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Beyond Right or Wrong: Supporting Teachers in Strengths-Based Approaches to Examining Student Work

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Abstract

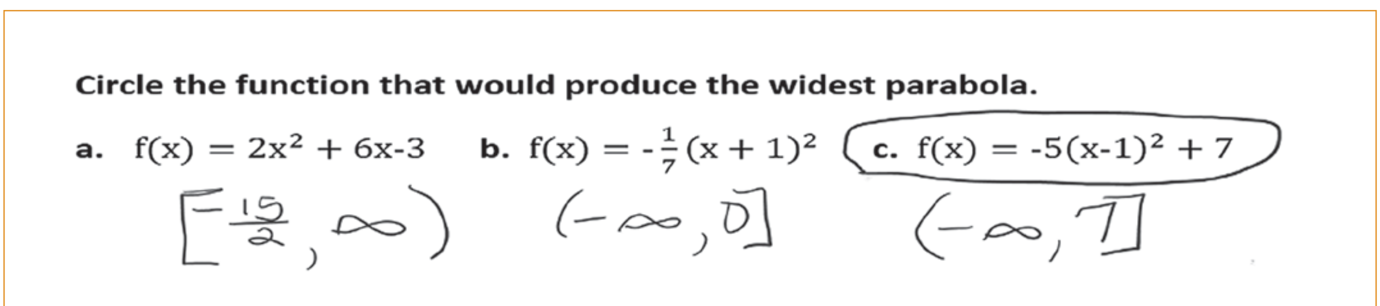
Recognizing the strengths of students through their written work takes time, practice, and intentionality. In this article, we detail a set of questions that can be used to purposefully engage with student written work in a strengths-based way. Derived from the exploration of experienced mathematics educators' mathematical knowledge for teaching quadratic functions, the questions place value on student thinking while providing the opportunity for teachers to enhance and extend their own mathematical knowledge for teaching. Mathematics leaders can use these questions to facilitate meaningful learning experiences for teachers, professional learning communities, and large group professional development activities.

When you look at the student work presented in Figure 1, what do you see?

Living in a world of high-stakes testing, from end-of-course assessments to college admissions tests, it is not surprising that teachers often focus on correctness when examining student work. Even with numerous years of teaching experience, sometimes a quick glance at the work in Figure 1 may simply reveal an incorrect response. However, what if we were to look a little closer and purposefully seek to understand the mathematical ideas demonstrated in the student's work, what would we see?

Purposefully seeking the mathematical understandings embedded in students' written work is key to taking a strengths-based approach to examining written work. A strengths-based perspective on teaching and learning emphasizes the "positive aspects of student effort and

FIGURE 1. *Student Work Sample 1*



achievement" (Lopez & Louis, 2009, p. 1). When examining student work from this perspective, the student is valued by recognizing what the student knows and has demonstrated while not ignoring what has not been recorded. It requires moving beyond simply identifying what is right or wrong. A strengths-based approach allows teachers to recognize where support is needed and determine ways to build upon the student's understandings (McCarthy et al., 2020). While this may require a degree of intentionality, we agree with Philipp (2008) that "we best help a learner by starting where he or she is and building upon his or her current understanding" (p. 23). Hence, paying attention to and building upon students' ideas can lead to more effective instruction and increased student learning (Bishop et al., 2014; Fennema et al., 1996; Kobett & Karp, 2020).

Focusing on the mathematical ideas present in student work not only benefits the student, but it has the potential to benefit the teacher (Wilson et al., 2013). As teachers engage with their students' written work, they may enhance and extend their own mathematical knowledge for teaching. Mathematical knowledge for teaching, or MKT, is the phrase that is commonly referenced to describe the knowledge used and needed by those providing mathematics instruction (e.g., Ball et al., 2008; Hill et al., 2008). It describes the knowledge that teachers rely upon to convey mathematical concepts to students in ways that are meaningful and useful. Over the last few decades, researchers have sought to identify, describe, categorize, and connect MKT to student learning. Through this work, researchers have linked increases in teachers' MKT to improvements in the quality of their instruction (Hill et al., 2008) and their students' achievement (Hill et al., 2005). With the importance of MKT recognized through research, it is understandable that "all teachers need continuing opportunities to deepen and strengthen their mathematical knowledge for teaching" (Conference Board of the Mathematical Sciences, 2012, p. 68). Utilizing a strengths-based approach to engage with student work can provide an opportunity for teachers to deepen and strengthen their MKT.

Student work can be a powerful mechanism for facilitating mathematics teacher learning (Kazemi & Franke, 2004). However, moving from an evaluative, diagnostic, or formulative approach to one that highlights students' strengths may require some guidance, particularly in an era of high stakes testing and accountability. Through our research with six accomplished mathematics educators committed to the idea that all student written work is valuable and worthy of careful review (Zimmerman, 2020), we surmised a set of questions that focus on a student's strengths and can guide the examination of student work. In the following sections, we first provide a brief overview of the study and then detail six questions that math leaders can use with teachers to focus on the assets and strengths that students bring to instruction. We then use the context of a professional learning community of teachers to illustrate the ways mathematics leaders might use the questions to support teachers in taking a strengths-based approach to mathematics teaching and learning.

Learning from Experts

Though MKT for elementary and middle grades teachers is an established area of research, studies of secondary mathematics teachers' MKT are less common (Howell, 2012; Howell et. al., 2016). To that end, we designed a study to explore and document the MKT for quadratic functions that accomplished mathematics educators use when examining student work (Zimmerman, 2020). Six mathematics educators (two high school teachers, two university teacher educators, and two university mathematicians) with considerable experience and recognized expertise were purposefully selected and invited to participate in the study. Combined, the participants had over 170 years of teaching experience, served as teacher leaders for their state and local school districts, amassed numerous awards and recognition at both the local and state level, regularly participated in and led multi-year professional development activities focused on teacher learning, and actively engaged in mathematics education research. Through a series of semi-structured interviews, the participants engaged with student work that represented various quadratic function concepts. In these individual interviews, each participant was presented with pieces of student work and simply asked to "think-aloud". By thinking aloud, we were able to capture each participant's initial thoughts and reactions to the student samples. Through phases of thematic analysis and iterative pattern coding, we identified six themes that were representative of the participants' demonstrated MKT.

Results from the study characterized the participants' MKT for quadratic functions as a dynamic relationship between their knowledge of mathematics and the ways they used this knowledge to make connections to related mathematical concepts, interpret and conjecture about student understanding, and consider how they might support students in continuing to learn. Further, as the participants used their mathematical knowledge to consider student learning and their teaching, they deepened their own understandings of quadratic functions.

Characterizing the participants' MKT was possible because they engaged with student work in ways that focused on the mathematical ideas demonstrated by the student. While the participants were aware of the study's purpose to explore MKT for quadratic functions, their engagement with student work focused on what students knew as opposed to what they did not know, and the participants used these strengths as a resource from which they might build future instruction. The participants moved beyond attending to correct or incorrect solutions to see information conveyed by students through writing as useful and powerful. For example, consider an excerpt from "Kurin's" interview discussion of the student work depicted in Figure 1:

> I think the width of a parabola is not that well defined without something to reference to. So, I would want to look at all 3 graphs together. Knowing about the different kinds of shifts and changes to functions based upon where you put coefficients, I would say it is the middle one, $f(x)=-1/7(x+1)^2$. So, I think maybe for this question I might say "circle the function that would produce the widest parabola at the same height" or maybe "at the same y-value." Will a student understand what I mean when I add that to it? Now, this student is not connecting what you want the student to connect to in terms of the widest parabola. When they report back the range, you know they are looking vertical instead of looking horizontal. I would ask the student to graph all three functions together and then point out to me in their picture, where they are looking to determine the widest parabola. Then I would just reorient them to the horizontal width instead of vertical. I think this is a place where a tool like DESMOS really comes in handy, where you can graph several of that same function family and change a single coefficient.

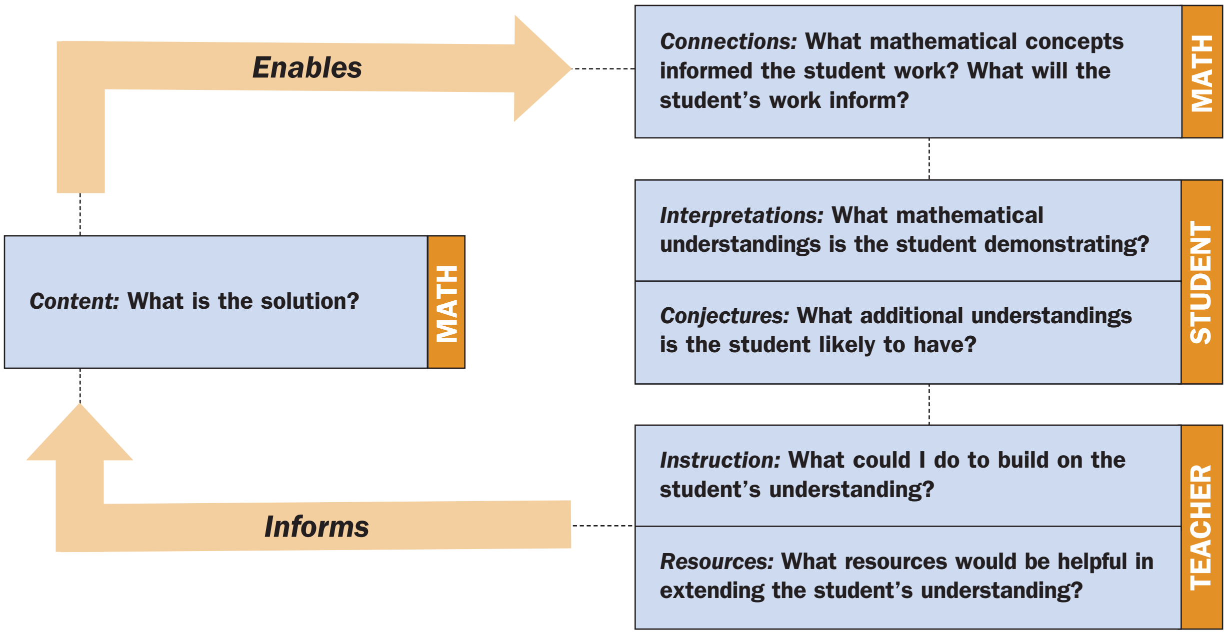
Intertwined with the solution to the problem, Kurin expressed concern regarding the problem itself. Kurin discussed possibly changing the wording of the problem and identified the range of a quadratic function as the mathematical idea represented in the student's work. Kurin conjectured that the student had a vertical perception of the functions graph and therefore used its range to identify the widest parabola. Kurin described how graphing might orient the student to width as a horizontal feature of parabolas. Rather than evaluating the response as incorrect or focusing exclusively on what the student did not know, Kurin identified the student's graphical understanding of the range of quadratic functions as a strength that they could use to reorient the student to determine the widest parabola.

Throughout the study, participants' discussions centered on student work and described how the understandings the students had recorded could be used in subsequent instruction. Their responses were not evaluative in nature but rather described a path for instruction that was based on what the student had demonstrated that they knew. The participants' strengths-based approach led them to identify the mathematical ideas embedded in the student work, recognize where support may be needed, and devise a plan to extend student understanding.

Similar to Kurin, other participants also tended to first discuss the mathematics of the problem before they engaged with the student's work. Then, their final remark focused on their role as a teacher and how they could support the student in continuing to learn. This pattern of focusing on the mathematics, then the student, and finally the teacher, was evident across all participants' interviews. They typically discussed the mathematics of the problem before deeply engaging with the student work. Once the student work was carefully analyzed, participants then turned their focus to what they might do to build from the student's understanding to further student learning.

Through our efforts to understand, describe, and categorize the participants' knowledge for teaching quadratics, we came to see the participants' responses as answers to six questions that embodied their strengths-based orientation to teaching and learning. It should be noted that the participants were not asked these six questions, rather it was their strengths-based responses that led us to the formulation of the questions. Summarized in Figure 2 (see next page), these questions related to the participants' MKT and the uses that we documented in our study. Each question had a focus on the mathematics of the task (math), the student's understanding (student), and the instructional steps that participants might take to build from the student's understanding (teacher). In what follows, we describe each question and offer examples of how they describe the work of our participants when examining student work. We then discuss how

FIGURE 2.
Participants' MKT Uses, Questions, and Question Focus



mathematics leaders might use these questions to support teachers in approaching student work from a strengths-based perspective.

Six Strengths-Based Questions for Examining Student Work

In this section, we elaborate on each question by discussing the link between the question and the different ways that expert mathematics educators from our study understood mathematics for teaching.

The Content Question: What is the Solution?

This question focuses on the mathematics content covered in the problem and is aligned with one's content knowledge. Although answering the *content question* does not require reviewing the student written work, an understanding of the mathematical concepts involved in a solution is foundational to understanding students' mathematical thinking. Such knowledge assists teachers in making sense of the student work, while establishing the level of understanding that is demonstrated by the student.

In our study, participants used content knowledge foundational to quadratic functions to generate solutions to the problems prior to interpreting student work. This is seen in Kurin's study of Figure 1 when they stated, "Knowing about the different kinds of shifts and changes to functions based upon where you put coefficients, I would say it is the middle one..." As evident here, an understanding of the solution to the given mathematics problem and multiple ways to reach it is critical to discerning the mathematical understandings present in student work.

The Connections Question: What Mathematical Concepts Informed the Student Work? What Will the Student's Work Inform?

The *connections question* also focuses on the mathematics and requires content knowledge. However, this form of knowledge situates the problem within a broader mathematical landscape by connecting prior, current, and future ideas. This question focuses attention to the concepts needed to devise a solution to the problem and those that will be built upon the concepts developed through completing the mathematics problem. For instance, knowing how to complete the square informs how to write a quadratic function in vertex form, enabling the identification of the vertex and coefficients that are relevant to the function's average rate of change; understanding the relationship between function coefficients and the average rate of change informs creating mathematical models of quantitative relationships and the exploration of derivatives.

During their interviews, participants discussed the mathematical concepts that might have informed a student's work and how those concepts could inform their learning of new mathematics. For example, linking concepts that could inform the student's work in Figure 1, "Jamie" remarked, "if they understand that the slope of a line – if it's greater than one, then it's steeper and if it's less than one, then it's less steep, they can easily transition that into their understanding of parabolas." By considering mathematical connections across the secondary mathematics curriculum and beyond, teachers can develop instructional plans that build from, and connect to, what students know and understand.

The Interpretations Question: What Mathematical Understandings is the Student Demonstrating?

The *interpretations question* guides teachers to examine student work for evidence of what mathematical understandings the student is likely to have. This question shifts focus from the mathematics to the student and allows teachers to make assertions about the student's mathematical knowledge based on evidence from their written work. For instance, a teacher might recognize the values of the functions at their vertices and closed brackets in the student's intervals in Figure 1 and based on this observation, claim that the student knows how to describe sets of real numbers, determine the vertex of a quadratic function, and perhaps how to complete the square. Further, noticing the relationship between the vertex of the quadratic function and the given intervals may provide additional insight into the student's thinking. Highlighting what a student understands and thinks provides teachers an array of cognitive resources that might be useful in subsequent instruction. In addition, a focus on what students know and can do mathematically on a task may also specify what ideas the student has yet to learn.

In our study, participants carefully studied the student work before discussing the possible mathematical understandings represented. They contemplated the details of the student work and used them to support claims about what they believed the student was thinking. Knowing how students think, such as Kurin's claim that "they are looking vertical instead of looking horizontal", is vital to incorporating student perspectives into instruction.

The Conjectures Question: What Additional Understandings is the Student Likely to Have? What Should They Learn Next?

The *conjectures questions* maintain a focus on the student and encourage teachers to consider how students arrived at their solution and hypothesize about other understandings the student may have that are not reflected in their work. These inferences can then assist teachers in identifying what students should learn next. The *conjectures questions* allow teachers to formulate a more complete picture of what the student knows, which in turn helps them to prepare for instruction by considering the various questions, strategies, or difficulties that students may encounter. For example, a teacher who believes the student selected function c because -5 is the smallest of the three quadratic coefficients may conjecture that the student should next develop an understanding of how multiplying a function by various constants affects the rate of change of its values.

In our interviews, participants build from their interpretations to conjecture what mathematical understandings the student had and what new idea would most likely advance student learning. They discussed experiences with former students with understandings they believed to be like those reflected in the student work they were examining. For example, as "Cameron" analyzed the response in Figure 1, they stated, "our textbook has certain questions where they are given quadratic functions and they are asked what the intervals for which the function is increasing and decreasing. I think that is what this student is doing when I see their written intervals."

The Instruction Question: What Could I Do to Build on the Student's Understanding?

The *instruction question* shifts attention to the teacher and focuses on what teachers might do to continue or enhance student learning. Though it is likely the question that teachers ask themselves most, postponing an instructional decision until after considering evidence of what a student knows is an aspect of strengths-based teaching (Lopez & Louis, 2009). Rather than focusing potential instructional moves on "fixing" what is incorrect about a student's thinking, this question encourages teachers to leverage a student's knowledge as a foundation for their subsequent teaching. For example, Kurin's decision to graph the three quadratic functions and compare them at the same value builds from her assertion that the student understands the range of each quadratic function and an inference that the student had envisioned the graphs of the functions and was focusing on resulting parabolas vertically.

During our interviews, participants addressed what they would do only after carefully determining what knowledge was demonstrated in the student work and contemplating how the student could have arrived at their response. For example, initially Cameron thought the student work represented increasing or decreasing intervals, but after close examination, changed their observation and stated,

> Wait, on second thought, that interval notation, they are looking for the range. I might give this student some simpler functions. So, I think this student might just need a simpler set of functions to compare, to get the idea of width across. Once they can see that, then I would introduce more complicated functions.

The Resources Question: What Resources Would Be Helpful in Extending the Student's Understanding?

The *resources question* directs attention to the tools a teacher might provide to support students in using what they know to build a new understanding. When considering this question, teachers draw upon their knowledge of instructional materials and resources that promote student engagement and understanding, such as Kurin's idea of utilizing DESMOS to assist the student in determining the parabola with the greatest width by adjusting the quadratic coefficient.

Participants in our study discussed a variety of tools that they would use to build from or extend student understanding based on their interpretations and conjectures. They discussed graphing calculators, online applets and graphing tools, as well as open curriculum sources that they believed would advance a particular student's learning of quadratic functions. This is seen in "Jeremy's" remarks as they stated,

> I would encourage graphing on a calculator because I don't want the graphing to be the exercise. I need the graph to be the tool to show them what their misconception is. I would have them get out the graphing calculator or go to the DESMOS app. I wouldn't want them to graph it by hand - that's not the point here. We aren't teaching graphing; we are teaching the differences. In DESMOS, it allows sliders – with the slider value as the leading coefficient, have them change the value and see what happens.

These six questions that embody the work of teaching encourage a thoughtful and productive engagement with student written work. Collectively, the questions provide a strengths-based guide to uplift and build upon the mathematical ideas of the students, while simultaneously providing opportunities for teachers to reflect upon and expand their own knowledge relevant to the content, student, and teaching. The content and connections questions provide teachers opportunities to expand their knowledge of content and content across the curriculum. The interpretations and conjectures questions allow teachers to grow their understanding of student thinking, relevant to specific content. Finally, the instruction and resources questions push teachers to think about content specific instruction and tools, hence increasing the knowledge of both.

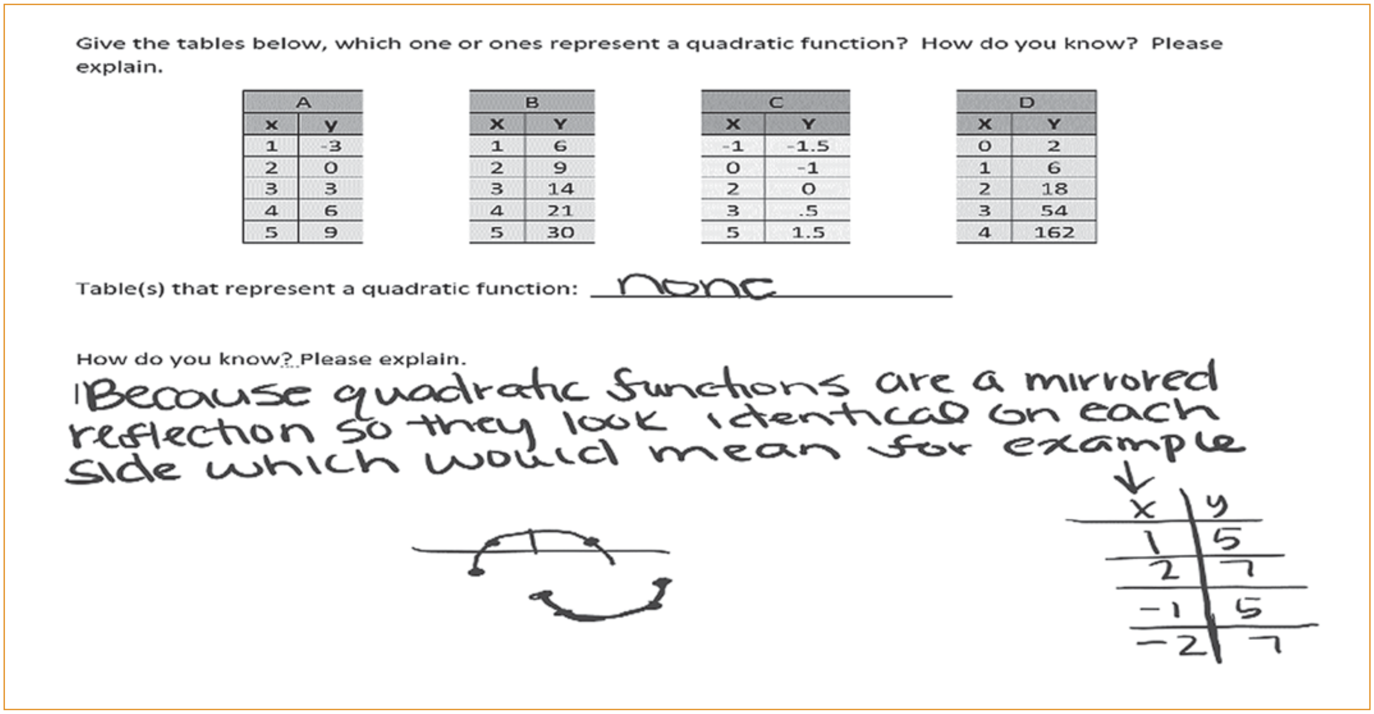
Utilizing the Six Strengths-Based Questions to Examine Student Work

In this section, we illustrate how mathematics leaders might use the questions to assist teachers in adopting a strengths-based approach to teaching mathematics. Consider a mathematics leader facilitating a professional learning community (PLC) of high school algebra teachers. Using the questions, leaders can support teachers in focusing on what students know and how to build upon their knowing. As a part of their regular review of student work, a leader might ask a PLC to consider the student work in Figure 3 by first focusing on the mathematics, then the student, and finally focusing on how they might support further learning.

Focusing on Mathematics with the Content and Connections Questions

How teachers think about a problem directly impacts how they interpret the student's written response. For example, thinking of quadratic functions as parabolas in the Cartesian plane might suggest a graphing approach for the problem in Figure 3, whereas thinking of quadratics as functions with a linear rate of change might lead to an examination of differences between successive differences in values of the range over consistent intervals of the domain. Therefore, before examining the specifics of the student response, leaders can encourage teachers to reflect on their solution, methods, and the mathematical ideas that precede and follow from the concepts by posing the *content and connections questions*.

FIGURE 3. *Student Work Sample 2*


Give the tables below, which one or ones represent a quadratic function? How do you know? Please explain.

A	
x	y
1	-3
2	0
3	3
4	6
5	9

B	
x	Y
1	6
2	9
3	14
4	21
5	30

C	
x	Y
-1	-1.5
0	-1
2	0
3	.5
5	1.5

D	
x	Y
0	2
1	6
2	18
3	54
4	162

Table(s) that represent a quadratic function: none

How do you know? Please explain.

Because quadratic functions are a mirrored reflection so they look identical on each side which would mean for example

x	y
1	5
2	7
-1	5
-2	7

By asking teachers to develop and share a solution using the content question, leaders can highlight the different ways of approaching the problem. For example, teachers might discuss looking for patterns of covariation, a strategy of examining first and second differences for linear and constant rates of change, identifying key function features in tabular representations, or creating a graphical representation. In facilitating a discussion where teachers publicize and discuss different approaches to solving the problem, leaders can create learning opportunities for teachers to enhance and extend their MKT for teaching quadratics.

After surfacing knowledge of quadratic functions useful in solving the problem, leaders might use the *connections questions* to shift teachers' focus to articulate prerequisite knowledge to the mathematical concept(s) required by the problem. Detailing the concepts needed to identify quadratic functions from tabular representations provides a foundation for insights into the gaps or connections that might be present in the student's work and an opportunity for the PLC to consider the prior knowledge needed to successfully complete the task. For example, teachers might discuss the importance of recognizing patterns to understand rates of change or identifying intervals of increasing and decreasing values of a function. Similarly, the connections questions can assist teachers in linking the mathematical concepts that are their current instructional focus to those that will be enabled by them in the future. Through discussions of how analyzing multiple representations and analyzing rates of change are essential for understanding function families in future courses for example, leaders can create opportunities for the PLC to deepen these understandings of vertical alignment in the mathematics curriculum and how their instructional choices build from previous learning and enable learning in the future.

Focusing on Students with the Interpretations and Conjectures Questions

Teachers can learn from the students' work through professional dialogue with their colleagues. However, the process of learning from student work requires a level of intentionality that moves the review beyond just being right or wrong. For instance, reviewing the student response in Figure 3 could result in the work being evaluated as incorrect. However, purposefully seeking out the mathematical ideas embedded in student work would reveal that the student knows that symmetry is a characteristic of quadratic functions, but possibly thinks that quadratics are only symmetric about the y-axis. Leaders can help teachers deeply engage with the student thinking demonstrated through written work by utilizing the interpretations and conjectures questions.

By asking teachers to attend to the mathematical understandings represented in student work using the *interpretations question*, leaders can facilitate a discussion of the various interpretations that teachers posit, while encouraging them to provide evidence of student knowledge and identify strengths in the work. Whereas some insights into student thinking may be shared across community members, such as the student knowing the graphical representation of quadratic functions, it is those insights that are uncommon that provide PLC members opportunities to broaden their understanding of student thinking and experiences, hence expanding MKT.

As the PLC works to interpret the mathematical meanings conveyed in the student work, leaders can introduce the *conjectures questions* to guide the PLC to theorize about prior instruction and experiences that could have influenced the student's response. Making inferences about student thinking based on evidence that are beyond what is presented in the written work can refine teachers' interpretations and may lead them to recognize why the solution made sense to the student. For example, careful review of Figure 3 reveals that the student can represent symmetry to the y-axis in both tabular and graphical form. Also, the student demonstrated a graphical representation that is not symmetrical about the y-axis. Hence, teachers might conjecture the student has not had the opportunity to analyze tables for quadratic functions where elements of the domain are not proximal to the vertex located on the y-axis. After exploring the mathematics represented in the student's work and then contemplating how the student arrived at their solution, leaders can then shift the discussion to considering how teachers might expand student understanding and further their learning.

Focusing on Teaching with the Instruction and Resources Questions

In PLC discussions, teachers have opportunities to consider different instructional choices that are appropriate for students based on the knowledge they have demonstrated in their written work. However, as diverse as student knowledge is, so are the instructional choices of teachers. PLCs are environments where teachers can learn from their colleagues by sharing what they would do based on the student's demonstrated understandings. Having multiple instructional paths in mind can help teachers ensure that the learning needs of all students can be addressed.

Using the *instruction question*, leaders can assist teachers in expanding their pedagogical knowledge by encouraging PLC members to consider and share different instructional choices they might take based on the work. For example, teachers may decide to have the student graph all of the tables in Figure 3 while others may want to encourage the student to determine the rates of change in the tables and extend the tables to see which functions have a local minimum or maximum. Identifying and evaluating different instructional moves with colleagues can assist teachers in determining how to build what students already know to meet their instructional goals.

As a PLC discusses instructional paths likely to be productive for students based on their written work, leaders can encourage teachers to consider what resources are available to scaffold student learning and to weigh their relative affordances and constraints. For example, using a graphing calculator to plot function values for each table might focus the student whose work is displayed in Figure 3 to examine changes in each function's average rate of change. While the use of a graphing calculator in this way might support a discussion of how quadratic functions have a linear rate of change, it is more difficult to build from the student's understanding of a quadratic function's line of symmetry. Alternatively, using a math action tool such as DESMOS or GeoGebra to generate dynamically linked multiple representations of a quadratic function in vertex form with sliders for its parameters would build upon the student's knowledge of symmetry but might not support a discussion of average rates of change. By facilitating PLC discussions around different pedagogical choices and resources, leaders can support teachers in expanding their instructional repertoires and their understandings of how to build from what students already understand.

Discussion

Building new understandings from current conceptions is a foundational principle of learning (National Academies of Sciences, Engineering, and Medicine, 2018; National Research Council, 2000). Examining student work provides an opportunity for teachers to identify the mathematical concepts that a student understands and consider how they might use them to support new learning. By focusing on what students know and considering them as strengths, teachers can create instructional experiences that build upon and extend student understandings.

The questions for examining student work presented in this paper provide mathematics leaders a guide for supporting teachers to engage productively with records of student understanding. Based on the ways accomplished mathematics educators analyzed student work in our study, the questions encourage teachers to draw upon and use their mathematical knowledge for teaching and, in doing so, create opportunities to learn from students. By encouraging teachers to identify what students know and consider ways that they use those understandings in future instruction, the questions also create opportunities for teachers to deepen their own mathematical knowledge for teaching.

We illustrated how mathematics leaders might use the questions to analyze student work in a PLC setting, but we believe the questions could be useful in other contexts such as one-on-one coaching cycles or developing common assessments. Over time and in multiple contexts, the questions can support teachers in developing a routine that first considers the mathematical content of a task, followed by focusing on evidence of student knowledge, and then articulating and evaluating instructional next steps. By considering student knowledge as an asset for teaching, the questions can support teachers in developing and strengthening not only their mathematical knowledge for teaching but also a strengths-based perspective on teaching and learning. ✪

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