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VERTICAL LESSON STUDY TO BRING COHERENCE IN PRIORITIZING STUDENT CONTRIBUTION AND VOICE

ONE CURRICULUM COMMITTEE'S PERCEPTIONS OF HIGH-QUALITY MATERIALS

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VERTICAL LESSON STUDY

VERTICAL LESSON STUDY TO BRING COHERENCE IN PRIORITIZING STUDENT CONTRIBUTION AND VOICE

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ABSTRACT

This article tells the story of a team of K-6 teachers who engaged in action research through Lesson Study to build equitable classroom structures through discourse-rich vertical tasks. Founded within the key recommendations of Catalyzing Change (NCTM, 2020), our community explored ways to prioritize student voice and distribute student mathematical contributions across more students within correlated patterning tasks.

Keywords: lesson study, equity, math discourse, vertical task.

The National Council of Teachers of Mathematics (NCTM)'s *Catalyzing Change in Early Childhood and Elementary Mathematics: Initiating Critical Conversations* (2020) called for the mathematics teaching community to engage in a conversation to bring equitable teaching practices to the forefront of our mathematics teaching and learning. An imperative component of this work is a school's shared and coherent vision of what equitable mathematics is and, importantly, how centering equity in mathematics makes students feel. An equitable mathematics classroom is one in which "Every child is capable of learning important mathematics with depth of understanding if provided with sustained opportunities that support children in reaching their full potential in mathematics" (NCTM, 2020, p. 14) by attending to how teachers and students position one another as capable of doing mathematics. Ideally, teachers specifically heed the content and vertical progression of standards to assess where students are, build their mathematical understanding, increase their confidence, and support their mathematical identities as doers of mathematics. *Doing mathematics* involves teaching practices that maintain high levels of cognitive demand for each and every student (Smith & Stein, 2011) and are grounded in mathematical discourse (NCTM, 2014).

Lesson Study (LS), as described by Lewis et al. (2012), is a cyclical investigation within a teacher-centered inquiry that uplifts teachers as co-researchers. Teacher members develop a community of practice (Robinson & Leikin, 2012) in which professional learning is grounded in a specific lesson and centers student thinking data with particular attention to how lesson components act in tandem with what students are doing or learning. The public and collective action of Lesson Study allows a team of educators to witness the lesson firsthand while providing a reflective space for critique and refinement (Lewis et al., 2012). To improve practice, Lewis et al. (2012) stress the importance of educators having the opportunity to observe their peers and take risks trying out new instructional strategies. LS allows for this space as the cycle includes both planning and implementing an intricately designed action research lesson with a post-lesson analysis of student learning (Shimizu & Kang, 2022). Ultimately, LS makes collegial and student thinking visible (Lewis et al., 2012). Schipper et al. (2022) report on research reviews that reveal LS as a professional development model that builds individual teacher knowledge and a positive mindset for mathematics teaching. We utilize the term "co-researchers" as an embodiment of all members of the LS Team—coach, teacher, and university partner—and to represent the deep levels of learning and practitioner research as we engaged in Lesson Study.

One of the prerequisites for a vision of teaching and learning for equity is ensuring teachers understand the mathematical content and processes to better assess and leverage students' strengths to advance their learning (Kobett & Karp, 2020). The research goal of this LS was to support equitable student participation through discourse to cultivate positive mathematical identities. Aguirre et al. (2024) conceptualize *mathematics identity* as "the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives" (p.14). The authors stress the importance of teachers recognizing the impact of each instructional decision on a child's identity and indicate the interconnectedness of equity work and classroom cultures that expand opportunities for young children to demonstrate competence (Aguirre et al., 2024). Thus, using the key recommendations set forth by *Catalyzing Change in Early Childhood and Elementary Mathematics: Initiating Critical Conversations* (NCTM, 2020), our learning process centered on creating equitable mathematics classrooms through 1) broadening the purposes of learning math, 2) creating equitable structures in math, 3) implementing equitable mathematics instruction, and 4) developing deep mathematical understanding. NCTM (2020) expands on the notion of *doing mathematics* through the processes and practices of:

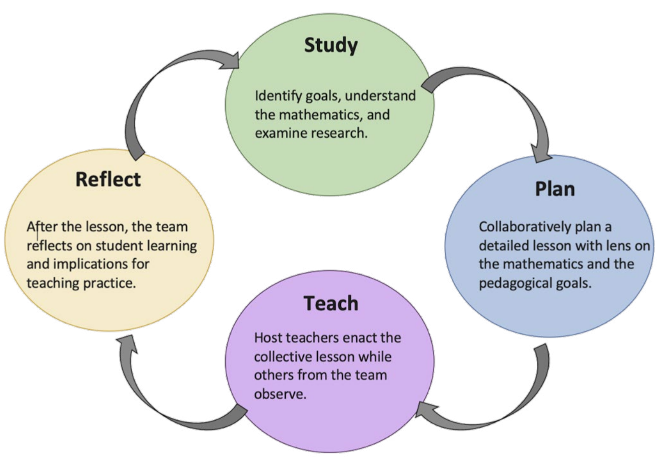
1. Representing and connecting,
2. Explaining and justifying,
3. Contextualizing and decontextualizing, and
4. Noticing and making use of mathematics structure.

This article presents a collaborative professional learning initiative endeavored by a team of kindergarten through grade six teachers. The co-authors writing this story include members of our team: a first-grade teacher (Amy), a third-grade teacher (Jenny), a fourth-grade teacher (Kaitlin), a sixth-grade teacher (Jacqueline), a mathematics coach (Holly), and a collaborating professor of mathematics education (Jennifer). Our team came together after Holly and Jennifer first discussed the idea of engaging in a vertical Lesson Study (LS). Holly reached out to teams to see if one (or more) members from each grade level might be interested in meeting after school for a few weeks to learn together and observe one another. Five teachers agreed (representing grades K, 1, 3, 4, 5, 6) to participate as a vertical learning community and spent the subsequent month using a LS model (Lewis, 2002; Suh et al., 2019) to study, plan, enact, and debrief lessons that promoted mathematical discourse and illuminated equity-centered elements to catalyze change. The LS team met after school weekly throughout the spring semester and had the opportunity to push into each host classroom to observe during the LS cycle.

THE VERTICAL LESSON STUDY CYCLE

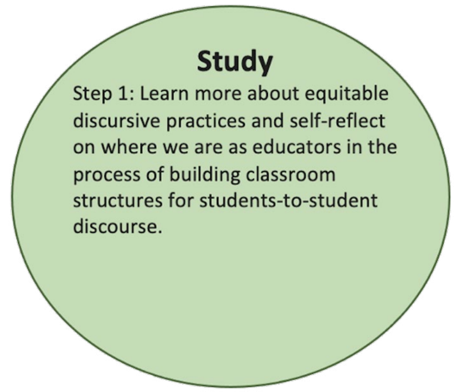
In Figure 1, we detail the protocol our vertical community used, focusing on equity and the key recommendations set forth by NCTM (2020). In the text that follows, we include detailed examples of our team's experience within each of the protocol's steps and its impact on our mathematical teaching and learning.

Figure 1
Protocol for vertical lesson study grounded in discursive practices



Note: Adapted from *About Lesson Study*, by The Lesson Study Group at Mills College, 2022, https://lessonresearch.net/about-lesson-study/what-is-lesson-study-2/.

Study: Creating and Implementing Equitable Mathematics Instruction



The LS process creates a community of vulnerability (Suh et al., 2021), allowing teachers to anchor learning in their wonderings shared as a collective unit and to strengthen their teacher mathematical identity (Aguirre et al., 2024; NCTM, 2020). Vertical LS provides opportunities not only to deepen teachers' and coaches' understanding of the development of children's mathematical knowledge through the analysis of student work but also allows for the vertical team to work toward a common teaching practice and bring coherence school-wide collectively (Suh et al., 2019). Lewis (2015) describes LS as improvement science, in that educators "choose an improvement aim, agree on how they will recognize improvement, identify the changes that might procedure improvement and test these changes in LS cycle" (p. 57). In this way, in the "Study" phase of the Lesson Study, the team identifies a problem of practice, in this case a student goal around increasing participation through discourse and learning about discursive practices.

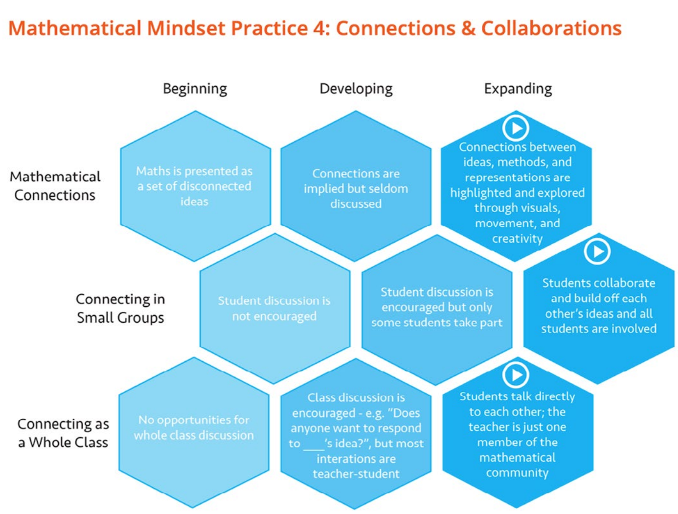
By placing the educators at the forefront of the LS "study" goals and research, they were empowered to explore areas of mathematical pedagogy that were meaningful to them as instructors of mathematics. With this model, we established a teacher community (NCTM, 2020; Robinson & Leikin, 2012) of collaborative mathematicians.

In our initial meeting, we learned collectively about the effective teaching practices outlined by NCTM, and the teachers chose to focus on, "Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments" (2014, p. 29). Our team chose to attend to this principle because it enabled us to consider how our classrooms could construct spaces with opportunities for students to engage in sense-making and deep reasoning. This notion is central to NCTM's first key recommendation, broadening the purposes of school math by "developing deep mathematical understanding as confident and capable learners" (2020, p. 11). Positive and discourse-rich classrooms allow each student to feel successful and proud (NCTM, 2020). Strong discourse structures elicit students' ideas and strategies, creating an equitable space for students to interact with their peers and value multiple contributions. Hierarchical status among students, such as differences in "smartness" or ability perceptions, diminishes (Zavala & Aguirre, 2023). Thus, we intended the LS would highlight the use of discourse to consider how students position one another as capable mathematicians, allowing our LS team to explore routines that make discourse an expected and natural part of mathematical thinking and reasoning (Aguirre et al., 2024). Students in this space are confident enough to ask questions and engage in mathematical argumentation, which enhances their mathematical learning. In sum, our research goal was to bring intentionality in our discursive practice to prioritize student voice and distribute mathematical intellectual contributions (Aguirre et al., 2024) across more students.

After choosing the focus on discourse, teachers asked questions when considering the principle—What is mathematical discourse? What isn't it? This open dialogue led to exploring researched-informed resources. The coach and the math educator provided resources to consider, such as a chapter from the *5 Practices for Orchestrating Productive Mathematics Discussion* (Smith & Stein, 2018) and three key functions in facilitating meaningful discourse (Staples & King, 2017) in this more exploratory phase of studying.

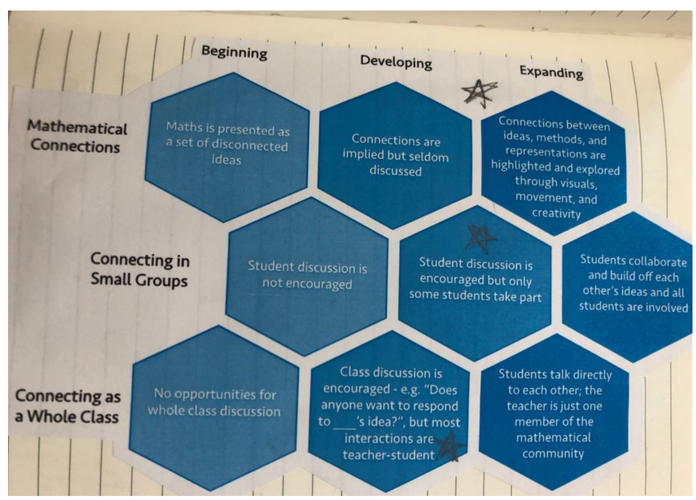
One teacher, drawing on the district's use of Jo Boaler's (2016) book *Mathematical Mindsets*, decided to further explore Youcubed (n.d.). Youcubed is a website designed for teachers, parents, and students, with many ready-to-go resources often used throughout our district-level curricula resources. Our district had recently supported teachers in taking the virtual Teacher's Course that focused on the brain, productive struggle, and how to create classroom communities that do not dichotomize children into those who "can" and "can't" (Boaler, 2016). Boaler indicates "responsibilities" of an equitable classroom, one inclusive of working on groups where "different thinkers are helped, both by going deeper and by having the opportunity to explain work, which deepens understanding (p. 138)." Within the Youcubed (n.d.) website, the teacher discovered "Hexagons for Mathematical Mindsets" rubric-like visuals designed to be reflective and non-evaluative tools (Figure 2) (Youcubed, 2018).

Figure 2
Rubric used to self-assess discursive practices in the math classroom (Youcubed.org, 2018)



The hexagon self-assessment was designed for teachers to gauge their practice using rubrics around Mathematical Mindset Practices. Our LS Team loved the idea of taking the Youcubed site's (2018) advice of using the rubrics to "understand where you are now" and "consider where you want to be" around the Mathematical Practice of "Connections and Collaboration" (p. 2), as a good fit for this practitioner study as our team examined both the literature and our current reality (Knight, 2016) of discursive practices. In the subsequent LS meeting, the coach and the math educator asked teachers to engage in self-assessment, situating their classrooms within the rubric of "Math Connections," "Connecting in Small Groups," and "Connecting as a Whole Class" (see example in Figure 3) (Youcubed, 2018, p 4). "Math connections" included a progression for the presentation of the mathematics itself, whether as a disconnected set of ideas or in a rich way that included visuals, creative strategies, and a structured experience for students to make connections. "Connecting in small groups" ranged from classroom contexts where very little discussion occurred to mathematics dependent on student collaboration and ideas in small groups. "Connecting as a whole class," similarly, focused on opportunities as a whole class for students to build off each other's ideas as monumental in a mathematical community.

Figure 3
Teacher sample of self-assessment in researcher journal



Across these categories, our context's "beginning" stages included worksheets or procedure-based games as a part of a guided math structure. Oftentimes, noted in the discussion from LS Teachers, students had very little opportunity to talk deeply about mathematics in these kinds of situations, with true understanding of the concepts through flexible and connected strategies. Further, many math centers/stations were either silent stations or ones where students might interact to talk about their answer, but not about their strategies and how strategies connected. It seemed there were fewer spaces where teachers opened opportunities for children to build mathematical understanding *together*. Every teacher self-assessed around the range of "developing," (Figure 3) with one teacher indicating that they were mostly "beginning" this journey. We realized through the self-assessment that a focus on discursive practices was a prime way to focus our learning to create and implement equitable structures as our LS goal.

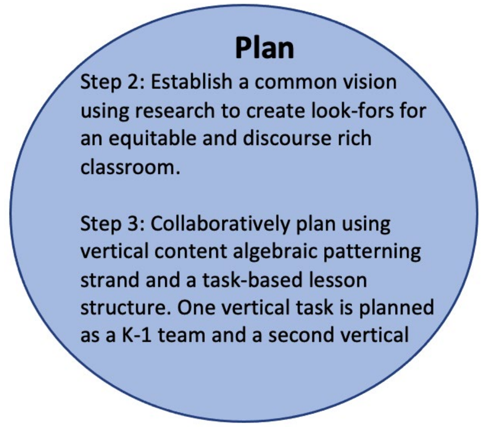
From there, teachers began unpacking their wonderings about facilitating meaningful mathematical discourse. They journaled, posing specific questions about their practice as they pondered what moving forward in the hexagon rubric would mean. For example, a teacher journaled about her wonderings and curiosities regarding student discourse sharing, "I would like to improve student-to-student discourse and ensure that all students are engaged in discussion and thinking. What happens to student learning when students are doing most of the talking? I am really curious about how I can get students to do this." As teachers enacted the other phases of the Lesson Study, this rubric acted as a way for them to reflect on their personal goals and questions within the larger LS context. Teachers continued to journal over the course of each LS meeting in response to their initial questions as they worked to move towards "expanding" discourse connections individually. In this way, the "study" of oneself occurred throughout the entirety of the Lesson Study.

During the "study" phase of LS, our vertical team continued to investigate action steps for each co-researcher's personal goals grounded around student discourse and collaboration. We spent two sessions in the "study" phase delineating how we would increase meaningful discourse in ways reflective of the themes that emerged from teacher learning goals. The conversation included discussions of equity and what it means for every student to have an opportunity to investigate mathematics deeply— Is this happening in classrooms currently? How often? What do the conversations sound like and how deep are the conversations? We considered how discourse is not just a "show and tell" but explicit in developing a shared meaning of mathematical ideas (McGatha & Bay Williams, 2018). Through our discussion "meaningful and equitable discourse" was defined as efforts to prioritize student voice and distribute mathematical contributions across more students through student to student discourse.

This notion of observing and listening to student thinking instead of relying on high-stakes assessment supported the second key recommendation from NCTM (2020), creating equitable structures, where the organization advocates assessment as a method of gathering evidence of children's mathematical thinking to inform learning and teaching. In reflecting on our next steps, teachers indicated that our school had classrooms of children who may not have seen themselves as "math people." Therefore, our plan to embed mathematical discourse needed to send the message, "You are a mathematician." In summary, our "study" phase consisted of the following learning processes, guided by the team:

1. We built common language and background knowledge around equitable and effective math pedagogy.
2. identified a target practice to explore in depth as a community.
3. continued to build common language and background knowledge about the target practice.
4. reflected on current practices and set goals for the LS.

***Plan:* Broadening the Purpose of Learning Mathematics**



In the "plan" phase of Lesson Study, teachers selected a rich task and planned for discourse using a task-structure (Smith & Stein, 2011) format. Since rich tasks were newer to several members of the LS team, the coach and math educator pulled together a bank of tasks from NCTM as a jumping off point to this selection. Since the team realized that current classroom structures were not always meeting the needs of cultivating deep mathematical discourse, as facilitators we created a lesson plan template to support teachers in thinking through planning for a task. The plan required thinking through a launch, monitoring student thinking in small groups and through purposeful questions, and selecting student groups to share their ideas in connection to the math goal (Smith & Stein, 2011; Van de Walle et al., 2019) (See Appendix B).

Planning for Discourse

During this phase, the LS team utilized our learning and reading from the "study" phase to create a list of crucial practices we deemed necessary as part of a mathematical community for equitable discourse. Teachers first worked in partnerships to brainstorm look-fors based on the previous sessions' readings (e.g., NCTM, 2020; Smith & Stein, 2011; Staples & King, 2017; Youcubed, 2018), investigations, and self-assessments. We then looked across the lists for themes, grouping ideas together and narrowing down to five important look-fors, which we called "Key Practices to Create an Equitable Discourse-Rich Classroom": (1) Opportunities for student-to-student discourse (2) students explaining their own mathematical thinking (3) students commenting on the mathematical thinking of their peers, (4) students using sentence frames to support their discourse and (5) students asking each other questions. As a team, we embedded these look-fors into a checklist, deciding it was also necessary to include a space for anecdotal evidence (see Appendix A). The anecdotal space not only encouraged LS members to take detailed notes, but it also gave our debrief sessions a more vibrant and evidence-based approach as teachers were able to draw on specific instances that stood out in classrooms. We utilized the discourse monitoring tool from Appendix A in all our LS classrooms, K-6, as a learning tool for our team.

While monitoring student discourse was an essential learning component across all grade levels, facilitation of student discourse based on the mathematical content varied in our kindergarten and first-grade classrooms compared to our upper-level classrooms as seen across the vertical progression of standards. In the primary grades, patterning started with repeating patterns and simple growing patterns, then progressed to connecting multiplicative patterns to linear functions by sixth grade. Thus, our next steps included making two important decisions: first, collaboratively planning instruction around a rich task; and second, making sense of student engagement with vertical mathematical content.

NCTM (2020) expresses the urgency for catalyzing change by broadening the purpose of learning mathematics. Students should recognize mathematics as a beautiful and creative study through the real-world discovery of concepts in meaningful instruction. To aid in the planning for a classroom grounded in students' creation of mathematical ideas, a general task-based plan, adapted from Smith et al. (2020), assisted teachers in orchestrating classroom discussions by posing questions of how they might introduce a task, allow for independent think-time, consider purposeful partnering for sharing ideas, and connect student strategies to culminate the task (Figure 4). Grade-level bands planned within this cycle together, in tandem with the task-based lesson planning template, while also explicitly considering the discourse monitoring tool.

As we considered the purposeful partnerships, manipulatives and purposeful questions that encourage students to describe their strategies and ideas, we considered questions to discover children's thinking. The teams developed open-ended questions to embed in the tasks, such as:

- What did you do to start the problem?
- Can you tell me more about that?
- Why did you choose to…?
- Is this a pattern? How do you know?
- How does your pattern relate to the multiplication table?
- How is your pattern growing?
- How are these connected?

Our team also considered how we might purposefully partner students based on their strategies and solutions to enhance peer discussions. Here, teams brainstormed sentence frames to support the necessary peer connections that students would need to make if partnered with someone strategically based on their strategy. Several classrooms began using the sentence frames before our LS implementation so that students would be familiar with the stems. Examples of the sentence frames included:

- I agree with _____ because…
- _____'s is connected to ______'s because…
- I see the pattern growing by….

Figure 4
Process of planning and implementing mathematical tasks

Note: Adapted from *5 Practices for Orchestrating Productive Mathematics Discussion,* (Smith & Stein, 2011).

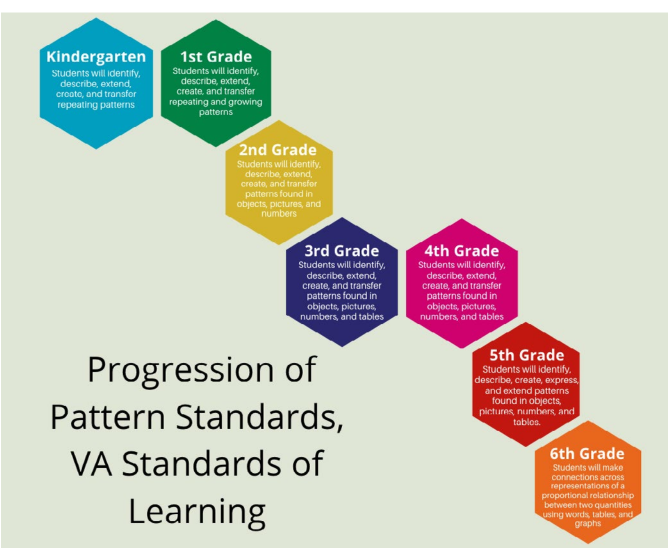
Anticipating student responses (Smith & Stein, 2011) allowed us to be intentional in the connections between strategies and mathematical ideas. For example, knowing how a kindergartener or first grader might build a pattern created an opportunity for teachers to recognize relationships among different strategies to understand what kinds of big mathematical ideas might arise from the chosen tasks to structure possible partnerships and whole class reflections. This type of planning was different for teachers than other kinds of instruction they had done in the past as they shifted from delivering procedural skills to noticing and naming various strategies and emphasizing the discourse of connections between strategies. For several of the teachers, this was the first time centering the students as the doers of mathematics, where students would not be working towards a single solution. Instead, the heart of the task would be in student thinking, analyzing, discourse, and reflecting while the teacher facilitated this open space.

Planning for Vertical Mathematics Concepts

An additional key recommendation for catalyzing change in early childhood and elementary classrooms includes the development of deep mathematical understanding (NCTM, 2020). To determine the best content strand for a vertical LS in tandem with the focus on discourse for meaning-making, the team looked across the standards in the district pacing for the semester to find content commonalities across the pacing timelines, landing on the strand of Patterns, Functions, and Algebra (PFA). PFA also worked for the team as the choice of vertical articulation because we felt that it had access points for us as mathematics educators to understand the mathematics in a cohesive manner from kindergarten to 6th grade because of the connections within patterning that could be seen as students moved from repeating patterns, to growing patterns, to multiplicative reasoning in patterns in different forms, to ratio tables and proportional reasoning. The learning progression was examined from kindergarten through sixth grade, as seen in Figure 5.

Figure 5

Progression of pattern standards from kindergarten to sixth-grade, Virginia Standards of Learning (2016)



Next, our collective broke into grade-band teams to examine their specific standards more closely: a kindergarten and first-grade team and a 3rd through 6th-grade team. Choosing a task that targeted our learning goal of providing students with many opportunities for discourse allowed for deep discussions about the nature of the problems chosen by the grade bands. To have discourse we, inherently, would need a mathematical space where students could talk about mathematical ideas and strategies. We wondered:

- What kinds of tasks provide opportunities for access and scaffolds but also for extension to even deeper levels?
- How can we best adapt a task to meet each grade's patterns, functions, and algebra learning targets?
- What do we anticipate students doing with the task, and how will that help us better understand their mathematical understanding?

Primary task. Figure 6 shows the kindergarten and first-grade task, created by the team to highlight the many kinds of repeating patterns children could make. Using the Kindergarten and First Grade pattern standards (see Figure 5), the team hoped to open a range of possibilities for creating a repeating pattern, but also connected to real world context. The team wondered about where in real-life primary students might see patterns, and what might excite them to create a pattern and decided on a task that had students create a bulletin board border. Kindergarten and first graders could create any repeating pattern they wished, as long as it repeated at least three times to create the border The task also included a "missing part" as an extension, with the hopes of extending student thinking to consider what a possible core of the pattern could be to fix a "torn-down" part of a bulletin board. The team was very intentional in putting in the purple shaded box to increase the level of complexity of the pattern and to open possible solution strategies.

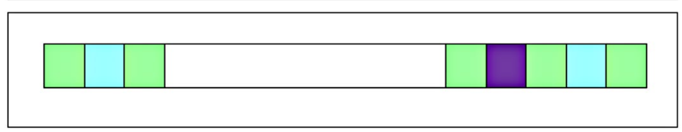
Figure 6

Kindergarten and first-grade task

It's time to make a new border for our classroom bulletin board! Your pattern must repeat at least 3 times! Choose any tool you'd like to create your border.

Can you create another border with at least three attributes?

Some of the bulletin board border has been torn down! Can you figure out what parts of the pattern are missing?



The kindergarten and first-grade team anticipated an initial default to ABC patterns and students having difficulty identifying a different type of pattern that still had three attributes, but brainstormed others such as AABC, ABAC, and ABCC as potential cores students might explore. Additionally, teachers wondered about students' creation of patterns outside of using colors for the core of the repeating pattern. We extensively discussed what part of the pattern we might "tear down" from the bulletin board. Do we cover one complete repetition? Ultimately, we decided that it might lead to more interesting conversations and student use of strategies if we "tore down" part of the bulletin board pattern that started mid-core, where students might not be able to see the whole core of the pattern from the first term, instead determining a potential core in later terms. After making final adjustments to our planning process and considering the intersection of the task, patterning content, and student mathematical discourse, the kindergarten and first-grade teachers were ready to implement.

Intermediate task. The third through sixth-grade team chose a task that connected the third-grade content of multiplication and growing patterns with sixth-grade content of linear functions and proportionality (see Figure 7). As mentioned in the “study” phase, our school district had begun to embed several task resources into our curriculum and schools, so the intermediate team began with Boaler’s (2018) Mathematical Mindset task books. In the third-grade version, Jenny noticed that a task called “Tile and Table Patterns” (Boaler, 2018, p. 222-230) connected growing patterns to multiplication, which would be beneficial to the third graders in her classroom. The task required students to connect a growing “tower pattern” to a hundreds chart by drawing arrays, extending the pattern and using creative color-coding to investigate the relationships between rectangles and numbers on the chart. Jacqueline pointed out that this type of growing pattern was also a linear function, and that the linear relationship could actually be seen as students connected multiplication arrays with a hundreds chart. The vertical team recognized the transition from “repeating pattern” foci in kindergarten, to “growing pattern” foci in the older grades. We also had interesting planning discussions about what actually “repeats” in growing patterns and linear functions, as there is a common repetition of a rule that causes a multiplicative relationship to occur. Teachers were curious to see how an eight-year-old might access or think about this task similarly or differently from an eleven-year-old.

The intermediate team anticipated that students would begin to create patterns that grow in multiples, for example, 3 x 1, 3 x 2, etc., and grow by doubling. The sixth-grade teacher noted that students may have a misconception about connecting this type of thinking to a ratio table and considered the types of additive thinking that might develop instead. For example, a student might create a ratio table that adds up by threes and also create sums of numbers representing what should be the number of rectangles (see Figure 7), indicating a disconnect between the student’s understanding of ratio tables and multiplicative reasoning. We also wondered about students drawing their patterns starting in different places in the multiplication table and how that would impact their understanding of the connection between the two. Further, the team discussed the language students might use when considering their growing patterns: Do they use multiplicative or additive language? Are they beginning to generalize an overall rule or describing from term to term? Anticipating student responses to the chosen low-floor, high-ceiling task allowed our intermediate LS group to take the next step in collaboration to reach their professional learning goal for student discourse, particularly in the moment of teacher. Now, they would know what to look for in order to elevate student-to-student discourse opportunities in small and whole-groups.

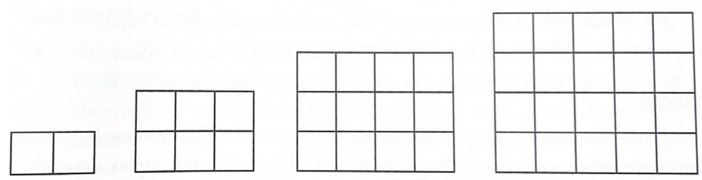
Figure 7

Third, fourth, and sixth-grade task

Tile and Table Patterns (Boaler et al., 2018)

Snapshot
Students create growing patterns with rectangles and then locate those rectangles on the multiplication table to see what new patterns this reveals.

Activity	Time	Description/Prompt	Materials
Launch	10–15 min	Show students the tower pattern and ask what patterns they see. Discuss how the pattern grows. Then ask them to locate these rectangles on the multiplication table. Discuss what patterns they see now, and predict where the next rectangle in the pattern will be in the table.	• Tower Pattern sheet, to display • Multiplication Table sheet, to display
Explore	30–40+ min	Partners create their own growing rectangle patterns on grid or dot paper and then locate their rectangles on the multiplication table, extending the pattern as far as it goes on the table. Partners try this with different patterns, color-coding them on the multiplication table and making observations.	• Grid or dot paper (see appendix), multiple sheets per partnership • Square tiles • Multiplication Table sheet, at least one per partnership • Colors
Discuss	20 min	Partners present the most interesting pattern they created, along with where it is located on the multiplication table. Discuss the patterns student see in the multiplication table, and generate questions that students are wondering about now.	• Multiplication Table sheet, to display • Colors



Tower pattern showing horizontal and vertical growth

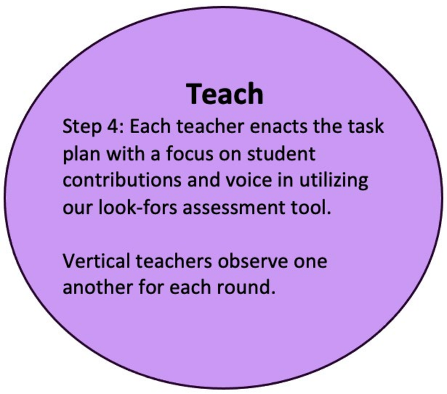
10	10	20	30	40	50	60	70	80	90	100
9	9	18	27	36	45	54	63	72	81	90
8	8	16	24	32	40	48	56	64	72	80
7	7	14	21	28	35	42	49	56	63	70
6	6	12	18	24	30	36	42	48	54	60
5	5	10	15	20	25	30	35	40	45	50
4	4	8	12	16	20	24	28	32	36	40
3	3	6	9	12	15	18	21	24	27	30
2	2	4	6	8	10	12	14	16	18	20
1	1	2	3	4	5	6	7	8	9	10
x	1	2	3	4	5	6	7	8	9	10

A 2 x 3 rectangle is shown with a point representing the area at the number 6. A 5 x 6 rectangle is shown to have an area of 30.

In planning for discourse paralleled with vertical mathematical content, the LS team created task-based lessons that would offer rich opportunities for students to participate in mathematics in ways they may not have had before. In summary, our "plan" phase consisted of the following learning processes, guided by the team:

1. We used our research from the "study" phase to plan a monitoring look-for chart;
2. Identified and adapted tasks from a bank of resources, including those from the district;
3. Anticipated student responses to the tasks to better understand the mathematics; and
4. Created a task-based lesson plan for implementation.

***Teach:* Connecting Deeply with Mathematical Understanding**



In the "teach" phase of LS, teachers enacted their selected tasks while the co-researcher team observed, utilizing the discourse monitoring tool. Each member of the team used the monitoring tool, focused on children's actions and language, while in the host-teacher's classroom. Each teacher from the LS team received an invitation to observe the other teachers on the LS team. So, in our model, every teacher was an observer and every teacher was a host.

Across the enactments of the lessons, we noticed the ways that students were doing mathematics across NCTM's (2020) key recommendations. The co-design team noticed that our observations of the ways students engaged in *doing mathematics* aligned with NCTM's (2020) vision in that students engaged in 1), representing and connecting, 2) explaining and justifying, 3) contextualizing and decontextualizing, and 4) noticing and using mathematical structures. In the following sections we utilize these practices as a way to frame what our LS team noticed within the "teach" phase. We employ these processes and practices to structure discussion of the discursive practices that our LS team noticed within the "teach" phase.

Representing and Connecting

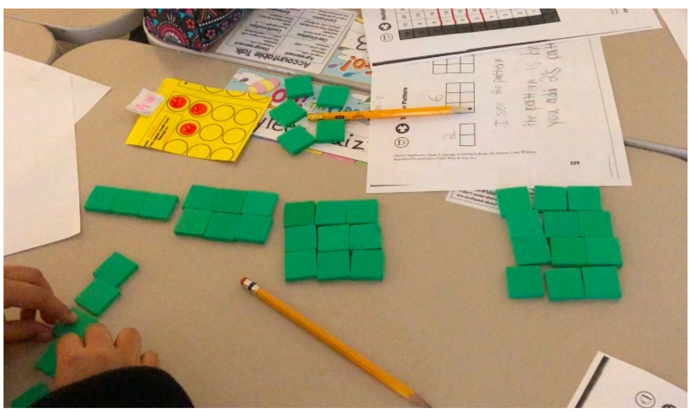
We observed students engaged in *doing* mathematics in various ways beyond simply solving an "extend this pattern" problem. Students represented their work with multiple representations and connected the different models of patterns. When given a variety of tools to use, we saw students' thinking and solutions in different ways. For example, Figure 8 shows a kindergarten student working on transferring patterns. By using the manipulatives, they made a direct transfer of this pattern, first using the same colors as is seen on the paper pattern. Importantly, teachers created a manipulative-rich environment, and because students had access to a variety of tools they were able to transfer the pattern in a new way. We see the child using the same ABCD core, but now with new manipulatives and then colors. On the bottom row, the student recreates the blue, purple, green, orange core as orange, red, yellow, tan. It's also clear that this student focused on the color as a pattern, rather than the type of manipulative being important because they used a red square and a red trapezoid to both represent the "red" part of the core which indicates their focus on the attribute of colors.

Figure 8
Kindergartener transfer of patterns across mediums

The openness of the task instilled mathematical agency as students chose strategies that made sense while justifying their reasoning to their peers. The variety of strategies allowed the teacher to purposefully partner students who were solving in different ways so that they could learn from one another through discussing how their representations of the bulletin board border were the same or different In the moment, the host teacher decided to partner students who had similar cores but used different materials to make the core. Kaitlin indicated in her anecdotal notes that it was powerful to see how this student justified the ABCD core to another child who was not sure that the mix of manipulatives still represented the pattern because in their pattern, they used all unifix cubes. By noticing that his partner's pattern was different than his through the connecting prompts, mathematical questions occurred: Do you have to always use all the same materials to represent a pattern? Or can we still see the core? It also provided space for the teacher to connect the small group discussions with the whole group's reflection on mathematical thinking and strategies to move beyond just "extending" the pattern to thinking deeply about the core and what exactly is repeating by facilitating conversation around several student representations and continuing to ask, how are these the same? How are they different? How do they all show us a repeating core?

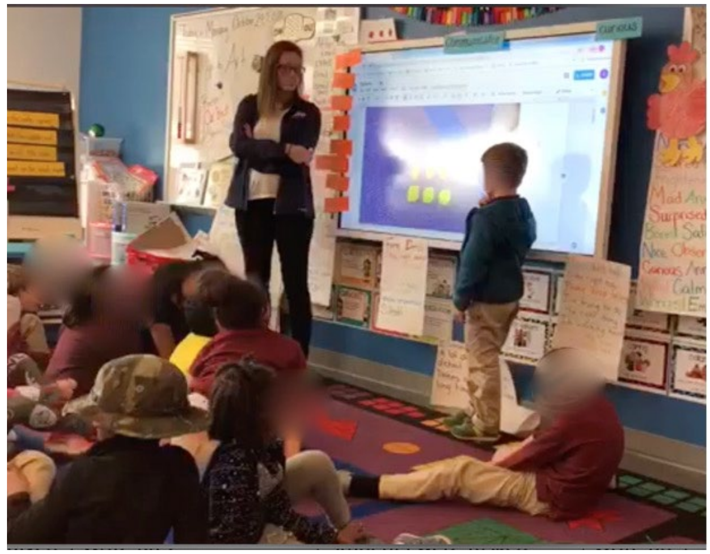
When students can choose to represent their math thinking with different tools, children make essential connections and generalizations. In the third-grade classroom, children used tiles to recreate the growing tower pattern (Figure 9). After creating this pattern with tiles, collaborative pairs worked to transfer their growing patterns to a multiplication table, which allowed students to make important connections between the physical tool of tiles and the abstract drawings of the representations. As students connected these two different representations, conceptual understanding developed as students were able to understand how the numbers in the multiplication chart connected to the total number of physical tiles and the equal rows and columns indicative of the multiplicative relationship.

Figure 9
Third grade students recreate tower pattern with tile



Explaining and Justifying
Much of our notes and discussions during the "teach" phase focused on the ways students moved between explaining and justifying to figure out the *mathematics* with a partner, to then explaining and justifying their group's mathematical models and ideas in a whole-group setting. A first-grade child, for example, shared how he and his partner saw a unique pattern representation with the class (Figure 10). He said, "We saw blue-blue-blue, orange-orange-orange, yellow-yellow-yellow." The teacher asked a very purposeful and pre-planned question from the lesson plan, "does that make this a pattern?" The student thought about his reasoning during a whole-class turn-and-talk and decided, no, it does not after bouncing some ideas off of his turn-and-talk partner. Then, he looked at the pattern from another angle and said, "I changed my mind. I see it this way: yellow-orange-blue, yellow-orange-blue." Because he explained his thinking and had to follow up to justify it, he was able to revise his mathematical ideas alongside his peers and express with more precision where to find the core of the pattern.

Figure 10
First grade student explains a unique mathematical noticing to his class



Similarly, a fourth-grade child in Figure 11 justified her mathematical ideas to her classmates by explaining her thinking about the growing pattern in the intermediate tiling task. Other students made connections, asked clarifying questions, and added to her thinking. As this student explained and justified her thinking, she represented the pattern in different colors, stating, "I noticed a pattern with the numbers in yellow. Those numbers are how many tiles are in each figure of the pattern." What she was noticing was that the entire quantity of the arrays could be captured by the upper right hand number in the multiplication table because of the relationship between the row number and column number. She also indicated that each square that she built was inside the larger the squares because the highlighted yellow number fell diagonally below the larger square number. The host teacher decided to open the conversation to the students listening from the carpet. The teacher asked, "What ideas do you have about her mathematical noticing and how she represented the diagonal?" After a turn-and-talk to reflect on the representation, comments from other students included, "I didn't see it that way. Now I understand why those numbers are important to the pattern." This student was doing mathematics while justifying her thinking through explanation and representation.

Figure 11
Fourth grade student justifying what she noticed about the growing pattern to the class

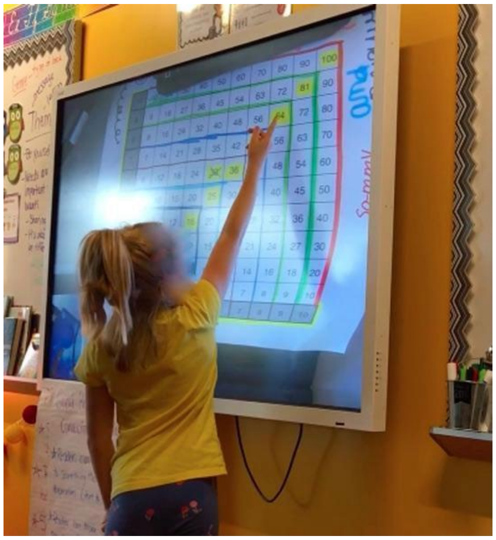
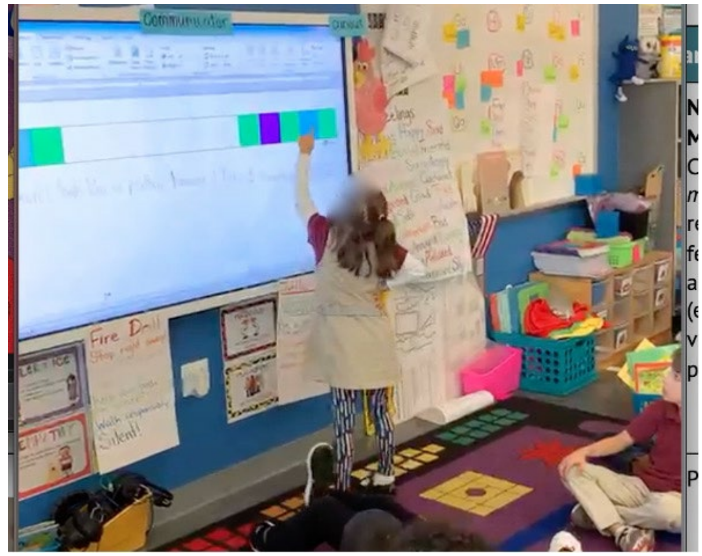


Figure 12
First-grade student explaining what would make the most sense for a bulletin board border



Contextualizing and Decontextualizing

We often noticed that students explained the ways that they were fluidly moving between the contexts, representations, and sense-making. In a first-grade class, the host teacher prompted partners to discuss their patterns by noting, "I noticed you both decided on different patterns for the missing part of the bulletin border. Could you all talk about that? Can they both work?" One partner explained what she thought the "hidden" pattern might be, while another student countered her ideas (see Figure 12). An observing teacher recorded the peer conversation on their monitoring tool.

Student A: The pattern could be blue, green, blue, green, then change in the middle to include the purple.

Student B: I don't think so. Normally bulletin boards have the same paper all the way across the bottom. I don't think it would change.

They likely engaged in mathematical argumentation because of their interest in the concept of the problem--- not simply identifying cores of given patterns in a low-level task, but instead were interested in solving the "mystery" of the torn down bulletin board. Because they were using the context of a bulletin board border to make sense of patterning, they were able to use this type of reasoning to better make sense of the mathematics and the core of a pattern. The students recognized that, most often, a bulletin board pattern has the "same paper across the bottom," which pushed them to have conversation about what would make the most sense as the continuing pattern.

Providing students with tasks where they can make mathematical connections (such as between a growing pattern and a multiplication table) and are interested in solving the problem creates high student engagement and collaboration. For example, a fourth-grade partnership engaged in mathematical discourse about the problem while questioning ideas and building a shared understanding of the pattern while connecting to the more familiar context of multiplication (see Figure 13). The fourth grade host teacher noticed that Leo was shading just the growing number pattern for counting by 2s (i.e. 2, 4, 6, 8), while Max was creating the rectangle arrays for the pattern. She wondered about these two ideas and decided to partner them together so they could discuss.

Max (pointing to the hundreds chart): So this is like multiplication. Like when we count by twos or fives. You know when we skip count and that's multiplying?

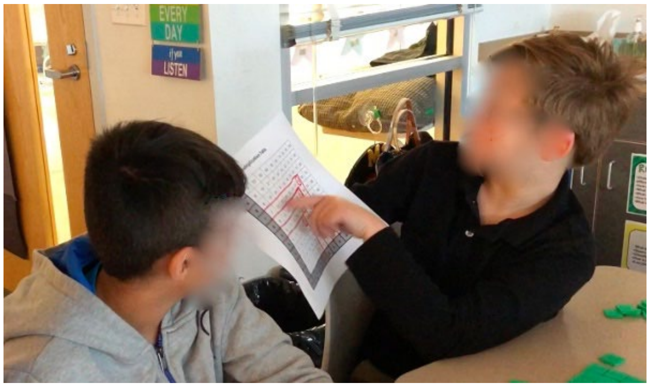
Leo: Wait. But I see 2, 4, 6, and 8 on this chart. But I don't see the multiplication.

Max: See how I made these rectangles. That shows the 2 x 4 which is the 8. But it's getting bigger and following this pattern.

Leo: So there's the two more (points to the larger rectangle which represented the array for 10).
Max: Yeah and it equals 10, so it makes this line across.

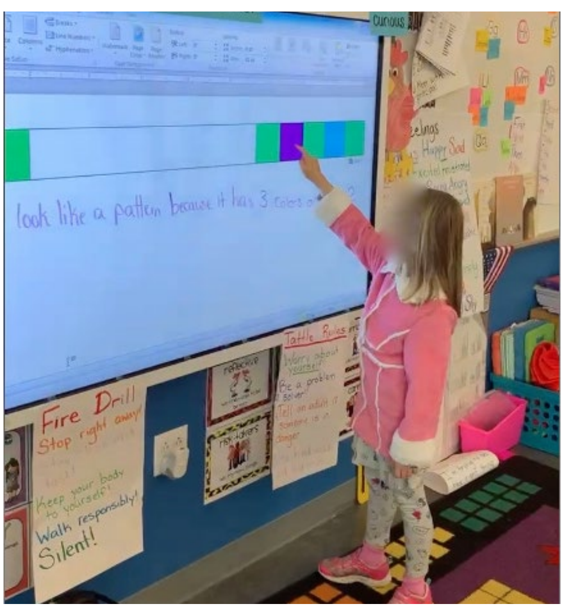
With further conversations, the multiplication chart, a common tool for the students, acted as the vessel for the partnership to notice the "line" created if you follow the diagonal up the page when exploring different square arrays. The group noticed that it seemed to stretch one row "up" and one column "out" to create the diagonal line.

Figure 13
Fourth-grade students used the hundreds chart pattern to recognize a diagonal line



Noticing and Using Mathematical Structures
Across our observations, we noted how children from kindergarten to sixth grade discussed mathematical structures and used them to justify strategic thinking. For example, a first-grade student used her math vocabulary to explain her thinking (Figure 14). She acknowledged that the core of the torn-down pattern appeared unfinished but applied her knowledge of patterns to develop a solution. "I see that green-blue-green is at the beginning, and I see it again over here. I think that this purple comes at the end of the core." She used the structure of patterns to establish her solution. Another child incorporated math vocabulary into her explanation, noticing that there *had* to be a core for it to be a repeating pattern, stating that the class needed to "decide what the core is." The class debated what that extra purple box meant for the core and how it might play out with the space missing in the pattern, using their understanding of repeating cores to make sense of this new and challenging situation.

Figure 14
A first-grade student used her understanding of "core" to decide where the purple might fall



During the whole-group reflection on the growing patterns task, fourth-grade students also came together to discuss key mathematical ideas. An important part of the LS team's collective lesson was specifically choosing student work to compare mathematical ideas with the whole class. The fourth grade teacher invited several students to share how they figured out what kinds of growing patterns existed on the multiplication table. One student explained how her model was different from another group's model because they created the towers horizontally, while the partnership she worked in represented the towers vertically on the hundreds chart (Figure 15). The teacher elevated this moment in the conversation as a noticing of mathematical structures asking, "Can we represent the growing pattern either way? Why or why not? What does this help us to understand about multiplication?" This student's noticing led to conversation revealing early conceptualizations of the commutative property through array models.

Figure 15
Fourth-grade child noticed that a group created a horizontal model of the pattern, which contrasted with her vertical representation

Across these examples from the "teach" part of our Lesson Study, students represented patterns while connecting models to deeper conceptualizations of patterning all related to the opportunities for small group and whole group discourse. Discursive practices emerged in sense-making around how models connected to the core of the pattern or how much each figure grew, and children generalized their thinking to find ways to extend the pattern. With a wide range of strategies and ideas discussed during small group thinking, the teacher facilitated a whole group discussion with diverse strategies and focused on the math context of connecting patterns to the multiplication chart. As students began to explore patterns within a real-world context, they looked at the problem as a whole and isolated the needed information, noticing and using pattern structures, which stood out to our LS team. As host teachers facilitated our group lesson plan, we collected much information that helped us to see that *how* we implement a task or lesson is just as important to *what* is in the lesson itself. Our "teach" phase consisted of the following processes:

1. Each grade level teacher (K, 1, 3, 4, 5, 6) hosted an enactment of the lesson in their classroom while other members of the LS team observed;
2. The observation team utilized the discourse observation tool to notice and note how students engaged in discursive practices.

***Reflect:* Valuing Student Contributions and Promoting Discourse**

Reflect

Step 5: Reflected on each enactment/round of task implementation and final reflection on the Lesson Study process for continuing our journey in building equitable mathematics classrooms. Teachers also reflected in their journals throughout the process.

A critical component of our vertical LS learning was continual observation, reflection, and revision of our educator perspectives and mathematical pedagogies. While the K-1 and 3-6 grade bands initially planned independently of each other (though collaboratively within the bands), our observations happened across all grade levels. Kindergarten and first-grade teachers had the opportunity to observe upper-grade instruction, and vice versa. We were able to gain a deeper understanding of the progression of the pattern standards by seeing student engagement, discourse, and problem-solving in action. After observations, the group engaged in a collective reflection. In the "reflect' phase, teachers discussed their collaborative work and evaluated ways to enhance their lesson and approach.

We followed the "reflect" protocol proposed by The LS Group at Mills College (2022) which began with taking time to look over data. Data artifacts included student work samples from the tasks and the discourse monitoring tool with anecdotal notes, along with any other notes that the team felt was important to our learning. We used this data in conducting our post-observation discussion, letting the host teacher speak first, then the rest of the team, to talk about what stood out about our collective lesson plan and the facilitation in the classroom. We ended our reflective sessions by consolidating our learning to think about what we wanted to tweak before the next iteration of LS observation and what we wanted to carry into our daily practice. Looking back on the process, teachers had a chance to recognize their growth and how the process impacted their own practice as mathematics teachers, both through the structure of LS itself in becoming reflective practitioners committed to growing and in the process of building discourse-based, equity centered classrooms.

Lesson Study as a Chance to Become a Reflective Educator
Teachers indicated throughout our reflective sessions and within their journal entries that the actual process of engaging in a vertical LS grew them as reflective and learning-centered practitioners. When asked how LS impacted her learning, Jacqueline shared how observing lessons and peers across different grade levels was powerful in helping her to recognize teacher moves that might elicit student strategies and ideas,

I was amazed at the valuable insight I gained about my own practice by observing another teacher in action from a grade level two years below mine. By focusing on student discourse and the strategies used, I was able to watch other teachers implement strategies in their own unique ways and contemplate ways in which I could implement/adapt those strategies with my own students at the sixth-grade level.

Further, the consistent journaling throughout the LS process helped teachers reflect on how their planned questions enhanced student discourse. Jacqueline acknowledged how writing down questions about her own practice helped to guide the things she looked for in classrooms,

First, writing down my questions of how I could implement strategies to increase student discourse, get students sharing their strategies/thoughts, and have students responding and reflecting on their peers' strategies helped me to hone in on these ideas and create a solid plan for implementing them.

Jacqueline went on to explain how journaling immediately following a lesson or debrief helped her to make visible the negotiation of teacher facilitation and children's discourse in mathematics. Embedded in the Lesson Study, teachers journaled both in the moment as they were observing and as part of our debriefing. One shared, "Reflecting in my journal after a lesson helped in recording my immediate thoughts-what worked well, what helped students to talk, and what steps I needed to take next so that they weren't lost in the shuffle of all the other teacher tasks." Not only did journaling as a reflective practice carry on in Jacqueline's teaching, she also found it beneficial for her students.

I felt the benefit of writing my thoughts, questions, and summaries of the lessons so greatly that it is actually a practice I implement for my students as well, giving them time at the end of a lesson or activity to write in their interactive notebooks about their own reflections of the math learning taking place.

Interactive writing and journaling, a form of written discourse, provided time and space for both teachers and students to make sense of their learning stemming from an emphasis on mathematics discourse during the task.

LESSON STUDY AS A CHANCE TO PRIORITIZE STUDENT VOICE AND DISTRIBUTE MATHEMATICAL CONTRIBUTION THROUGH STUDENT TO STUDENT DISCOURSE

Keeping a focus on the common vision of creating an equitable discourse-rich classroom from the planning stage allowed for teachers engaged in the Lesson study to work on prioritizing student voice and contribution through student to student discourse. Jacqueline looked back on her journal entries to see how the notion of her role as a mathematics learning facilitator shifted over the course of the lesson study. She indicates how, as LS "teach" and "reflect" sessions progressed, she witnessed the transfer of students moving from passive receivers of math knowledge, to creators and contributors of math knowledge. This captures a huge goal of our LS as we grew as a team to create spaces where children could contribute and have a voice that brings more ownership in their learning there by developing positive mathematical identities (Aguirre et al., 2024),

> *In one journal entry I wrote, 'the best part is that my students are formulating conceptual ideas for themselves!' I witnessed (and continue to witness) students developing a deeper understanding of concepts through engaging with one another, sharing strategies, being able to talk through and then building upon their own understanding. My role, as a facilitator, is to showcase student thinking to help students conceptualize, connect, and discover math. The impact this had in my classroom is seen in the increased level of engagement in my students, students' confidence in their math abilities rise, and my favorite part, the noise level in my classroom.*

Jenny also explained how observations of vertical classrooms impacted her mathematical knowledge for building a discourse-rich community. Jenny noted the need for student-to-student discourse, especially as children built models to represent their thinking, and discussed how she went even further to create more scaffolds to access peer discourse.

> *Following each of my observations, I was able to revisit, adjust and improve my planning of the lesson and eventually the delivery. I saw the benefit of conversation in connection with building a model. I decided to create sentence starters that would provide students ways to tell about their thinking. I also had sentences starters of ways to ask questions in response. This created a conversation about each pattern. My goal was to hear the students not just share their own ideas, but to really challenge their classmates to talk through their thinking. This influenced students to talk more and to reach a level of mindfulness of their why.*

Through the probing questions and sentence stems, she allowed for students to work on their explanations and justification and to orient students towards other students' thinking. In this way, the discourse moved beyond just increasing student voice to distributing intellectual ownership across multiple students. LS not only led to reflection within the moment, but propelled teachers into thinking about their daily practice and what might come next within their specific contexts and future tasks. Further, Jenny noted,

> *My classroom is a place that fosters conversations. I find myself speaking more with questioning sentences and less with telling sentences. I encourage my students to also use questioning sentences to learn from each other. I purposefully plan to observe my students and encourage cognitive self-awareness.*

As teachers reflected both within and after the LS sessions, they became increasingly aware of not only the benefit in creating discourse-rich environments, but also the ways students learned mathematics deeply--- pivotal shifts towards creating and implementing equitable structures and reaching towards NCTM's (2020) key recommendations. In summary, we found two major benefits of LS as a professional learning experience and efforts in increasing student discourse. First, LS is a way for teachers and teacher leaders to hone their practice by learning from the community. Teachers observed and integrated new to them strategies such as sentence frames, and consistently reflected on their growth as educators through the process. Second, LS allowed teachers to really focus on their goal of creating discourse rich environments. Highlight the various takeaways they had about HOW they learned to do this through the reflection process. I think the idea of "forward thinking" and planning is a really important success to highlight as you close this section. We cannot always do lesson study, so we want the impact to go beyond the single lesson itself and to create a ripple effect.

Next Steps and Implication for Lesson Study Implementation to Catalyze Change

Lesson Study and the learning process around student discourse documented shifts in teachers' practice and thinking that reflect more equitable mathematics experiences by increasing student voice as well as distributing intellectual contributions across more students. Together, we were able to practice forming discourse-rich classroom communities and employed our new knowledge to determine the next steps for the classrooms in our school. Our individual and collective reflections allowed us to refine the tasks for other iterations to deepen our knowledge of what it means for students to *do mathematics* through the lens of NCTM's key recommendations (2020). One of the prerequisites for creating a more equitable learning space where student math thinking is honored and mathematics learning is distributed and experienced by each and every student is ensuring teachers better understand notions of "doing mathematics" and "student discourse," which is what the vertical LS structure afforded to teachers. Teachers who understand both the mathematics and discursive structures for topics they teach are better able to provide rich opportunities in which students can engage. Our patterns task gave every student an entry point to engage in mathematics at their own level, but everyone's thinking was elevated due to learning as a collective. Creating space to connect the ideas that emerged in small groups with the whole group discussion allowed the teacher to assign competence in mathematical ideas. The robust array of strategies and diversity of thinking allowed the class to explore the mathematical concept's intricacies further, engage in one another's mathematical knowledge, and empower them with access as the knowers and doers of mathematics.

Without the LS opportunity, entrenched with peer observation, reflecting questioning, planning, and revising, teachers may not have had such a robust and job-embedded learning experience. LS provided a safe professional learning opportunity to bring the essential effective PD components (Desimone, 2009) which include 1) content focus—activities that are focused on algebraic thinking and how students learn that content 2) active learning: opportunities for teachers to observe, receive feedback, analyze student work 3) coherence: content, goals, and activities that are consistent with the school curriculum and goals, teacher knowledge and beliefs, the needs of students, and school 4) sustained duration: PD activities that are ongoing and 5) collective participation: groups of teachers across grades to participate in PD activities together to build an interactive learning community.

Engaging in a vertical LS gave us the opportunity to think beyond the learning targets of our particular grade levels, providing a vision of what a mathematical concept looks like across multiple grade-levels. Knowing what comes in the future, and what students learned previously, can establish a more holistic understanding of mathematical teaching and learning for teachers and, consequently, the students in their classrooms. Looking along a vertical content progression of patterns, functions, and algebra allowed us to connect students' mathematical knowledge to equitable discursive practices to find strength in student sense-making and understanding. The possibility of a vertical LS requires a commitment to teacher development and professional learning, as well as the continued opportunity for collective planning, observation, and reflection.

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Appendix A

Checklist Used for Lesson Study Assessment of Student Discourse

Evidence of Student-to-Student Discourse

Evidence	Y/N	Notes
Opportunities during task and whole-class reflection for students to engage in meaningful discourse.		
Students explaining their mathematical thinking.		
Students commenting on their peers' mathematical thinking.		
Students using sentence frames to talk about their peers' thinking (e.g.): ● I agree because… ● I disagree because… ● Their thinking is similar to mine because… ● I wonder about… ● I notice that…		
Students asking each other questions.		

Appendix B

Lesson Study Lesson Plan Template (adapted from Van de Walle, et al., 2019)

Team Members:	
Lesson Date:	
Grade Levels:	
Title of Lesson:	
Research Theme:	
Rationale for choosing the topic:	

Task Lesson Plan

Title of lesson:

PART 1: PRIOR TO IMPLEMENTING THE MATHEMATICAL TASK; Van de Walle et al., "Before Phase" (10-15 min)	
Essential Knowledge and Skills: Content and Cognitive level of standard(s)	
Essential Question	
What are your mathematical learning goal(s) for the task? **Include behavior and conditions.**	
Criteria for mastery and formative assessment tool—how will you monitor student understanding as they engage in the task?	
What task will you use for the lesson? Provide a short description of the task. ● What tools will students have available to them? ● What additional scaffolds might you provide?	
Anticipating: *Work through the problem yourself.* Describe possible **anticipated student solutions and strategies** that may occur during the task, as well as **possible misconceptions**. *-What will students do or say that lets you know how students are thinking about the mathematical ideas?* *-What will children be doing or saying if they understand or if they do not understand?*	

Launching the task/lesson (activator): Address each of the teacher actions for the Before Phase of the lesson plan. How will you: ● activate prior knowledge? ● be sure the problem is understood? ● establish clear expectations?	

PART 2: DURING TASK IMPLEMENTATION; Van de Walle et al., "During Phase" (30-50 min)	
Monitoring How will you monitor and keep track of key strategies used by students during the task? Which strategies will you monitor? *(This will connect with your selecting plan below.)* Create an observation chart or checklist or other monitoring tool that is connected to the EKS (formative assessment). *-What should the observer pay attention to as the children work and talk?* *-What is the evidence that students understand the mathematics?* What questions will you ask to— ● help students be organized to work on the task while you are free to confer? ● help a student get started or make progress on the task? ● focus students' thinking on the key mathematical ideas in the task? ● informally assess students' understanding of key mathematical ideas, problem- solving strategies, or the representations? ● move students to think deeply about the mathematics and advance students' understanding of the mathematical ideas? ● provide appropriate support without taking over student thinking? ● provide appropriate extensions?	

PART 3: SHARING AND DISCUSSING THE TASK; Van de Walle et al., "After Phase" (10-20 min)	
Selecting What possible solutions do you want shared during discussion? Explain. (These are the strategies you monitored.)	

Sequencing In what order do you want to present the student work samples? Consider your mathematical goals and learning trajectories. How will: ● students share their work? ● you listen actively without evaluation? ● you sequence who will share to highlight your mathematical goals of the lesson?	
Connecting (Summarizer): What specific questions will you ask so that students will: ● make sense of the mathematical ideas that you want them to learn? ● expand on, debate, and question the solutions being shared? ● make mathematical connections among different strategies that are presented? ● summarize main ideas and identify connections to future mathematics problems?	