathematics learning. First, we identify instructional strategies that mathematics education research indicates may benefit students with identified exceptionalities. We then dig more deeply into how to support students with identified exceptionalities to engage in the text-heavy mathematics problems (i.e., context-rich word problems) characteristic of standards-based curricula (for example, see the sample problems available at https://cpm.org/lessons and https://cpm.org/try-this). Next, we address the dilemma of social and academic status in inclusive classrooms before summarizing research on co-teaching and introducing the framework of Universal Design for Learning.

Instructional strategies for broadening access

This section describes what research tells us about how to include students with identified exceptionalities in standards-based curriculum and instruction rather than only in discrete skills-based tasks. Participation in teamwork, problem-solving, and whole-class mathematical discussions is necessary to promote equitable learning and to foster 21st Century Skills. Citing the scholarship of Jo Boaler and others, Lambert and Sugita (2016) claim that "when students are engaged in problem-solving and mathematical discussion rather than memorization, they become equally efficient in calculation and better prepared to transfer knowledge and problem-solve" (p. 348, citing Boaler 1997; Boaler & Staples, 2008; Silver & Stein, 1996). Of course, some students with identified exceptionalities may require scaffolds to participate in these ways.

Mathematics education research is beginning to identify such scaffolds. A study by Lambert et al. (2020) identified culture-building teacher moves that supported the engagement of a student with autism in standards-based mathematics (appendix Table 1). As the result of a literature review on qualitative studies set in standards-based curriculum contexts, Lambert and Sugita (2016) found several strategies that hold promise for supporting students with identified exceptionalities (specifically, learning disabilities) in problem-solving and mathematical discussions (appendix Table 1). Browder et al. (2012) found that the use of graphic organizers helped students diagnosed with moderate intellectual disabilities (verbal and non-verbal) to engage in grade-level, standards-based content, including word problems. In Browder et al.'s study using graphic organizers as a scaffold, 11–13-year-olds were able to engage in the following mathematics:

1. ***Algebra:*** Solve simple one-step equations that relate to stories about daily events.
2. ***Geometry:*** Identify and describe the intersection of figures in a plane. Draw line segments and a coordinate plane to demonstrate spatial sense for familiar contexts like grocery stores.
3. ***Measurement:*** Develop numbers sense for real numbers. Develop flexibility in solving mathematical problems by selecting strategies and using appropriate technology. Use the next dollar strategy to solve problems related to everyday transactions.
4. ***Data Analysis:*** Collect, organize and display data to solve problems from familiar events. (Browder et al., 2012, p. 378)

The strategies for supporting engagement in standards-based curricula identified in these studies (see the appendix) have been shown to support meaningful learning because they increase participation (Lambert & Sugita, 2016) and thus also lead to identities as mathematical thinkers and doers. For example, students who have never offered more than one-word responses during whole group instruction have been shown to shift participation by the end of the year to have equal rates of engagement to their nondisabled peers after ongoing participation in particular mathematics routines (Lambert & Sugita, 2016, p. 359, citing Foote & Lambert 2011).

Importantly for mathematics education leaders, research indicates that teachers struggle to learn how to provide some types of scaffolds more than others. Pfister et al. (2015) found that curricular materials were able to support teachers to engage in important scaffolds such as using manipulatives and helping students to focus on the important aspects of the lesson. However, more interactional micro-scaffolds such as stimulating discourse, cognitive activation (e.g., What do you notice? What did you have to do so that …?), and handling errors productively were much harder for teachers because they required in-the-moment decisions responsive to student needs. Examples of what these five scaffolds look like when they are done well (and not) can be found in the rubric in Table 2 (see the appendix). Mathematics education leaders must create ongoing, embedded professional learning opportunities for teachers to support them in learning to provide scaffolds for students with identified exceptionalities while still providing opportunities for productive struggle in tasks with high cognitive demand.

Strategies for starting word problems

One of the most challenging instructional demands on teachers in inclusive, standards-based classrooms is supporting students to interpret word problems sufficiently enough that they are able to access the mathematics. In other words, teachers in such contexts must find ways to support students with literacy such that they are able to get started on mathematical tasks. Unfortunately, there is extremely thin research on how to specifically support students with identified exceptionalities with literacy challenges encountered in word problems in ways that do not cut-off inquiry (Lambert & Tan, 2016). This paucity of research is partially a result of research taking different approaches to studying neurotypical and neurodiverse students' struggles with cognitively demanding tasks. Lambert and Tan (2016) found that research on neurotypical students tends to focus on problem-solving while research on students with identified exceptionalities tends to focus on word problems. Research that focuses on the former aligns with standards-based instruction characterized by inquiry while research that focuses on the latter aligns with schema-based instruction characterized by an explicit, teacher-mediated approach (Jitendra et al., 2013; Lambert & Tan, 2016). It is important to question why this distinct difference in methodologies for students with and without diagnosed disabilities exists and whether the exclusion of studying the problem-solving of students with identified exceptionalities has negative consequences for those students (Lambert & Tan, 2016, p. 1061).

This disparity is unsurprising because most research on students with identified exceptionalities occurs in special education and not in mathematics education, not because it is impossible to do. Research in special education supports students with word problems through schema-based instruction, which involves unpacking the problem's structure (e.g., the pieces of a linear equation) before the student explores the problem (Browder et al., 2018; Jitendra et al., 2015). This, in essence, removes the inquiry. Providing students with any kind of scaffold that tells them what to do with a task's mathematical problem inherently lowers the cognitive demand and short circuits the productive struggle that supports meaningful mathematics learning. Recent modified versions of schema-based instruction are more complex but similarly inhibit opportunities for conceptual learning, for example by providing students with graphic organizers that are specific to a problem-type, explicit instructions, "rules taught as chants with hand motions representing the underlying problem structures," and more (Browder et al, 2018, p. 223). Attending to the schema of a word problem is important, but to support inquiry, it should occur as discussion about different problem-types after students have engaged in problem-solving with different problem types. Allowing students to collaboratively create graphic organizers and other visual representations may support the learning of students with and without identified exceptionalities.

One exploratory study (Moscardini, 2010) indicates that a potentially productive way to mitigate literacy challenges is through scaffolding the beginning process of problem-solving in ways that do not kill inquiry. This can be done by restating word problems (while retaining the cognitive demand) and re-reading word problems in small chunks that students model step-by-step. According to Lambert and Sugita (2016), "this reduces students' difficulties with language but does not reduce the cognitive demand of the mathematics task" (p. 358).

In a manual designed for teachers, Cole et al. (2000) suggest strategies such as making oral recordings of the text so that students can listen to and rewind the written material instead of reading it, providing students with the word problem text or recording in advance so that they can become familiar with the context of the problem, and discussing the problem context in teams (Aiquraini & Gut, 2012).

Another strategy that supports all students, including those with identified exceptionalities and those learning English as a second language (emerging bilinguals), is fully unpacking a problem's context in the lesson launch without pre-teaching. For example, in a problem about compound interest, teachers could elicit students' prior knowledge about what compound interest means without discussing how to calculate it. This is a matter of ambitious, equitable instruction in all classrooms (Jackson & Cobb, 2010).

Strategies from this section on supporting students with literacy such that they are able to begin problem-solving with word problems are summarized in the appendix in Table 1.

Mitigating status issues in heterogeneous, inclusive classrooms

Because teamwork is a key component of standards-based curriculum and instruction, teachers must carefully attend to issues of status so that students with identified exceptionalities are not marginalized and stigmatized by their peers. For example, students diagnosed with learning disabilities are excluded from making mathematical decisions when they are delegated non-mathematical responsibilities in teamwork (e.g., material management; Baxter et al., 2001). Exclusion from meaningful work on the mathematical aspects of the task is likely to negatively stigmatize mathematical competence.

This exclusion can be mitigated by giving students explicit instruction on how to work together so that students with higher proficiency levels do not take over the mathematical thinking for students with identified exceptionalities (Bottge et al., 2002; Cohen & Lotan, 2014; Horn, 2012). For a list of such instructional strategies, see Table 1 in the appendix.

One important and ongoing role of mathematics education leaders will be to help teachers become aware of their own biases about the abilities of their students; indeed, mathematics education leaders may need to examine their own biases about the abilities of students with identified exceptionalities. Teachers' attitudes and values around ability and mathematical competence influence the way they teach and thus also influence their students' attitudes and values, which can lead to unequal learning outcomes and exclusionary identity development for students with identified exceptionalities (Thurber et al., 2018 citing Davis, 2010, p. 58). As an example of how teacher biases can impact students with identified exceptionalities, these students can be called on fewer times than students who have not been diagnosed with a disability (Bottge et al., 2002). This may be one of the greatest challenges for mathematics education leaders, to create environments where teachers feel confident in their students' potential for learning and in their own ability to teach in heterogeneous classrooms. Creating strong co-teaching environments in which teachers collaboratively examine and hold each other accountable for their assumptions about and treatment of students may be one way that mathematics education leaders can begin this work.

Philosophical alignment and interdependence in co-teaching relationships

While co-teaching is not possible in all contexts, many conjecture that co-teaching is a productive way to support equitable instruction for students with identified exceptionalities in inclusive classrooms. Unfortunately, we know little about what productive co-teaching looks like and the kinds of student outcomes it supports (Friend et al., 2010; Rexroat-Frazier & Chamberlin, 2019). This is partially due to the fact that there is no shared definition of co-teaching in educational research (Rexroat-Frazier & Chamberlin, 2019). Despite this lack of coherence in the research on co-teaching, we do know a little about when co-teaching is productive or harmful and what kinds of collaboration between general and special education teachers might be promising for student learning in inclusive classrooms.

First, a promising definition of co-teaching was proposed by Sileo and van Garderen in 2010: "an instructional delivery model applicable to teaching students with disabilities in least restrictive integrated classroom settings in which general and special educators share responsibility for planning, delivering, and evaluating instructional practices for all students" (p. 14, as cited in Rexroat-Frazier & Chamberlin, 2019; p. 173). In this definition, teachers work together in an inclusive classroom by collaborating on multiple dimensions of instruction, including planning, teaching, and assessing for all students, not just students who qualify for special services. Not only might such co-teaching help mitigate status issues, but it can also allow for coherent, integrated support for students with identified exceptionalities.

This definition of co-teaching can be operationalized in the classroom in many different ways, with some more productively leveraging special education teachers' strengths than others. For example, a few possible co-teaching approaches are, "one teaching, one observing; one teaching, one circulating; team teaching; parallel teaching/ split class and team pull out." The dominant co-teaching approach is one teach, one observe. In this approach, the general education teacher leads and the special education teacher supports (Rexroat-Frazier & Chamberlin, 2019, p. 175). This is not ideal. Ideally, co-teaching more fully leverages the strengths of special education teachers such as through team teaching. Of course, both general education and special education teachers will need to engage in professional learning to support this kind of work and will need time outside of the classroom to work together towards identifying their mutually beneficial goals.

Akyuz and Stephan (2020) identified planning and instructional practices for co-teachers that can help students with identified exceptionalities build autonomy. First, they described ways that co-teachers facilitate students' learning of the lesson objective. Three co-planning practices for supporting the learning goal are (1) creating mathematical tasks rich in imagery, (2) unpacking the learning goals, and (3) reflecting on the learning goals to guide further discussion. A related co-instructional practice is strategically selecting students to share their ideas based on how their solutions contribute to the learning goals. Second, Akyuz and Stephan described ways to support students to persevere through cognitively challenging points of the lesson. A co-planning practice is reflecting on students' progress on the learning goal, and a co-instructional practice is supporting social and sociomathematical norms, where social norms are the ways students expect and are expected to interact with each other and sociomathematical norms are the ways students expect and are expected to create and justify their mathematical work (Yackel & Cobb, 1996). Finally, Akyuz and Stephan identified two ways that co-teachers can modify their instruction to support students to deal with cognitive challenges that might arise: (1) by supporting visualization with gestures and tools, and (2) by restating students' words in clearer language.

Even when co-teaching is an option, there are many barriers to being able to implement the co-teaching practices described above. First and foremost, often neither general education nor special education teachers receive pre-service or in-service education on how to co-teach. Unfortunately, as mentioned earlier, research on the necessary knowledge and skills that teachers need to collaborate in inclusive classrooms is also underdeveloped. Other barriers to co-teaching include co-planning time, scheduling, caseload, administrative support, academic content knowledge, high-stakes testing, and co-teacher compatibility (Cook & Cook, 2020, p. 144).

This last barrier is especially important to overcome. Co-teaching should not be forced, as teacher attitudes in co-teaching situations can impact the tone for classrooms and impact student learning (Rexroat-Frazier & Chamberlin, 2019, p. 178, citing Sakiz, Pape, and Hoy, 2012). Desirable attitudes include mutual caring, interest, concern, encouragement, and high expectations. In addition, co-teachers should have relatively equal professional standing, co-teaching is not an opportunity for mentorship, and should negotiate their respective roles so that expectations for distributions of labor and responsibility are explicit (Rexroat-Frazier & Chamberlin, 2019, citing Walther et al., 1996). Finally, co-teachers should be relatively aligned with their philosophy of education, meaning how they view the purpose of their profession (Rexroat-Frazier & Chamberlin, 2019 citing Magiera et al., 2005). In sum, conditions are right for co-teaching to be productive when both teachers are willing and eager to work together to plan, teach, and assess; are equals in professional standing; have clearly defined each teacher's responsibilities in the classroom; and similarly orient to the purpose of teaching.

This definition does not apply to paraeducator aides. In classrooms fortunate enough to have paraeducators, it is important to ensure that their presence does not interfere with the classroom teacher's sense of instructional responsibility for students with identified exceptionalities or that the paraeducator's proximity to students interferes with peer-to-peer relationships or foster dependence (Cook & Cook, 2020). In a National Science Foundation-funded study of paraeducator professional development for supporting learning in mathematics classrooms, Storeygard et al. (2018) found that paraeducators benefit from having (1) a safe and encouraging learning environment where they can explore a lesson's mathematics before they encounter it with students, (2) access to the mathematics curriculum and opportunities to understand the curricular goals and learning philosophy, and (3) opportunities to engage in problems conceptually and reflect on how conceptual approaches differ from procedural approaches. With such support, paraeducators can contribute much more than routine monitoring and clerical tasks; they can fill in where teachers are spread thin by providing additional support for students with identified exceptionalities. For a summary of this section's takeaway strategies, see Table 3 in the appendix.

Designing for disability-first rather than differentiation

Historically, curriculum and instruction have been designed for the average learner. However, this average learner is an ideal type, it is imaginary (Rose, 2016). Because the average learner is imaginary, the reality is that fair instructional practices are not the same instructional practices for each student. Thus, curricula must be designed to support instruction that flexibly meets students' unique needs, especially in the case of students with multiple exceptionalities (Hartmann, 2015).

This is unique from differentiation, which some argue leads to within classroom tracking, and thus also to exclusion and stigmatization (Bannister, 2016). Many argue that differentiation through increased direct instruction and practice and through supporting students in their preferred learning style is beneficial to students. Yet, learning-styles instruction has no consensus in research (Bannister, 2016; Pashler et al., 2008). What research has shown is that interventions should focus on increasing participation in collaborative mathematics problem-solving (Lambert & Sugita, 2016).

Instead of differentiating in the traditional sense, differentiation can be front-loaded in curricula through flexible designs that support multiple-ability engagement. This is called Universal Design for Learning, or UDL. Consider this description from Thurber et al. (2018):

> UDL is an educational framework that emphasizes the use of flexible goals, methods, materials, and assessments in order to provide effective instruction to a diversity of learners. Rather than approaching accessibility as an afterthought or only on a case-by-case basis, UDL principles help instructors to design courses that address the needs of diverse learners from the start so that all students may benefit. (Thurber et al., 2018, emphasis added)

Thus, UDL is meant to increase the accessibility of participation and content through the design of curricula. Examples of UDL include designing for opportunities for multi-modal engagement with mathematical concepts (e.g., algebra tiles) and for participation structures that distribute labor in ways that support students with identified exceptionalities to engage in cognitively demanding tasks (e.g., team roles).

While it may not be appropriate for UDL to replace all case-by-case accommodations and modifications, UDL makes learning more accessible to all students through design. For example, UDL supports students to learn through collaboration. Collaboration is more challenging for some students with identified exceptionalities, still, it has both short- and long-term benefits. In the short term, collaboration can increase student motivation. This in turn has long-term effects, since increased participation supports student learning. In addition, in the 21st century, collaboration is an essential skill that can contribute to learning both content and social strengths such as learning to work effectively with others. In particular, collaboration can support the learning of students with identified exceptionalities through increasing opportunities for one-on-one support through the mentoring of peers (CAST, 2018). Of course, collaboration needs to be carefully structured, such as through implementation of team roles (e.g., see the Team Support Guidebook at https://cpm.org/teamsupport) in order to mitigate status issues, as previously mentioned, such as through sentence starters that support students to ask each other for help, team roles that foster multiple-ability participation, and teamwork norms and rubrics (CAST, 2018). Adopting a curriculum designed in alignment with UDL is a great way to begin supporting teachers to make changes to their instruction because it makes teaching for inclusion less subversive. In other words, instead of swimming upstream as they work around curricula designed to require differentiation in the traditional sense, teachers using UDL aligned curricula can focus on expanding their visions and enactment of equitable, ambitious instruction in their inclusive classrooms.

Summary

So, how can mathematics education leaders and educators support students with identified exceptionalities to experience productive struggle during collaborative problem-solving on cognitively-demanding tasks?

First, mindsets must shift from thinking about disability as a kind of diagnosable brokenness and lack (the medical model) to focusing on each student's existing capacity for learning, their special abilities, and their potential and how environment designs and culture may foster learners' ability to meet their potential or squander it (the cultural model). As Susan Robinson, a distinguished alum of Penn State University and CEO/founder of Global Health AspirAction, and a person with a genetic visual impairment, says in her Ted Talk, "The term 'disability' detonates a mindset of less than" (Robinson, 2016), a clear conflict with growth mindsets shown to be important for mathematical achievement (Boaler & Dweck, 2016; Bostwick et al., 2017). Redressing this is especially important for students of color as they are systematically disenfranchised, stigmatized, and underserved by the education system. By refocusing on students' strengths and building on their current understandings, teachers can build a classroom culture that fosters growth mindsets in support of all students' learning.

Second, students with identified exceptionalities can experience productive struggle during cognitively-demanding tasks when they are supported through ambitious instructional strategies with aligned assessments that highlight what students do know, some of which are identified in Tables 1 and 2 in the appendix.

Third, access to word-heavy mathematics tasks can be increased with small modifications that may benefit all students, such as by recording a read-aloud of the problem that students can rewind and relisten to in their groups. More strategies can be found in Table 1.

Fourth, it is critical to attend to status issues in inclusive classrooms that use teamwork to engage students in cognitively demanding tasks. Table 1 provides multiple research-bsed strategies for mitigating status issues.

Fifth, co-teaching may support all students' learning in inclusive classrooms by re-distributing the labor of teaching across two teachers with differing, complementary expertise. Research indicates that co-teaching may be most productive when both teachers are willing and eager to work together to plan, teach, and assess; are equals in professional standing; have clearly defined each teacher's responsibilities in the classroom; and similarly orient to the purpose of teaching (i.e., their philosophy of education). A summary can be found in Table 3 in the appendix.

Finally, mathematical tasks should be designed to have multiple entry points and engage students through multiple modalities with their peers in cognitively-demanding problem-solving tasks. This approach is in line with UDL (Lambert, 2020). UDL is generated by designing from the margins first; designing for disability first is a way to benefit all learners. This flips the traditional approach to designing for the imaginary average learner upside down, and it may have promising results. A proponent of this approach, Elise Roy, has given lectures on this design stance at leading design firms such as Microsoft, NASA, AIGA, and the U.S. Institute for Peace (Roy, 2015). Surely if innovators such as NASA see value in designs that re-able those who have previously been dis-abled, there may be value in exploring disability-first designs for learning as well.

By maintaining asset-based perspectives of and high expectations for students with identified exceptionalities, including those diagnosed with severe disabilities, mathematics education leaders and the educators they support can expunge barriers to conceptual learning and foster scaffolds for meaningful engagement for all students. ✪

Acknowledgments

We would like to thank Dr. Rachel Lambert for her support in this literature synthesis. We drew heavily on her scholarship, and her feedback helped us fine-tune our argument. We also thank CPM Educational Program for supporting the development of this manuscript, both conceptually and through funding.

References

Aiquraini, T., & Gut, D. (2012). Critical components of successful inclusion of students with severe disabilities: Literature review. *International Journal of Special Education, 27*(1), 1.

Akyuz, D., & Stephan, M. (2020). Co-teaching practices that build autonomy for students with learning disabilities in mathematics. *International Journal of Mathematical Education in Science and Technology*, 1-28. https://doi.org/10.1080/0020739X.2020.1795286

Annamma, S. A. (2017). *The pedagogy of pathologization: Dis/abled girls of color in the school-prison nexus.* Routledge. https://doi.org/10.4324/9781315523057

Annamma, S. A. (2018). Mapping consequential geographies in the carceral state: Education journey mapping as a qualitative method with girls of color with dis/abilities. *Qualitative Inquiry, 24*(1), 20-34. https://doi.org/10.1177/1077800417728962

Annamma, S. A., Connor, D., & Ferri, B. (2013). Dis/ability critical race studies (DisCrit): Theorizing at the intersections of race and dis/ability. *Race, Ethnicity and Education, 16*(1), 1-31. https://doi.org/10.1080/13613324.2012.730511

Armstrong, T. (2012). *Neurodiversity in the classroom: strength-based strategies to help students with special needs succeed in school and life [electronic resource]*. ASCD.

Bannister, N. A. (2016). Breaking the spell of differentiated instruction through equity pedagogy and teacher community. *Cultural Studies of Science Education, 11*(2), 335-347. https://doi.org/10.1007/s11422-016-9766-0

Baxter, J. A., Woodward, J., & Olson, D. (2001). Effects of reform-based mathematics instruction on low achievers in five third-grade classrooms. *The Elementary School Journal, 101*(5), 529-547. https://doi.org/10.1086/499686

Blanchett, W. J. (2006). Disproportionate representation of African American students in special education: acknowledging the role of white privilege and racism. Educational Researcher, 35(6), 24-28. https://doi.org/10.3102/0013189X035006024

Boaler, J., & Dweck, C. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages, and innovative teaching.* Jossey-Bass.

Bostwick, K. C. P., Collie, R. J., Martin, A. J., & Durksen, T. L. (2017). Students' growth mindsets, goals, and academic outcomes in mathematics. *Zeitschrift für Psychologie, 225*(2), 107-116. https://doi.org/10.1027/2151-2604/a000287

Bottge, B. A., Heinrichs, M., Zara Dee, M., & Hung, Y.-H. (2002). Weighing the benefits of anchored math instruction for students with disabilities in general education classes. *The Journal of Special Education, 35*(4), 186-200. https://doi.org/10.1177/002246690203500401

Browder, D. M., Jimenez, B. A., & Trela, K. (2012). Grade-aligned math instruction for secondary students with moderate intellectual disability. *Education and Training in Autism and Developmental Disabilities, 47*(3), 373-388.

Browder, D. M., Spooner, F., Lo, Y.-y., Saunders, A. F., Root, J. R., Ley Davis, L., & Brosh, C. R. (2018). Teaching students with moderate intellectual disability to solve word problems. *The Journal of Special Education, 51*(4), 222-235. https://doi.org/10.1177/0022466917721236

Cabana, C., Shreve, B., & Woodbury, E. (2014). Working toward an equity pedagogy. In N. i. S. Nasir, C. Cabana, B. Shreve, E. Woodbury, & N. Louie (Eds.), *Mathematics and equity: A framework for successful practice* (pp. 35–52). Teachers College Press.

CAST. (2018). *Universal Design for Learning guidelines version 2.2*. http://udlguidelines.cast.org

Cohen, E. G., & Lotan, R. A. (2014). *Designing groupwork: strategies for the heterogeneous classroom.* Teachers College Press.

Cole, S., Horvath, B., Chapman, C., Deschenes, C., Ebeling, D., & Sprague, J. (2000). *Adapting curriculum and instruction in inclusive classrooms: A teacher's desk reference* (2nd ed. ed.). Center on Education and Lifelong Learning, Indiana Institute on Disability and Community.

Connor, D. J., Gallagher, D., & Ferri, B. A. (2011). Broadening our horizons: Toward a plurality of methodologies in learning disability research. *Learning Disability Quarterly, 34*(2), 107-121. https://doi.org/10.1177/073194871103400201

Cook, B. G., & Cook, L. (2020). An examination of highly-cited research on inclusion. In J. M. Kauffman (Ed.), *On educational inclusion: Meanings, history, issues and international perspectives* (pp. 130-159). Routledge, Taylor & Francis Group. https://doi.org/10.4324/9780429344039-7

Courtade, G., Spooner, F., Browder, D., & Jimenez, B. (2012). Seven reasons to promote standards-based instruction for students with severe disabilities: A reply to Ayres, Lowrey, Douglas, & Sievers (2011). *Education and Training in Autism and Developmental Disabilities, 47*(1), 3-13.

Dietiker, L.; Baldinger, E.; Shreve, B; Kassarjian, M; Nikula, M. (Eds.)(2015). *Core Connections, Course 3* (2nd ed.). CPM Educational Program.

Friend, M., Cook, L., Hurley-Chamberlain, D., & Shamberger, C. (2010). Co-teaching: An illustration of the complexity of collaboration in special education. *Journal of Educational and Psychological Consultation, 20*(1), 9-27. https://doi.org/10.1080/10474410903535380

Garderen, D. v., Scheuermann, A., Jackson, C., & Hampton, D. (2009). Supporting the collaboration of special educators and general educators to teach students who struggle with mathematics: An overview of the research. *Psychology in the Schools, 46*(1), 56-78. https://doi.org/10.1002/pits.20354

Gregory, A., Skiba, R. J., & Noguera, P. A. (2010). The achievement gap and the discipline gap: Two sides of the same coin? *Educational Researcher, 39*(1), 59-68. https://doi.org/10.3102/0013189X09357621

Guerrero, A. D., Rodriguez, M. A., & Flores, G. (2011). Disparities in provider elicitation of parents' developmental concerns for US children. *Pediatrics (Evanston), 128*(5), 901-909. https://doi.org/10.1542/peds.2011-0030

Gutierrez, R. (2013). The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education, 44*(1), 37-68. https://doi.org/10.5951/jresematheduc.44.1.0037

Hammond, Z. L. (2014). *Culturally responsive teaching and the brain: Promoting authentic engagement and rigor among culturally and linguistically diverse students.* SAGE Publications.

Hartmann, E. (2015). Universal Design for Learning and learners with severe support needs. *International Journal of Whole Schooling, 11*(1), 54.

Horn, I. S. (2012). *Strength in numbers: Collaborative learning in secondary mathematics.* National Council of Teachers of Mathematics.

IDEA regulations, C. F. R. (2016). *Individuals With Disabilities Education Act, 20 U.S.C. §§ 1400 et seq. (2006 & Supp. V. 2011).*

National Governors Association Center for Best Practices, & Council of Chief State School. (2010). Common Core State Standards for Mathematics. Authors. http://www.corestandards.org

Jackson, K., & Cobb, P. (2010). Refining a vision of ambitious mathematics instruction to address issues of equity. Paper presented at the annual meeting of the *American Educational Research Association*, Denver, CO.

Jitendra, A. K. (2013). Understanding and accessing standards-based mathematics for students with mathematics difficulties. *Learning Disability Quarterly, 36*(1), 4-8. https://doi.org/10.1177/0731948712455337

Jitendra, A. K., Dupuis, D. N., Rodriguez, M. C., Zaslofsky, A. F., Slater, S., Cozine-Corroy, K., & Church, C. (2013). A randomized controlled trial of the impact of schema-based instruction on mathematical outcomes for third-grade students with mathematics difficulties. *The Elementary School Journal, 114*(2), 252-276. https://doi.org/10.1086/673199

Jitendra, A. K., Petersen-Brown, S., Lein, A. E., Zaslofsky, A. F., Kunkel, A. K., Jung, P.-G., & Egan, A. M. (2015). Teaching mathematical word problem solving: The quality of evidence for strategy instruction priming the problem structure. *Journal of Learning Disabilities, 48*(1), 51-72. https://doi.org/10.1177/0022219413487408

Klingner, J. K., & Vaughn, S. (1999). Students' perceptions of instruction in inclusion classrooms: Implications for students with learning disabilities. *Exceptional Children, 66*(1), 23-37. https://doi.org/10.1177/001440299906600102

Lambert, R. (2015). Constructing and resisting disability in mathematics classrooms: A case study exploring the impact of different pedagogies. *Educational Studies in Mathematics, 89*(1), 1-18. https://doi.org/10.1007/s10649-014-9587-6

Lambert, R. (2020). Increasing Access to Universally Designed Mathematics Classrooms. *Policy Analysis for California Education*, PACE.

Lambert, R., Chun, M., Davis, J., Ceja, K. L., Aguilar, K., Moran, P., & Manset, L. (2019). "My dyslexia is like a bubble": How insiders with learning disabilities describe their differences, strengths, and challenges. *Learning Disabilities: A Multidisciplinary Journal, 24*(1), 1-18. https://doi.org/https://doi.org/10.18666/LDMJ-2019-V24-I2-8765

Lambert, R., & Sugita, T. (2016). Increasing engagement of students with learning disabilities in mathematical problem-solving and discussion. *Support for Learning, 31*(4), 347-366. https://doi.org/10.1111/1467-9604.12142

Lambert, R., Sugita, T., Yeh, C., Hunt, J. H., & Brophy, S. (2020). Documenting increased participation of a student with autism in the standards for mathematical practice. *Journal of Educational Psychology, 112*(3), 494-513. https://doi.org/10.1037/edu0000425

Lambert, R., & Tan, P. (2016, November 3-6, 2016). Dis/ability and mathematics: Theorizing the research divide between special education and mathematics. Paper presented at the 38th annual meeting of the *North American Chapter of the International Group for the Psychology of Mathematics Education*, Tucson, Arizona.

Lambert, R., & Tan, P. (2019). Does disability matter in mathematics educational research? A critical comparison of research on students with and without disabilities. *Mathematics Education Research Journal, 32*(1), 5-35. https://doi.org/10.1007/s13394-019-00299-6

Lewis, K. E. (2016). Beyond error patterns: A sociocultural view of fraction comparison errors in students with mathematical learning disabilities. L*earning Disability Quarterly, 39*(4), 199-212. https://doi.org/10.1177/0731948716658063

Lewis, K. E., & Fisher, M. B. (2017). Clinical interviews: Assessing and designing mathematics instruction for students with disabilities. *Intervention in School and Clinic, 53*(5), 283-291. https://doi.org/10.1177/1053451217736864

Lewis, K. E., & Lynn, D. M. (2018). Access through compensation: Emancipatory view of a mathematics learning disability. *Cognition and Instruction, 36*(4), 424-459. https://doi.org/10.1080/07370008.2018.1491581

Magiera, K. & Zigmond, N. (2005) 'Co-teaching in middle school classrooms under routine conditions: does the instructional experience differ for students with disabilities in co-taught and solo-taught classes?' *Learning Disabilities Research & Practice, 20*, pp. 79–85. https://doi.org/10.1111/j.1540-5826.2005.00123.x

Morgan, P. L., Farkas, G., Cook, M., Strassfeld, N. M., Hillemeier, M. M., Pun, W. H., & Schussler, D. L. (2017). Are black children disproportionately overrepresented in special education? A best-evidence synthesis. *Exceptional Children, 83*(2), 181-198. https://doi.org/10.1177/0014402916664042

Morgan, P. L., Farkas, G., Cook, M., Strassfeld, N. M., Hillemeier, M. M., Pun, W. H., Wang, Y., & Schussler, D. L. (2018). Are Hispanic, Asian, Native American, or Language-minority children overrepresented in special education? *Exceptional Children, 84*(3), 261-279. https://doi.org/10.1177/0014402917748303

Morgan, P. L., Staff, J., Hillemeier, M. M., Farkas, G., & Maczuga, S. (2013). Racial and ethnic disparities in ADHD diagnosis from kindergarten to eighth grade. *Pediatrics (Evanston), 132*(1), 85-93. https://doi.org/10.1542/peds.2012-2390

Moscardini, L. (2010). 'I like it instead of maths': How pupils with moderate learning difficulties in Scottish primary special schools intuitively solved mathematical word problems. *British Journal of Special Education, 37*(3), 130-138. https://doi.org/10.1111/j.1467-8578.2010.00461.x

Nasir, N. i. S., Cabana, C., Shreve, B., Woodbury, E., & Louie, N. (2014). *Mathematics for equity: A framework for successful practice.* Teachers College Press.

Pashler, H., McDaniel, M., Rohrer, D., & Bjork, R. (2008). Learning styles: Concepts and evidence. *Psychological Science in the Public Interest, 9*(3), 105-119. https://doi.org/10.1111/j.1539-6053.2009.01038.x

Pfister, M., Moser Opitz, E., & Pauli, C. (2015). Scaffolding for mathematics teaching in inclusive primary classrooms: A video study. *ZDM, 47*(7), 1079-1092. https://doi.org/10.1007/s11858-015-0713-4

Rexroat-Frazier, N., & Chamberlin, S. (2019). Best practices in co-teaching mathematics with special needs students. *Journal of Research in Special Educational Needs, 19*(3), 173-183. https://doi.org/10.1111/1471-3802.12439

Robinson, S. (2016). *How I fail at being disabled* [Video file]. TED: Ideas worth spreading. https://www.ted.com/talks/susan_robinson_how_i_fail_at_being_disabled

Rodis, P., Garrod, A., & Boscardin, M. L. (2001). *Learning disabilities and life stories.* Boston, MA: Allyn and Bacon.

Rose, T. (2016). *The end of average: How we succeed in a world that values sameness.* Harper Collins.

Roy, E. (2015). *When we design for disability, we all benefit* [Video file]. TED: Ideas worth spreading. https://www.ted.com/talks/elise_roy_when_we_design_for_disability_we_all_benefit

Schoenfeld, A. H. (1988). Problem solving in context(s). In R. Charles & E. A. Silver (Eds.), *The teaching and assessing of mathematical problem solving: Research agenda for mathematics education* (Vol. 3, pp. 82-92). Lawrence Erlbaum Associates.

Smith, M. S., & Stein, M. K. (2018). *5 practices for orchestrating productive mathematical discussions* (Second edition. ed.). The National Council of Teachers of Mathematics, Inc.

Storeygard, J., Mutch-Jones, K., & Ngongi-Lukula, S. (2018). *Doing the math with paraeducators—Hands on!* TERC. https://www.terc.edu/doing-the-math-with-paraeducators-hands-on-winter-2018/

Sullivan, A. L. (2011). Disproportionality in special education identification and placement of English language learners. *Exceptional Children, 77*(3), 317-334. https://doi.org/10.1177/001440291107700304

Tan, P., Lambert, R., Padilla, A., & Wieman, R. (2019). A disability studies in mathematics education review of intellectual disabilities: Directions for future inquiry and practice. *Journal of Mathematical Behavior, 54*, 100672. https://doi.org/10.1016/j.jmathb.2018.09.001

Thurber, A., Bandy, J., & Participants in the 2017-2018 Disability and Learning Community at Vanderbilt University (2018). Creating accessible learning environments. http://cft.vanderbilt.edu/guides-sub-pages/creating-accessible-learning-environments/

Vaughn, S., & Klingner, J. K. (1998). Students' perceptions of inclusion and resource room settings. *The Journal of Special Education, 32*(2), 79-88. https://doi.org/10.1177/002246699803200202

Warshauer, H. K. (2014). Productive struggle in middle school mathematics classrooms. *Journal of Mathematics Teacher Education, 18*(4), 375-400. https://doi.org/10.1007/s10857-014-9286-3

Wehmeyer, M. L. (2019). *Strength-based approaches to educating all learners with disabilities: Beyond special education.* Teachers College Press.

Wehmeyer, M. L., Shogren, K. A., & Kurth, J. (2021). The state of inclusion with students with intellectual and developmental disabilities in the United States. *Journal of Policy and Practice in Intellectual Disabilities, 18*(1), 36-43. https://doi.org/10.1111/jppi.12332

Wood, T. (1999). Funneling or focusing? Alternative patterns of communication in mathematics class. In H. Steinbring, M. G. Bartolini-Bussi, & S. A. (Eds.), Language and Communication in the Mathematics Classroom, a special issue of *Journal for research in mathematics education* (Vol. 30, p. 243). National Council of Teachers of Mathematics.

Woodward, J., & Montague, M. (2002). Meeting the Challenge of Mathematics Reform for Students with LD. *The Journal of Special Education, 36*(2), 89-101. https://doi.org/10.1177/00224669020360020401

Union of Physically Impaired Against Segregation, & The Disability Alliance (1975). *Fundamental principles of disability.* Authors.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458-477. https://doi.org/10.2307/749877

APPENDIX

Table 1: Strategies for engaging students with identified exceptionalities in productive struggle through collaboration on cognitively demanding tasks

Support Area	Description
Problem-solving*	
Multi-modal curriculum design	Provide students with choices about what materials are used to solve problems (equations, drawings, algebra tiles, connecting cubes, base-ten blocks, etc.)
Consistent routine	For example, in the case of CPM Educational Program's lessons, a consistent routine is (1) teacher-led lesson launch, (2) individual or team problem-solving, and then (3) a whole-class discussion in which students present their strategies and solutions.
Teacher scaffolds for problem-solving	Scaffold the starting problem-solving by restating word problems (while retaining the problem type) and re-reading word problems in small chunks that students model step-by-step. This reduces students' difficulties with language but does not reduce the cognitive demand of the mathematics task.
Equitable teamwork	Mitigate marginalization by providing additional support to teams (this often requires teacher professional development)
Mathematical discussion*	
Student rehearsal of strategy shares	For example, allow students to rehearse the strategy they will share out in a discussion by providing them with a paraeducator, allow students to use FlipGrid to record a strategy, etc.
Access to manipulatives and notebooks	Allow students to use their notebooks as a record of their problem-solving for as long as they need in order to support their participation in discussion; allow students to use manipulatives such as algebra tiles rather than equations to model their mathematical thinking during discussions.
Teacher questioning	Hold students accountable for explanations of their strategies by asking multiple follow-up questions.
Participation**	
Begin with relationships	Establish strong relationships from the beginning of the year, especially through finding shared interests.
Strengths-based views of exceptionality	Verbally notice both mathematical and social strengths when talking to and about students. Do so using non-medical language (e.g., "shy" instead of "non-verbal"). Ask questions to elicit thinking and then help students build from their current understandings. Pay attention to specifically what is challenging for students, such as verbal participation. Consider asking students' permission before sharing their thinking in front of a team or the whole class.
Scaffolded discussion with peers	Intervene in teamwork to support students to share out. Direct students to work with specific peers and physically move their notebooks or papers to be next to each other, then check in on their progress. Hold peers accountable even if students are quiet talkers and thus hard to hear.
Collaborative shares	Have students share out in pairs so that students who do not prefer verbal interaction can still participate.

Table continued on next page

Table 1: Strategies for engaging students with identified exceptionalities in productive struggle through collaboration on cognitively demanding tasks (continued)

Support Area	Description
Participation *(continued)***	
Make norms of mathematical discussion very explicit	Have the class define and describe what discussion looks and sounds like and create a durable, visible record of this discussion. For example, have the class work collectively to create a chart with an eye on one side ("looks like") and an ear on the other ("sounds like") with each side filled in. Students may generate ideas such as: • ***Mathematical discussions are:*** when you talk about math and what it can do, talking about how we use strategies, when two or more people have different answers, sharing ideas with others • ***Looks like:*** Notebooks out, eyes on the speaker, showing work to each other, taking notes on what other people are saying, agree on an "agree symbol" • ***Sounds like:*** "I agree with you because…," "I respectfully disagree with you because…," "This is how I did my work," "I don't understand the strategy," "Can you repeat that?," "I'd like to add on to what you said," "My strategy has a connection with yours," and "Can you explain more?" (p. 505, direct quotation from a student conversation poster image)
Notice students' participation	Some students may participate differently than others. For example, instead of raising their hand high, a student may raise just one finger slightly. Be sure to notice and respond to such participation as quickly as possible.
Get started on word problems	
Explore problem context	Create an engaging launch that fully explores the problem context, but does not lower the cognitive demand (Jackson & Cobb, 2010); consider exploring the problem context in teams (Cole et al., 2000)
Rephrase	Rephrase word problems without lowering the cognitive demand (Lambert & Sugita, 2016)
Read in small chunks	Re-read word problems in small chunks that students model step-by-step (Lambert & Sugita, 2016)
Oral recordings	Make oral recordings of the text so that students can listen to and rewind the written material instead of reading it (Cole et al., 2000)
Provide problems in advance	Provide students with the word problem text or recording in advance so that they can become familiar with the context of the problem (Cole et al., 2000)
Work in pairs	Allow students to work in pairs with a supportive peer (Lambert et al., 2020)
Teams create visual representations	Require teams to collaboratively create graphic organizers and other visual representations of the problem (modified from Browder et al., 2018)
Equal-status interactions in teamwork***	
Value rough-draft thinking	Create a classroom culture that values mistakes and rough-draft thinking (Nasir et al., 2014)
Teamwork accountability	Use accountability structures that hold each team member accountable for the group's shared work (Nasir et al., 2014)

Table continued on next page

Table 1: Strategies for engaging students with identified exceptionalities in productive struggle through collaboration on cognitively demanding tasks (continued)

Support Area	**Description**
Equal-status interactions in teamwork* *(continued)***	
Visibly random teams	Use random assignment of team roles (Nasir et al., 2014)
High press questioning	Press all students for high levels of justification (Nasir et al., 2014)
Group worthy tasks	Use ‘‘group worthy’’ tasks (Cabana et al., 2014; Cohen & Lotan, 2014) by: • focusing on the big ideas of a lesson. • providing tasks that afford multiple solution pathways and/or require multiple representations. • providing tasks that require multiple intellectual abilities—finding information, problem-solving, basic skills, or material organization—such that no single individual can possess all of them.
Multiple-abilities framing	List out the intellectual abilities the task requires to students and then say something like, “None of us has all of these abilities that are required for this task. Everyone has some of these abilities, and so everyone will have something important to contribute to our shared work today. Listen carefully to one another, as you will all be important resources for your group.” (Bannister, 2016, p. 342)
Assign competence	Make public, positive, evaluative statements that recognize specific intellectual contributions that students with identified exceptionalities make during teamwork (Horn, 2012). This can be done for other low-status students as well, such as students who are marginalized along lines of gender, race, social class, physical attractiveness, and prior academic performance (Bannister, 2014; Cohen & Lotan, 2014).

* Strategies are near direct quotations from Lambert and Sugita (2016, p. 357-358)
** Strategies from Lambert et al. (2020, p. 508-509)
*** Strategies summarized in Bannister (2016)

Table 2: Rubric for scaffolds. (Pfister et al. 2015).

Scaffolds	**Teacher Actions Rubric Examples**			
Scaffolding Questions	**Goals**	**(0)**	**(1)**	**(2)**
Micro-Scaffolds:				
Cognitive Activation				
Compare! What do you notice? What did you have to do so that …?	Poses clear, content-related, meaningful, challenging questions and problems, provides stimulation for describing or substantiating facts, observations, etc. Enables the establishment of relationships between contents Stimulating discourse	Set tasks with small steps Told the students which actions they have to carry out Posed questions that require a one-number answer Carried out actions with the manipulatives	Carried out actions with the manipulatives himself Often told students solution steps Sometimes requested observation, description, or substantiation of facts and findings Sometimes requested a comparison of solution strategies	Constantly requested students to verbalize and substantiate their solution steps Allowed problems (even correctly solved ones) to be discussed Invited the formulation of insights and observations
Handling Errors Productively				
Where are you stuck? What are you considering? How did you find it out? How can we find out whether that's correct?	Recognizes the learning potential or difficulty of a situation Intervenes in the learning processes in a supportive manner Endeavors to understand the students' solution strategies or reflections Supports students in tackling problems independently Checks the students' understanding following the intervention	Demanded that certain procedures be carried out When students were uncertain, he told them how to continue Rubbed out mistakes and wrote down the solution himself Pointed to what was written on the blackboard	Provided hints for using the structure of the Dienes blocks Requested the students to try the problem again with help of the manipulatives Demanded more careful work (not specifically mathematical)	Requested verbalization of the procedure Requested substantiation and proof Provided feedback on systematic procedures Let insights from a mistake be explicitly formulated, or the mistake to be "named" Established connections with other solved problems or problems that have not yet been solved

Table continued on next page

Table 2: Rubric for scaffolds. (Pfister et al. 2015). *(continued)*

Scaffolds	**Teacher Actions Rubric Examples**			
Scaffolding Questions	**Goals**	**(0)**	**(1)**	**(2)**
Micro-Scaffolds *(continued)*:				
Stimulating Discourse				
Describe what you have done! Can you explain that in more detail? Can we solve/write it differently?	Invites the students to comment on contributions or actions of others Responds to students' contributions Initiates reflections on solution strategies	Asked for numbers, results Let the students finish sentences he has started Spoke most of the time	Formulated central findings for students Primarily asked for results or subsequent steps Let the students finish teacher sentences Sometimes let students determine next steps Rarely included student ideas into discussions	Asked for reflections Let thought processes and insights be presented Mostly does not interrupt the students' contributions
Macro-Scaffolds:				
Using Manipulatives				
Can you show that with manipulatives? Can we place/do it differently?	Employs manipulatives to support the learning process Allows facts to be represented actively using manipulatives Emphasizes the understanding of structural relationships or the systematic use of manipulatives	Let students name the units of the Dienes blocks Mostly manipulated the Dienes blocks himself Told the students what they should do with the Dienes blocks Mostly wrote down the problem solution by himself	Encouraged students to use the Dienes blocks in a structured manner Let the structure of the Dienes blocks be used clearly for the grouping or de-grouping process and for recording (interim) results In part, he established the relationship between manipulatives, representations, and notations Addressed the difference between an empty number line and a number line	Let notation forms and arithmetic steps be compared Worked out the characteristics or differences of the manipulatives, representations, and notations clearly on several occasions (e.g., difference between an empty number line vs. and a number line) Let different presentation forms be used for individual solution strategies
Target Orientation				
Describe the rule/pattern! Why does it have to be done like that?	Focuses on core content elements Demonstrates what is important, points out conventions Summarizes important findings, recapitulates these findings in his/her own words	Focused on carrying out the procedure correctly	Formulated central findings Always pointed out important things Recapitulated insights or relevant things	Summarized the students' thoughts "Translated" student contributions Let insights be formulated and summarized Worked out key characteristics and procedures

Table 3: Summary of reported strategies for supporting successful co-teaching relationships

Co-teaching supports	Description
Share responsibility	Support general and special educators to share responsibility for planning, delivering, and evaluating instructional practices for all students, not just students who qualify for special services (Rexroat-Frazier & Chamberlin, 2019; p. 173, citing Sileo & van Garderen in 2010 p. 14).
Co-plan	Support teachers to co-plan by (1) creating mathematical tasks rich in imagery, (2) unpacking the learning goals, and (3) reflecting on the learning goals to guide further discussion (Akyuz & Stephan, 2020).
Team-teach	Support teachers to team-teach to more fully leverage the strengths of special education teachers, including by both teachers (1) supporting visualization with gestures and tools and (2) restating students' words in clearer language (Akyuz & Stephan, 2020; Rexroat-Frazier & Chamberlin, 2019).
Reduce barriers	Reduce barriers to co-teaching: provide in-service education on how to co-teach and to gain academic content knowledge, provide co-planning time administrative support, reduce case-load, schedule appropriately, mitigate high-stakes testing, and ensure co-teacher compatibility (Cook & Cook, 2020, p. 144).
Teacher pairing	Ensure co-teachers have relatively equal professional standing and provide opportunities for them to negotiate their respective roles so that expectations for distributions of labor and responsibility are explicit (Rexroat-Frazier & Chamberlin, 2019, p. 175 citing Walther-Thomas, Bryant and Land, 1996).
Joint reflection on philosophies of education	Create time and space for co-teachers to examine, compare, and work towards alignment of their philosophies of education (Rexroat-Frazier & Chamberlin, 2019 citing Magiera et al., 2005).