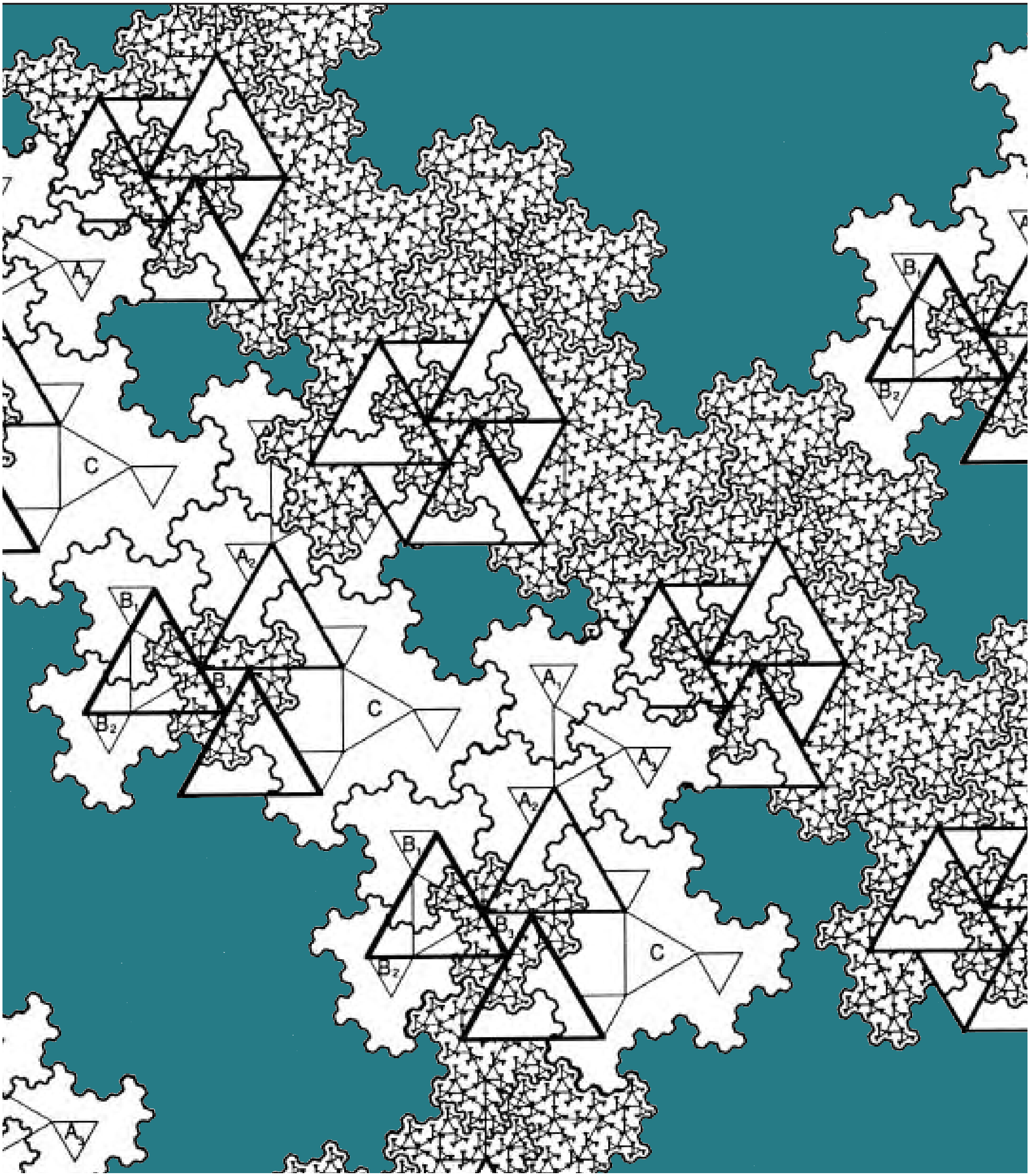
NATIONAL COUNCIL OF SUPERVISORS OF MATHEMATICS

NCSM JOURNAL

of Mathematics Education Leadership

VOLUME 7, NUMBER 1

SPRING 2004



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Looking Inside the Classroom: *Mathematics Teaching in the United States*

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Mathematics education has been in the spotlight for some time now. Over the past fifteen years, out of concern that an overemphasis on computation and algorithms had led to a misrepresentation of the discipline of mathematics, the National Council of Teachers of Mathematics (NCTM) has produced a series of national standards documents (NCTM, 1989, 1991, 1995, 2000). These documents make a case for more attention to problem solving and conceptual understanding as hallmarks of quality mathematics instruction. However, there continue to be differences of opinion about the extent to which mathematics instruction should be directed by the teacher and/or instructional materials, with some mathematics educators viewing guided discovery as appropriate, and others defining problem solving as only those instances in which students are engaged with open-ended questions for which they devise their own approaches.

Given the time required for instruction based on open-ended problem solving, some mathematics educators worry that students will not have opportunities to learn many important mathematics ideas. In some cases, use of hands-on activities, manipulatives, calculators, and real-world contexts has been equated with problem solving. Critics argue that using manipulatives or technology without rigor is far from mathematical; and that much of the problem solving that takes place in the discipline of mathematics remains a mental exercise, often without specific applications to real-world situations. In addition to these disagreements, some mathematicians, educators, and parents favor more direct instruction focused on explication of procedures and concepts followed by considerable practice on skills and applications. Within these differing stances regarding the best instructional approaches, there is a broader consensus that mathematics instruction is best when it aims at student understanding, not only understanding of mathematics disciplinary content, but also understanding the essential role of problem solving in mathematics as a discipline.

Very little information was available, until recently, about the extent to which teaching for understanding characterizes instruction in the nation's mathematics classrooms. Much of the information that exists on classroom practice comes from large-scale survey data. A strength of surveys is their capacity to provide information on the extent to which a variety of instructional strategies are being utilized, but they lack the capacity to describe the quality of instruction (Burstein et al., 1995; Mayer, 1999; Porter et al., 1993; Spillane and Zeuli, 1999).

A quarter century ago, the Case Studies in Science Education (Stake and Easley, 1978), a national observation study involving a cross-section of 11 U.S. school districts, described the conditions and needs of science, mathematics, and social studies education. The researchers reported that the mathematics instruction students experienced was quite varied in quality; while some of the observed mathe-

This report was prepared with support from the National Science Foundation under grant number REC-9910967. These writings do not necessarily reflect the views of the National Science Foundation.

matics classes stressed important concepts and were described as interesting to students, most overemphasized facts and memorization and were not seen as relevant to the students. Mathematics education observation studies since that time have generally either been quite small, or have been conducted in the context of the evaluation of a reform initiative, in both cases limiting the generalizability of the results.

The *Inside the Classroom* study provides new insight into the extent to which teaching for understanding is occurring in our nation's mathematics classrooms, complementing the self-report data on teacher preparedness and frequency of various instructional strategies, e.g., lecture, available from the 2000 National Survey of Science and Mathematics Education (Weiss et al., 2001). The study included observations of 184 mathematics lessons in 90 schools, selected to be representative of lessons nationally, as well as interviews with the teachers of those lessons. This article shares findings about the national status of quality mathematics instruction and the components of lessons that seem likely to promote student understanding.

Methodology

The study design for *Inside the Classroom* drew upon the nationally representative sample of schools that had been selected for the 2000 National Survey of Science and Mathematics Education. A subset of middle schools from the schools that participated in the 2000 National Survey was selected. To ensure that these sites would be as representative of the nation as possible, systematic sampling with implicit stratification was used. When a middle school agreed to participate, the elementary schools and high school(s) in the same feeder pattern were identified and one of each was randomly selected. Two mathematics teachers were then randomly selected from each school for classroom observations.

Observations were conducted by experienced mathematics educators trained in the use of the "Inside the Classroom Observation and Analytic Protocol." Researchers were asked to take detailed field notes during the observation, including describing what the teacher and students were doing throughout the lesson, and recording the time spent on various activities. Following the observation, the researcher interviewed the teacher about the lesson, focusing on why the particular content and instructional strategies had been selected.

Researchers completed the protocol using the data collected during the observation and interview, documenting the nature and quality of the observed lessons in a number of different areas, including the accuracy and developmental appropriateness of the mathematics content and the extent to which the classroom culture facilitated learning. The lessons were ultimately assessed on the extent to which they were likely to impact student understanding in mathematics and develop their capacity to "do" mathematics successfully.

The completed protocols were reviewed for clarity, comprehensiveness, and consistency by a senior Horizon Research, Inc. mathematics education researcher, and revised by the observer as needed. Data from the analytic protocols were weighted in order to yield unbiased estimates for all mathematics lessons in the nation. The weighted estimates of the frequency of classroom practices based on *Inside the Classroom* data are generally equivalent to those based on the 2000 National Survey sample, suggesting that estimates of lesson quality based on the observation data are an accurate depiction of what happens in the nation's mathematics classes.

The Quality of Mathematics Lessons Nationally

Inside the Classroom researchers rated the observed lessons on individual indicators in a number of areas, e.g., the quality of teacher questioning. Following the rating of individual components of the lesson, researchers were asked to provide an overall rating of the lesson. The scale observers used is divided into the following levels:

Level 1: Ineffective instruction
- a. passive "learning"
- b. "activity for activity's sake"

Level 2: Elements of effective instruction
Level 3: Beginning stages of effective instruction (low, solid, high)
Level 4: Accomplished, effective instruction
Level 5: Exemplary instruction

Lessons judged to be low in quality (those rated 1a, 1b, and 2) are unlikely to enhance students' understanding of important mathematics content or their capacity to do mathematics successfully. While low quality lessons fell down in numerous areas, their overarching downfall tended to be the students' lack of engagement with important mathematics. Examples of low quality lessons included:

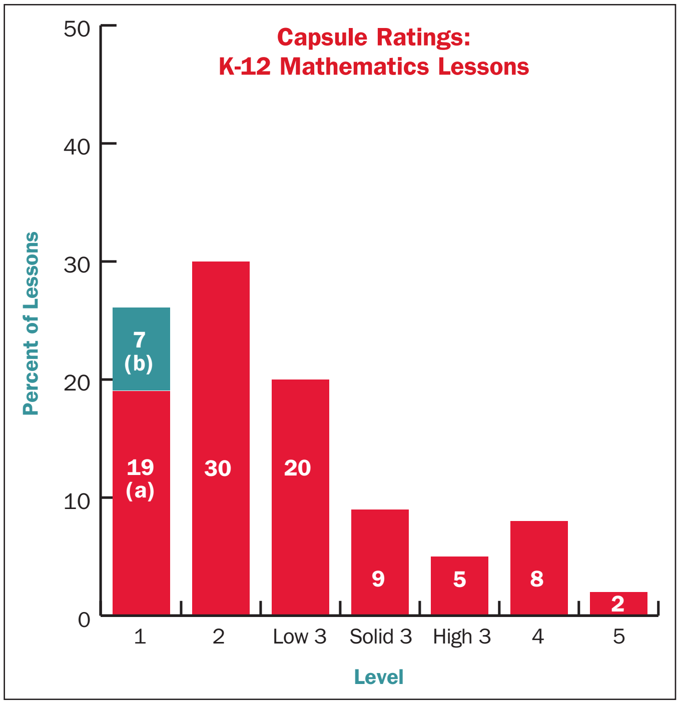

Figure 1

- A mathematics class where students spent most of the time playing a mathematics-related game with no attention to the mathematics concepts implicit in the game; and
- A mathematics lesson in which the primary purpose was to learn algorithms without concern for the meaning of the concepts represented by the algorithms.

At the other end of the scale, high quality lessons (those rated high 3, 4, and 5) were designed and implemented to engage students with important mathematics concepts; they were very likely to enhance their understanding of these concepts and to develop their ability to engage successfully in the processes of mathematics. Regardless of the pedagogy (e.g., investigations, teacher presentations, reading, discussions with each other or the teacher), high quality lessons provided opportunities for students to interact purposefully with mathematics content and were focused on the overall learning goals of the concept. Examples of high quality lessons included:

- A 3rd grade class where students worked individually on mathematics problems, with the teacher circulating and asking challenging questions to help them articulate their thinking.
- A middle school mathematics lesson where small groups of students developed strategies to find the volume of irregularly shaped objects and shared them with the rest of the class; and
- A lecture in an advanced placement calculus class, where the teacher derived the general exponential growth and decay formula and provided examples of how the formula was applied in the growth of bacteria populations.

Other lessons were purposeful and included some elements of effective practice, but also had substantial weaknesses that limited the potential impact on students. The specific areas where "middle quality" lessons fell down varied. Examples included:

- A lesson where the teacher spent a substantial amount of time describing the context of a problem, leaving too little time for the students to engage with the rich mathematics in it;
- A lesson where the teacher posed good questions, but moved ahead as soon as any student gave a correct answer, without checking if others were understanding; and
- A discussion that involved high-quality ideas, but was too fast-paced for many of the students.

Data from the *Inside the Classroom* study indicate that most mathematics lessons in the United States are low in quality, with a general lack of teaching for understanding. As can be seen in Figure 1, based on observers' judgments, only 15 percent of K–12 mathematics lessons in the United States would be considered high in quality, 29 percent medium in quality, and 56 percent low in quality. In the high quality lessons, students were fully and purposefully engaged in deepening their understanding of important mathematics content. Some of these lessons were "traditional" in nature, including lectures and worksheets; others were "reform" in nature, involving students in more open inquiries. In contrast, in the low quality lessons, which included both traditional and reform-oriented lessons, learning important mathematics would have been difficult, if not impossible.

Detailed analyses were conducted in order to learn more about the characteristics that distinguished lessons that seemed to promote student understanding from those that did not. A number of factors emerged, including the extent to which the lesson was able to engage students with the mathematics content; create an environment conducive to learning; ensure access for all students; use questioning to monitor and promote understanding; and help students make sense of the mathematics content.

Effective Lessons Provide Students with Opportunities to Grapple with Important Mathematics Content in Meaningful Ways

Certainly one of the most important aspects of effective mathematics lessons is that they address content that is both significant and worthwhile. Lessons using a multitude of innovative instructional strategies would not be productive unless they were implemented in the service of teaching students important content. Based on the lessons observed in this study, mathematics lessons in the United States are relatively strong in this area, with 69 percent of lessons judged to include significant and worthwhile content. (See Figure 2.)

Lessons Receiving High Ratings on Selected Indicators

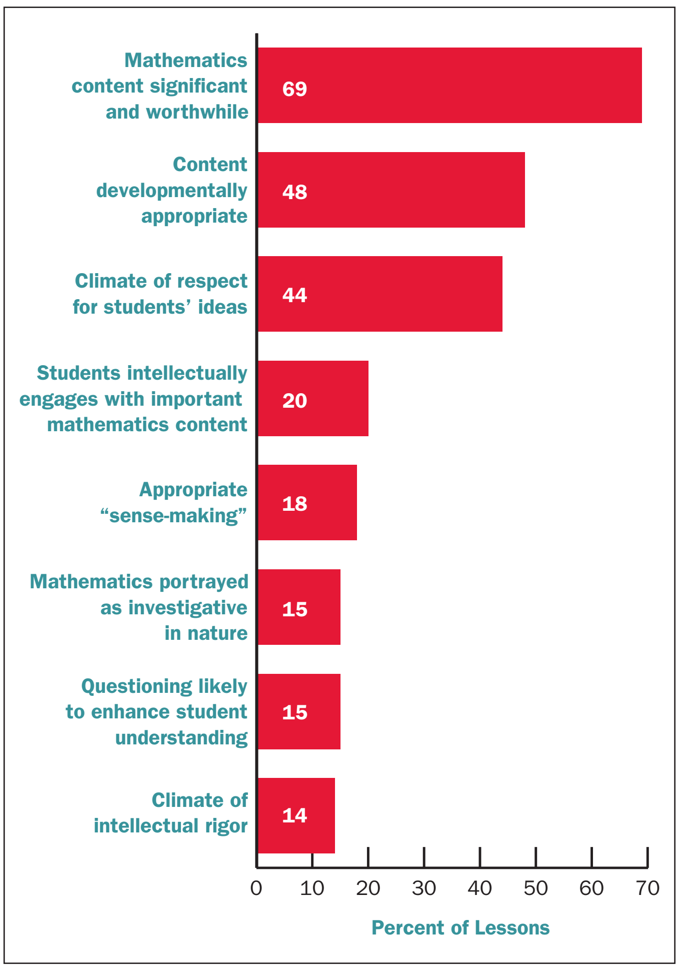

Figure 2

It is important to note that while the majority of mathematics lessons in the United States included important content, most lessons were nevertheless rated low. Clearly, while the inclusion of important content is necessary for high quality mathematics education, it is not sufficient.

Effective lessons include meaningful experiences that engage students intellectually with mathematics content. These lessons make use of various strategies to interest and engage students and to build on their previous knowledge. Effective lessons often provide multiple pathways that are likely to facilitate learning and include opportunities for sense-making. Unfortunately, students are not often intellectually engaged with important mathematics content, with only 20 percent of lessons rated highly in this area.

Lessons Should "Invite" Students to Engage Purposefully with Content

It is clear that teachers need a thorough understanding of the purpose of the lesson in order to guide student learning. It has also been argued that students need to see a purpose to the instruction, not necessarily the disciplinary learning goals the teacher has in mind, but some purpose that will motivate their engagement (Kesidou and Roseman, 2002). In the ideal, lessons will "hook" students by addressing something they have wondered about, or can be induced to wonder about, possibly but not necessarily in a real-world context. Many observed lessons failed to incorporate strategies to gain student interest and motivation; in many cases, lessons "just started," often with a warm-up problem that was unrelated to the rest of the lesson, or by the teacher handing out worksheets for the students to complete.

Teachers who succeeded at engaging students intellectually with mathematics content had various strategies for doing so. Some lessons that "invited the learners in" did so by engaging students in first-hand experiences with the concepts. For example, in a 7th grade lesson on fractions and percents, one student measured the height and arm spread of a second student, and the class was asked to use these numbers to express the relationship both as a ratio and as a percent. Other lessons invited the students in by using real-world examples to illustrate the concept vividly. Still others used stories, fictional contexts, or games to engage students with the content of the lessons. The following are examples of lessons that were particularly successful at motivating student interest and engagement:

A teacher of a 3rd grade mathematics class worked to develop an understanding of how parentheses may be used to direct order of operations in number sentences by involving students in writing number models for different ways a basketball team might score 15 points.

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In a high school Algebra I lesson, the teacher presented three line graphs showing data about two fictitious companies regarding productivity (intersecting lines), production cost (parallel lines), and sales (equivalent lines). She discussed each graph with the class and then asked the class to vote for the company they would hire based on the graphs.

Lessons Should Foster Students' Understanding of Mathematics as an Investigative Process

How mathematics is portrayed is key to student understanding of the discipline. Lessons can engage students with concepts so they come away with the understanding that mathematics is a dynamic body of knowledge, generated and enriched by investigation. Alternatively, lessons can portray mathematics as a set of algorithms to be memorized. Based on *Inside the Classroom* observations, only 15 percent of mathematics lessons nationally provide experiences for students that clearly depict mathematics as investigative in nature (rated 4 or 5 on a five-point scale). The following lesson is illustrative of those that highlighted the investigative nature of mathematics:

A 7th grade pre-algebra lesson began with the teacher introducing a new word problem. The purpose was to help reinforce the need for careful reading of problems, justification of strategies used and solutions presented, and the concept that there are multiple ways to approach solving a single problem. The students and teacher were engaged for a considerable time in a whole class discussion about strategies used to solve this single word problem with students presenting their solutions. The teacher stressed that there was "not a right way or a wrong way" to solve a problem, but "many ways to get into an investigation." Throughout the lesson, the teacher made statements like "I think it would be a good idea to make sure you can verify your answer with others in your group." and "I need you to convince me it's the right answer."

In contrast, many lessons presented mathematics as algorithmic in nature. The following example is typical:

According to the observer, "success in this 6th grade mathematics class hinged on students learning algorithms. Students were to learn rules and procedures, not the concepts behind them. Although the teacher had told them at the beginning of the lesson that moving the decimal place in both the divisor and dividend the same number of places was essentially the same as multiplying them both by the same power of 10, the message he gave students throughout the lesson was, essentially, "Just do it." When students pushed him for the reason they had to move the decimal, more than once the teacher responded: "The divisor must be a whole number."

In some cases, high stakes accountability may help explain why lessons tend to focus on a procedural view of mathematics. Based on *Inside the Classroom* observations, an estimated 18 percent of mathematics lessons include review/ practice to prepare students for externally mandated tests. On rare occasions, teachers were able to integrate test preparation fairly seamlessly into instruction that was geared toward learning of mathematics, as the following example illustrates.

The teacher passed out two worksheets to the students in an 8th grade pre-algebra class. The first one contained the mango problem, in which members of a family each take 1/3 or 1/5 of the mangoes in a basket until finally there are only three left. The task for students was to determine how many mangoes were originally in the basket. The second worksheet was for students to use to write down their solution to the problem; it included prompts such as "what I know," "strategy," and "steps."

The students worked independently; the teacher moved around the room and looked over shoulders, but said little. His questions encouraged students to think about what they were doing, and challenged them to articulate their ideas with more than a one-word answer.

The teacher noted that he was trying to continue with the planned curriculum while getting students ready for an upcoming benchmarks exam. The observer indicated that the lesson in fact provided a nice combination of test-preparation and a review of problem-solving strategies.

More often, the test preparation piece had the feel of an "add-on," and in some cases the entire lesson was focused on having students perform well on a high stakes test without also focusing on student understanding. The following example is typical:

The teacher of an 8th grade mathematics class reminded students that, "When you take the test, they might not give a specific unit, but all the units will be cubic." The teacher then turned to the topic of inequalities. She asked: "What's the opposite of an inequality?" Students responded: "An equality." The teacher said: "Okay, we're going to refer to these as inequalities. This is important because you can use inequalities to represent everyday situations. Why should you learn them? Because they're on the test."

Lessons Should Take Students from Where They Are and Move Them Forward

Although it is unlikely students are learning if they are not engaged, engagement is not enough; to develop student mathematical understanding, lessons need to be at the appropriate level, taking into account what students already know and can do, and challenging them to learn more. Approximately half of all mathematics lessons were rated high for the extent to which the content was appropriate for the developmental level of the students in the class. The estimated 17 percent of lessons nationally that were judged to be at the low end of the scale on developmental appropriateness were only occasionally too difficult, where it appeared that students lacked the prerequisite knowledge/skills, and the content seemed inaccessible to them. More often lessons were pitched at too low a level for some or all of the students. The following examples are typical:

According to the observer, "Some of the students in a 2nd grade mathematics class appeared to find the lesson too easy, and were handed worksheet after worksheet to keep them busy."

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The content of an 8th grade mathematics lesson seemed to be at too low a level for the students. Said the observer, "There were no instances in which the students seemed really stuck, when the process of moving to a deeper understanding of the content could occur. They were introduced to a new concept, they made sense of the definition, they applied it to different situations, but they didn't take the next step and see how this concept might be further explored."

Some lessons used multiple representations of concepts to facilitate learning, providing greater access to students with varying experiences and prior knowledge, and helping reinforce emerging understanding. One such lesson was observed in a 7th grade mathematics class:

The teacher introduced the concept of symmetry by first demonstrating the concept with examples. The concept development unfolded by engaging students in (a) exploring the concept, (b) investigating its application to familiar cases, (c) making connections to meaningful contexts, and (d) expanding it in a more challenging activity. Students were asked to write the alphabet in capital letters and find which letters have a line of symmetry. The teacher drew examples on the chalkboard A, B, C, D, E, to explain, demonstrate, and discuss possible lines of symmetry. Students then worked on their own for a few minutes, investigating the symmetrical properties of each letter, expressing some puzzlement about letters like N, Z, and H.

A discussion about symmetry in real world and familiar examples followed. The teacher presented examples that helped students make connections between symmetry and familiar contexts. Then she continued soliciting students' input of their own examples. The teacher welcomed their ideas and expanded the discussion around each example. In the last 15 minutes of the lesson, students worked on a hands-on activity designed to apply the concept of symmetry. Students were to draw the left side of a Christmas tree (on graph paper), add decorations of their choice, (e.g., half of a star), then exchange with their neighbor and draw the other half of their neighbor's tree.

Effective Lessons Create an Environment Conducive to Learning

Based on the observations in this study, a classroom culture conducive to learning is one that is both rigorous and respectful. Nearly half of mathematics lessons nationally received high ratings for having a climate of respect for students' ideas, questions and contributions. Ratings for rigor were much lower, with only 14 percent of mathematics lessons nationally judged to have a climate of intellectual rigor, including constructive criticism and the challenging of ideas. Table 1 shows a cross tabulation of the two variables; note that only 14 percent of mathematics lessons nationally are strong in both respect and rigor (with all of the lessons that were judged high in rigor also judged to be respectful to students), and 26 percent of lessons judged low in both areas.

Nineteen percent of mathematics lessons were categorized as respectful but lacking in rigor. *Inside the Classroom* observers used phrases like "pleasant, but not challenging" to describe such lessons. The following example is typical:

An observer described a 4th grade mathematics lesson where "the teacher was very enthusiastic, and encouraged her students to be the same. She gave lots of verbal encouragement to students as they worked…The culture suffered from a lack of focus on the intellectual content, however. The teacher appeared more intent on the students having a positive experience with mathematics through completing the task than really engaging with the concepts. The classroom was a welcoming environment for students, and there was a focus on 'learning,' but the level of learning expected seemed rather low."

Table 1

CROSS TABULATION OF CLIMATE OF RESPECT AND INTELLECTUAL RIGOR				
		Percent of Lessons		
		Intellectual Rigor, Constructive Criticism, and Challenging of Ideas Are Evident		
		Low	Medium	High
Climate of Respect for Students' Ideas, Questions, and Contributions	**Low**	26	2	0
	Medium	23	4	0
	High	19	11	14

Effective Lessons Help Students Make Sense of the Mathematics Content

Focusing on important mathematics content; engaging students; and having an appropriate, accessible learning environment set the stage for learning, but they do not guarantee it. It is up to the teacher to help students develop understanding of the mathematics they are studying. The teacher's effectiveness in asking questions, providing explanations, and otherwise helping to push student thinking forward as the lesson unfolds often appeared to determine students' opportunity to learn.

Researchers observed some extremely skillful questioning, where the teacher was able to use questions to assess where students were in their understanding, and to get them to think more deeply about the mathematics content. There were many more instances where the teacher asked a series of low level questions, with the focus primarily on the correct answer, rather than on understanding. Questioning was among the weakest elements of mathematics instruction, with only 15 percent of lessons nationally incorporating questioning that seemed likely to move student understanding forward. Lessons that were otherwise well-designed and well-implemented often fell down in this area.

When effective questioning was observed, the teachers used questions both to find out what students already knew and to provoke deeper thinking in helping them make sense of mathematics ideas. For example:

The observer reported that an 8th grade mathematics class was a very nice illustration of an interactive lecture, where the instructor asked for examples and justifications from the students as a means of assessing their understanding. "For example, when generating examples of tessellations around the room one student proposed the border of the bulletin board that was made of circles.

Student: 'How about the border?'
Students: 'No… that won't work.' (several students talk at once and reject this contribution)
Teacher: 'Why won't it work? Can the circle ever work?'

The discussion became focused on why the circle did not create a pattern that fit the definition of a tessellation. While the student who suggested the circle had been focusing more on patterns, the disagreement helped him redirect his analysis back to the definition of tessellations presented earlier."

More often observers noted that the teachers moved quickly through the lessons, without checking to make sure that the students were "getting it." As soon as one or two of the most verbal students indicated some level of understanding, the teacher went on, leaving other students' understanding uncertain.

By far, the most prevalent pattern in mathematics lessons was one of low-level "fill-in-the-blank" questions, asked in rapid-fire, staccato fashion, with an emphasis on getting the right answer and moving on, rather than helping the students make sense of the mathematics concepts. The following example illustrates this pattern as it played out in a high school mathematics lesson:

The observer reported that questions asked of students tended to be low-level and leading. The students were given the following system of equations:

$$6x + 5y = -2$$
$$5x - 4y = 31$$

The following "discussion" occurred:

Teacher: "What do we want?"
Students: "x and y"

Teacher: "What do I need to do to get x and y?"

Students: "Get rid of the first matrix."
Teacher: "What do I need to do to get rid of it?"
Students: "Multiply by the inverse."

Said the observer, "discussions during this lesson were much more about identifying steps to do than about justifying the steps by considering conceptual underpinnings."

Interestingly, observers reported that some teachers asked good questions, but were so intent on getting the right answer that they supplied the answers themselves, in effect short-circuiting student thinking. The following example is typical:

Said the observer of a high school calculus lesson, "When the teacher put a problem on the board and asked students to solve it, which they did in silence at their seats, the teacher often solved the problem on the board as they were working through the problem, or else waited about one minute and asked a student for input. On one problem the teacher asked for a student's input as to the next step toward the solution, but then disregarded the student's suggestion (which was one correct way to proceed) and went with his own strategy, saying: 'Yes, we can do that. But let's….' So the teacher solved the problem his way, even though he had asked for a student's strategy."

Teacher questioning is one way, but not the only way to help students understand the mathematics. The important consideration is that lessons engage students in doing the intellectual work, with the teacher helping to ensure that they are in fact making sense of the key concepts being addressed. The following example is illustrative of lessons that included appropriate "sense-making":

The purpose of a 2nd grade mathematics lesson was to allow students to demonstrate understanding of place value—ones, tens, and hundreds, and to practice with thousands place. The lesson emphasized numbers containing a zero, since this was something students found difficult. The lesson began with students working in groups of four. Each student in the group had a group member number. The teacher would give a digit for all the #1s to write on their marker board, then a digit for all the #2s, #3s, and #4s. The teacher would then give a number using all the digits and the students in the group would line up with their digits in the proper order to build the number. Students would look at each group's response and indicate their agreement with thumbs up or down.

The teacher encouraged students to question each other if there was an answer they didn't understand or didn't agree with. If a group did not represent the number correctly, the teacher would probe with questions to see if they could identify their error. She also asked students to respond to discrepancies that appeared among the groups' solutions. The class did several examples like this and then the students worked individually on more examples. After that the teacher had the students put their marker boards away, then wrapped up the lesson by asking, "What did we learn in math today?" Students gave responses like, "If there's a zero, you have to count it" after which the teacher asked for more explanation. She emphasized, "When we write numbers, the digits have to be in the right spot. Remember that the zeros are important, too. This will get easier as we go along."

Although researchers observed some lessons where students were helped to make sense of the mathematics content as the lesson progressed and/or at its conclusion, most lessons lacked adequate "sense-making;" only 18 percent of lessons received high ratings in this area. Many teachers seemed to assume that the students would be able on their own to distinguish the big ideas from the supporting details in their lectures, and to understand the mathematics ideas underlying their explorations. The following lesson descriptions illustrate inadequate sense-making in mathematics lessons.

Students in a 6th grade mathematics class were asked to complete a practice worksheet, which involved their measuring nine angles and identifying each as acute, right, obtuse, or straight. Said the observer, "Instead of students being encouraged to make sense of mathematics, students were to follow directions. Students were not asked to explain their thinking — either during the whole-class discussion or on the assessment. Mathematics was presented as a set of rules and procedures."

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The student in this Algebra class who put the equation $6x + 7 = -14y$ *into standard form on the board explained that she first subtracted 6x from both sides getting* $7 = -14y - 6x$*, which in standard form is:* $-6x - 14y = 7$*. Some students seemed confused, and asked the teacher if that was right. The teacher said it was, then solved it a different way, by first moving the y-term, getting the answer* $6x + 14y = -7$*. As she began solving it this way, some students seemed fixed on first moving the 6x—they didn't understand that either way was correct. The teacher concluded "So you can have two different answers."*

The observer noted that the teacher never mentioned that these two answers are mathematically equivalent.

In summary, while the aim of instruction in all cases needs to be understanding, based on the *Inside the Classroom* observations, there appear to be multiple approaches for achieving this goal. Observers saw lessons that were well-designed and well-implemented using lectures, manipulatives, or paper and pencil tasks to help develop student understanding of important mathematics concepts. Observers saw other lessons using each of these strategies that seemed unlikely to lead to student conceptual understanding. Factors that seem more instrumental than instructional strategies in promoting student opportunity for learning include the extent to which lessons engage students with important mathematics concepts; create an environment that is both respectful and rigorous; use questioning effectively; and help students make sense of the mathematics concepts being addressed.

Discussion and Recommendations

Teaching for understanding, most mathematics educators would agree, requires teachers who have a command of the important mathematics concepts being addressed, and who have the requisite knowledge and skills to help students develop their understanding of these mathematics concepts. Rather than focusing primary attention on which instructional strategies teachers use, student understanding would more likely be enhanced by ensuring first that instruction, regardless of instructional strategy, is purposeful; accessible; engaging to students; both respectful and rigorous; and maintains a clear and consistent focus on student learning of important mathematics concepts.

To the extent that teachers teach as they have been taught, they must experience teaching for understanding if they can be reasonably expected to teach for understanding. Similar logic certainly underlies calls for undergraduate mathematics courses to use cooperative learning and other "reform-oriented" strategies, but the findings from the *Inside the Classroom* study suggest that the key to instruction aimed at meaningful learning is not the particular strategies that are used, but rather engaging prospective teachers as learners with instruction that develops their conceptual understanding of mathematics.

Any instructional strategy can be implemented well, or implemented poorly. Working on open-ended problems that never lead to conceptual understanding is no more beneficial to learners than is sitting through inaccessible, uninteresting lectures. Of course, lectures do not have to be boring demonstrations of the use of algorithms or derivations of formulas. A well-conceived and well-delivered lecture can provide learners thoughtful explorations of important ideas. In theory, at least, a good lecture can engage learners in mathematical investigation by setting up an accessible yet challenging problem situation; identifying important questions that have been asked about the situation; discussing how they have been investigated, and which methods turned out to be useful pathways, and which were dead ends; and concluding with an explanation of how we now know what we know, as well as what we still do not know. If prospective teachers were to experience a variety of well-implemented instructional strategies in their pursuit of mathematics content understanding, and if their mathematics education courses attended explicitly to what constitutes high quality use of each strategy, they would likely be better prepared to implement high quality instruction in the mathematics lessons they will teach.

Even with excellent initial preparation, teachers need ongoing opportunities for continuing education, just as all other professionals do. Providers of teacher professional development can help teachers explore and enhance their vision of, and understandings about, effective mathematics instruction; and they can help teachers consider how to use their enhanced understanding to improve the design and implementation of their classroom lessons.

In addition, with the advantage of knowing which grades the in-service teachers are teaching, and often which student instructional materials are being used, professional development can be designed to provide very targeted assistance for teachers—clearly identifying the key concepts being developed in particular activities; sharing the research on student thinking in the specific content area; suggesting questions that teachers can use to diagnose student thinking and monitor student understanding; and outlining the key points that should be emphasized to help students make sense of the mathematics concepts. Teacher professional development activities, in turn, need to reflect the elements of high quality instruction with clear, explicit objectives; a supportive but challenging learning environment; and means to ensure that teachers are developing understanding. Modeling teaching for understanding and making its characteristic elements explicit in professional development will provide teachers additional opportunities to learn how to improve their own practice.

Professional development for mathematics teachers often focuses on, and advocates, particular instructional strategies, such as the use of manipulatives or cooperative learning groups. Instructional strategies, however, did not appear to determine the quality of the mathematics lessons observed in this study. We recommend, consequently, that professional development for mathematics teachers focus on aspects of effective instruction that cut across instructional strategies: learning goals that are both important and developmentally appropriate; examples and activities that capture students' attention and interest; an intellectual climate that both nurtures and challenges students; and, critically important, tasks, questioning strategies, and explanations that explicitly help students make sense of the concepts they are studying.

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