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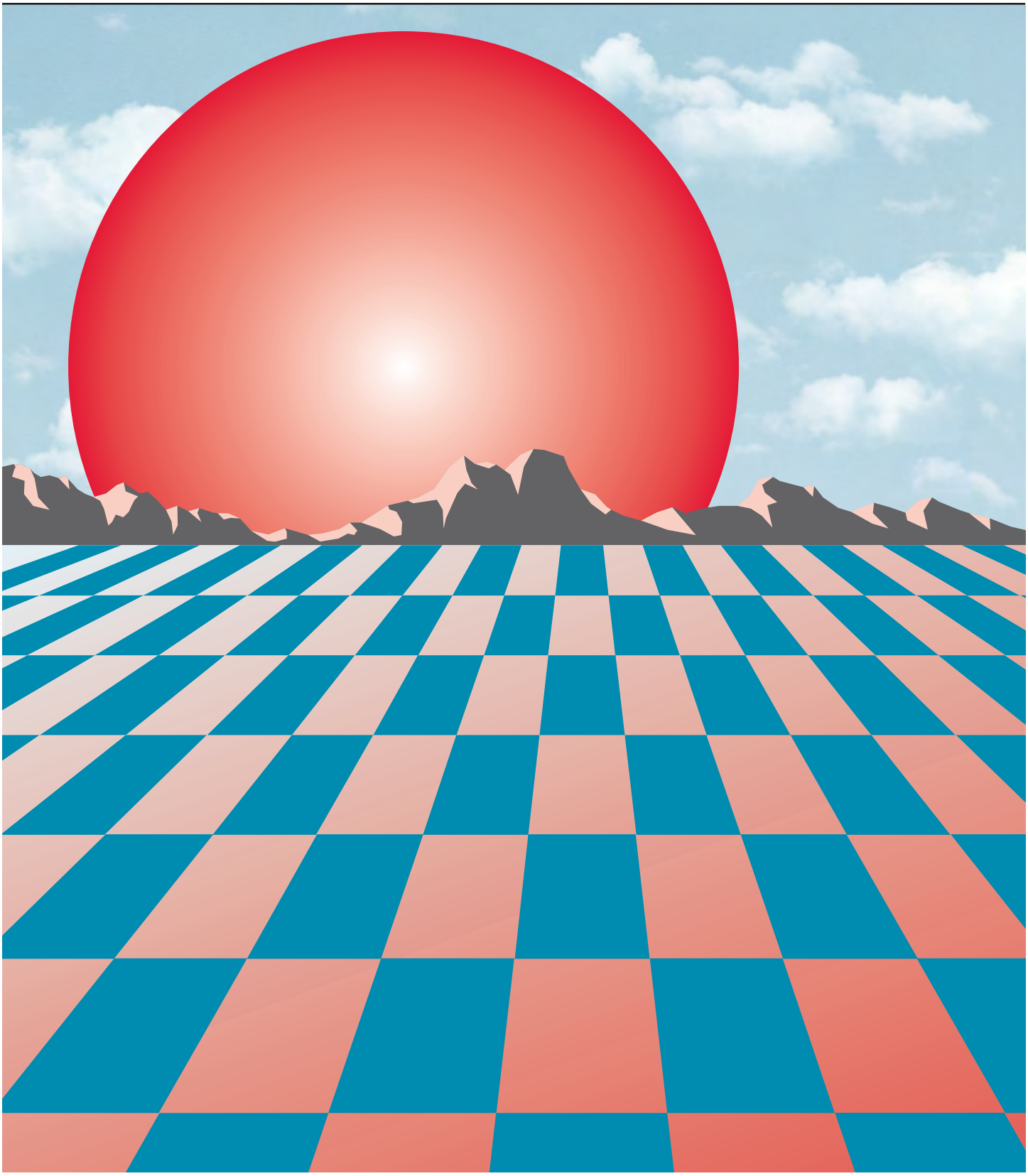

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Fidelity and Adaptation of Professional Development Materials: *Can They Co-Exist?*

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ABSTRACT:

This paper investigates the relationship between fidelity and adaptation. It is intended for both professional development materials consumers and developers. We will identify the core principles of a set of well-specified professional development materials — Learning and Teaching Linear Functions — and use these to illustrate our argument about the relationship between fidelity and adaptation. We will propose a continuum of adaptation practices, from fatal to productive, maintaining that productive adaptations are ones that are consonant with underlying values and, in that way, expressions of fidelity. By explicating this continuum of adaptation practices and by playing out various scenarios where adaptation is called for, we attempt to shed light on how well specified core principles within professional development materials can provide access and opportunity for productive adaptations by facilitators.[1]

There is often a gap between what authors of professional development materials intended to happen with their materials and what actually happens when used by others (LeFevre, 2004). The resulting inconsistency often gets blamed on the facilitator for not using the materials as "intended" — adapting them in ways that corrupt the goals of the authors. It may not be that simple — the reality is that all materials require some adaptation, but how does one do so in alignment with the authors' intentions? Hilda Borko encourages us as a field to study "whether the materials and resources provided by programs are sufficient to ensure that multiple users in diverse settings can maintain integrity with the designer's intentions". She recognizes that:

> "Designers of these programs as they attempt to scale up, will inevitably face the dilemma that policymakers face: On the one hand, mutual adaptation to the needs and conditions of local sites is essential if a program is to be implemented effectively; on the other hand, too much adaptation can mean that the overall intent of the program is lost." (Borko, 2004, pgs 12-13)

The purpose of this paper is to explicate the meaning of and relationship between fidelity and adaptation. The notion of fidelity will be illustrated by describing the core principles of one set of well-specified professional development materials — The Video Cases for Mathematics Professional Development: *Learning and Teaching Linear Functions* materials[2] (LTLF). These core principles will be used to demonstrate the relationship between fidelity and adaptation. In addition, a continuum of adaptation practices will be proposed — from fatal to productive, maintaining that productive adaptations are ones that are consonant with underlying values and, therefore expressions of fidelity. By explicating this continuum of adaptation practices and by playing out various scenarios where adaptation is called for, this paper will attempt to shed light on the design features and values that are most important to the LTLF materials in particular, and potentially to well-specified professional development materials in general.

[1] I am grateful to Barbara Miller and Judith Mumme for their insightful comments and suggestions to this paper

[2] Supported by the NSF project ESI-9731339

Fidelity and Adaptation

from contextually embedded communication, wherein Fidelity means acting in accord with the core principals explicated in the professional development materials. Adaptation means not using the materials strictly as written/scripted (Borko,2004, LeFevre, 2004). In adapting materials, some actions are consistent with the underlying values and others are not. More typically, fidelity and adaptation are defined in opposition to each other and get cast in overly simplistic and dichotomous notions of good or bad. This is quite often determined by one's role — as a curriculum developer, fidelity is a good thing and adaptation is perceived as inherently bad. Professional developers may be more likely to put a premium on creativity and attention to context that is only possible with adaptation and curriculum developers may want their curriculum followed as close to as written as possible. Regardless of one's role or view, we believe that adaptation is inevitable because it means to take seriously the context (i.e., setting, participants, facilitator) in which materials are used. Adaptation and fidelity can not only co-exist, but also exist productively when two critical conditions are present:

(1) When the materials are well specified.

(2) When adaptations are consistent with the underlying design and values of the professional development materials.

Well-Specified Materials

Borko reminds us that as designers of professional development programs attempt to scale up, they will inevitably face a dilemma — "on the one hand, mutual adaptation to the needs and conditions of local sites is essential if a program is to be implemented effectively; on the other hand, too much adaptation can mean that the overall intent of the program is lost" (Borko, 2004). In order to deal with this inevitable dilemma, designers of professional development materials need to explicitly state their underlying core values and principles so that facilitators have the opportunity to adapt their materials with fidelity. Without access to the materials' core principles, a facilitator only has access to the script (e.g, the proposed agendas, the series of activities, etc.). This is analogous to only having access to the student edition of classroom materials. A teacher in this case would not have access to additional information a teachers' guide might offer such as: overall unit and lesson goals, content explication, connections to other units, pacing charts, teaching tips, etc. Quite often adaptations go awry because there is no resource for making decisions in accordance with the authors' intentions. The user is left to navigate in the dark without a sense of direction or guidance.

Well-specified materials make it possible to use materials with fidelity because they explicitly communicate the underlying principles. They clearly lay out the core principles for use of the materials and a rationale for why the authors believe these are important. They often make explicit the facilitator demands of the materials and include clearly laid out tasks. In addition, the tasks and facilitation demands have been articulated with an eye toward making the design and values explicit. The well-specified nature of the materials make it *possible*, not *probable* for users to adapt with fidelity. Written specification of underlying principles does not necessarily mean that readers will interpret them with the same understanding as the authors intended, or that they will choose to use them (even if they understand them) in making adaptation decisions. But, we do argue that the written specification does create an opportunity to do so — without which makes adaptation with fidelity nearly impossible.

Learning and Teaching Linear Functions: Video Cases for Mathematics Professional Development (LTLF)

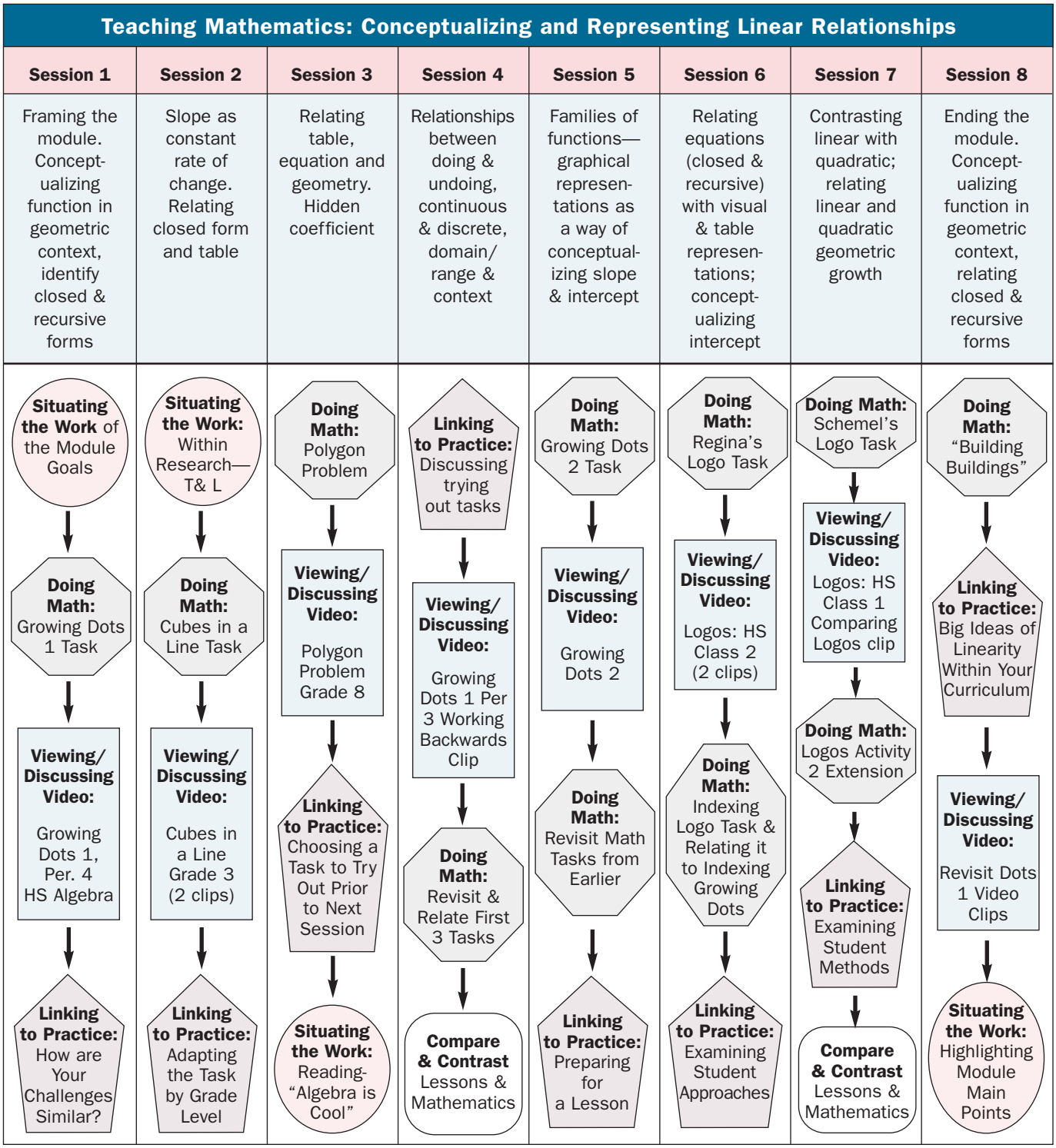
As a set of well-specified materials, the *Learning and Teaching Linear Functions materials*[3] are designed to help teachers deepen their understanding of mathematics content, students' mathematical thinking, and instructional strategies; as well as develop norms and practices for learning about teaching. The first of five modules, *Conceptualizing and Representing Linear Relationships,* is a sequential series of eight 3-hour sessions that are designed to enrich teachers' ability to teach linear relationships and deepen their own detailed knowledge of the distinctions and linkages among the various representations. Each session has at its core one or two digital video clips of a mathematics classroom. These clips are segments selected from real classroom footage of un-staged mathematics lessons, representing a range of grade levels, geographic locations and student populations. Each session is considered

[3] The Learning and Teaching Linear Functions materials are published by Heinemann at http://books.heinemann.com/products/E00682.aspx

a video case with the video episode as its centerpiece and includes four basic elements: situating the work, doing mathematics, viewing and discussing video, and linking to practice. The module map below helps to illustrate the activity flow of the module's eight sessions (figure 1).

The facilitation guide offers explicit and well-specified support for using the materials with fidelity. It includes such information as: a complete overview of the materials, explanations and rationale of the underlying principles and specific goals, sample agendas and guidelines for

FIGURE 1



sessions, lists of references and useful resources, tips for facilitation including caution points, mathematics commentaries and excerpts from a composite facilitator's journal chronicling the experiences of others having used these materials. These materials are carefully designed, multi-layered, cohesive modules of professional development curricula. Ball and Bass highlight the contribution these materials offer to the field.[4] Specifically, they note the support they provide for use by facilitators, "They provide practice-based resources for study and analysis, and they provide guidance and insight to the teacher developer using the materials."

Underlying Principles and Values of the LTLF Materials

In order to use these materials with fidelity, facilitators need to understand and honor the explicated three core principles of the materials: (1) serious and intentional use of video as a carefully chosen medium; (2) serious and intentional use of the mathematical work within the materials; and (3) respect for both the curricular nature of the materials and teachers as learners. Each of these three principles is further explicated below:

I. *Serious and intentional use of video.* These materials are designed around video as the medium. Since video is central to the intentions of the materials, in order to analyze the interactive nature of mathematics teaching and learning, NOT using the video would be a breach of values. Besides using the video, in order for facilitators to use these materials effectively in orchestrating discussion and probing teacher thinking, a deep understanding of what is inside each video clip is also needed. For example, the first session of the foundation module includes the following video clip:

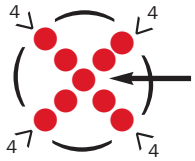
Kirk, a 9th grade Algebra teacher, poses the following task to his students with the goal of helping them visualize and conceptualize slope and y-intercept:

At the beginning | At 1 minute | At 2 minutes

Study the sequence of dots. Describe the pattern you see. Assuming the sequence continues in the same way, how many dots are there at 100 minutes? Find an equation for the number of dots at *t* minutes.

After working on the problem for a few minutes, Kirk brings the class together to discuss their solution methods. Danielle shares her answer of x • 4 + 1 *and Kirk asks her to illustrate how she connected her expression to the dots at the board. When Kirk asks for a different method, James shares his method.*

x • 4 + 1 **Danielle**



The center or 1 in the equation 4 should be all the dots except in the center. X is how many dots out from the center.

James x + 4

X is the previous picture. That plus 4 is the next picture.

At the beginning | At 1 minute | At 2 minutes

1 + 4 = 5

[4] Ball, D.L. & Bass, H. (2004), Foreword in Learning and teaching linear functions. Portsmouth, NH: Heinemann.

First of all, the facilitator needs to show the video and give ample time for analyzing and discussing it. Within this segment, the demands on a facilitator are many. A facilitator needs to recognize that both James and Danielle's methods are made public and potentially available for collective use by the whole class when they are asked by Kirk to come to the board and share their solution and method. They need to understand what each student method entails. They need to recognize the complexity and challenge in teaching students to represent, communicate and present their ideas during a whole class discussion. In addition to all of this, they need to predict and plan for how their teachers will respond to this segment and the ideas embedded in it and figure out how to increase their opportunities to learn the curricular trajectory of the materials. Although the materials provide support for developing these understandings, in order for a facilitator to gain these skills they would need to view for themselves the video clips multiple times, read over the commentaries and notes, figuring out the mathematical points, project how teachers may respond to the video clip, and prepare how to use the clips with their teachers.

II. *Serious and intentional use of the mathematical work within the materials.* As with the video, the mathematical work is an important feature of these materials and to NOT do it would be counter to the principles. But doing the mathematical tasks is not sufficient, facilitators of these materials should possess the ability to "decompress" mathematical ideas, particularly when such ideas appear to be straightforward mathematics. A facilitator would need to prepare for the mathematical content — both what is taught and what is embedded in the teaching of that content. Using the video clip example above to illustrate this point, a facilitator would need to know that James is using a recursive, iterative approach to finding the number of dots (noticing that you add four each time to the previous number of dots) and Danielle is using an explicit approach, looking at the relationship between the number of dots and the number of minutes. They would also need to understand that each is using the variable "x" to represent something different. While James is using x to represent the previous number of dots, Danielle is using the x to represent number of groups of four [though it is unclear what the 4's represent in her drawing and which group of fours she is talking about — the number of dots in each of the four "arms" of the picture or the number of groups of four dots "out from the center", or a combination of the two methods]. A facilitator of this case must also be able to recognize various approaches and methods beyond these two approaches, distinguish amongst them, and find correspondences between them. Likewise, in supporting teachers to conceptualize and represent slope and y-intercept, a facilitator must be able to sift through the fine grain details involved within these two seemingly straightforward elements of linear functions and unearth potential underlying conceptions and misconceptions. Again, the materials provide support for the development of these capacities, a facilitator would need to examine the student methods carefully, analyze the mathematical logic of each, and study the mathematical explanations and resources of the materials.

III. *Respect for both the curricular nature of the materials and teachers as learners.* Facilitators need to have a clear sense of the learning trajectory of the materials, their cohesive, scaffolded design and would need to use them as a curriculum, not a menu of activities. They would need to respect teachers as learners and a hone their ability to observe and assess the teachers in relationship to the curricular goals. This means that they would need to adapt the materials to fit the needs of their teachers, while at the same time respecting the curricular nature of the materials. For example, teacher participants are often unfamiliar with the thinking involved within recursive approaches to problems, having previously assumed that such solutions as James were simply incorrect. A facilitator would need to understand that as teachers explore the mathematical logic behind a recursive approach and its relationship to explicit strategies across multiple video case sessions, the opportunity exists for them to develop a deeper understanding of the distinction between and the relationship amongst recursive and explicit forms of linear relationships in particular and linear functions in general. Understanding that these ideas will emerge in multiple sessions in various ways, can ease the burden that facilitators often face in trying to cover everything in one session. In order for facilitators to gain this understanding, they would need to spend time examining the entire module and its cohesive, scaffolded design, so they have a sense of how it fits together. In addition, they would need to examine what each session brings in light of the overall goals.

In order to honor and respect the participants, the facilitator will need to spend time listening carefully to their teachers.

The Relationship between Adaptation and Fidelity

It is assumed that all facilitators make adaptations. Furthermore, it is assumed that not all adaptations are productive ones, nor are they of the same magnitude. In order to examine the various types of adaptations a facilitator might make, categories of adaptation along a continuum scale can help reveal use with fidelity. This scale has a range from fatal adaptations at one extreme to productive adaptations at the other, while in the middle lies the types of adaptations that do not impact the design of the materials negatively or positively.

CATEGORIES OF ADAPTATION

As the scale illustrates, there are various types of adaptation. The levels on the scale are not meant to imply discrete stages of use — a person could use part of the materials productively and part of them fatally. The scale is not intended to represent a linear progression from one stage to the next — a person could begin adaptation fatally but gain knowledge and jump to productive use, or a person could begin with "no impact" adaptations and make a "fatal" adaptation down the road. The scale is intended to suggest categories of adaptation by facilitators in using the materials. The scale represents a relationship between fatal and productive adaptations in terms of underlying values — a fatal adaptation reflects no fidelity to underlying values, and a productive adaptation reflects a great deal of fidelity to underlying values. It is assumed that "fatal" adaptations are not made intentionally. That is, they are not intended to be unproductive — but it can happen nevertheless. Fatal adaptations can be a result of facilitators choosing to do something contrary to the underlying principles or they can be a result of the fact that a facilitator had no access to the underlying values. Decisions to change things have consequences in relation to the underlying principles and values of the designed materials. At the two extremes, a fatal adaptation would be one that is in direct opposition to the underlying values and a productive adaptation would be one that is in concert with them. Indeed, no adaptation at all can also be problematic. Mechanical, scripted use of the materials to the point of an over-reliance on only what is contained in the written guides — the belief that if I as a facilitator, *use the materials exactly as written, with no adaptations, then I'll be effective* — is not using these materials as designed, since adapting the materials to participants is an explicit and critical component of effective use (an underlying value).

The three-point scale along a continuum described below:

I. *Fatal Adaptations.* Some adaptations create severe problems and reveal misconceptions of facilitators about the intended use of the PD materials. These adaptations can be considered fatal errors and seriously undermine critical components of the materials. Because they are contrary to the basic design or values of the materials, we categorize these adaptations as fatal.

II. *No Impact Adaptations.* Some adaptations are possible, and seem relatively neutral in that they don't have a big impact on use with fidelity. These "no harm, no foul" adaptations are categorized as no impact because they don't undermine the basic design or values of the materials, nor do they make the best use of them.

III. *Productive Adaptations.* Some adaptations by facilitators are productive in that they make the best use of the materials given the circumstances in which they are working. Adaptations are made that relate to particular participants in particular contexts, while at the same time keep an eye on the learning trajectory of the materials. Because the changes that are made are consistent with the design and underlying values, we categorize these adaptations as productive. In our view, thoughtful use of LTLF materials should involve productive adaptation.

The Nature of Adaptation

The LTLF materials will be used in this section of the paper as an example to consider the nature of adaptation. Two common cases of use will be described that require adaptations by a facilitator: 1) not enough time and 2) participants with limited content knowledge. Within each case are examples of No Impact, Productive and Fatal adaptations in order to highlight what is meant by these varying degrees of use with fidelity.

CASE 1: NOT ENOUGH TIME

Time is limited for professional development and facilitators are often faced with decisions about adapting materials to fit a set of time constraints. While the foundation module of the LTLF materials is designed for eight, three-hour sessions, many situations simply cannot accommodate this — district constraints for the number of after school sessions, or a need to utilize a district's designated professional development days. This situation requires adaptations to be made. These adaptations can impact use with fidelity. We explicate three categories below, beginning with no impact and then moving to fatal and productive impacts.

No Impact: No Impact adaptations are essentially such insignificant adaptations that they don't matter — they don't really change the experience. No impact on use with fidelity in the case of the LTLF materials could include a facilitator's decision to:

- Combine two, three-hour sessions into one full day professional development offering. This has no impact because the number of sessions stays the same and is aligned with the design and values of these materials.
- Cut the sessions to two and a half hours — eliminating the formal break and individual time for work on the mathematics. This decision only has no impact if the participants are given the math task prior to the session so that they have a chance to work on it.

Fatal: Fatal adaptations are changes made with little or no fidelity to the intent or the underlying ideas in the materials. Adaptations that would be considered fatal errors in relationship in using the LTLF materials, can include such facilitator decisions to:

- Eliminate crucial components within a session (e.g., eliminate all video or cut short the work on the mathematics). This adaptation is fatal because it is contrary to the design of the materials — leaving out any crucial element such as video or the mathematical work would severely limit the potential effective use of the materials.
- Eliminate full sessions. This adaptation is fatal because these particular materials were designed as a series of connected professional development experiences. By eliminating full sessions, the materials are not being used with fidelity to the underlying values because teachers might miss important ideas and because the sequence builds upon one another.

Productive: Productive adaptations are changes made with fidelity to the intent or underlying principles. Adaptations that would be considered productive in relationship to intended use of the VC materials include such facilitator decisions such as:

- Combine consecutive sessions that cohere well in a full day session. This adaptation takes seriously the mathematical flow and purposeful order of the materials and highlights the links between particular sessions. In this way, the adaptation is productive and in alignment with the underlying values of the materials.
- Re-design a full day session to think carefully about participant's energy level (e.g., more in morning than after lunch). This adaptation takes seriously the participants as well as the design of the sessions and module. It requires both knowledge of the materials and the participants and is therefore consistent with the underlying principles and values.
- Design for partial online work in order to shorten face-to-face time, without compromising overall time. This adaptation can be productive if the nature of the online work is consistent with the underlying values of the materials such as: online discussions of linking to practice activities, comparative analysis across sessions, or mathematical work. This ensures consistency with the values in that the work is still part of the work, only in an online venue.

CASE 2: PARTICIPANTS HAVE LIMITED CONTENT KNOWLEDGE

A common situation that facilitators of professional development face is the issue of their participants limited content knowledge. In order to be responsive to their participants needs, this situation demands adaptations of the LTLF materials. These adaptations can result in varying degrees of use with fidelity. We explicate three categories below, beginning with no impact and then moving to fatal and productive adaptations.

No Impact: No impact on intended use (use with fidelity) in the case of the LTLF materials could include a facilitator's decision to:

- Reorder activities to focus first on linking to practice. This is characterized as "no impact" since the activities reordered (linking to practice, see figure 1) do not change the underlying design and values. However, if a decision was made to reorder the mathematical work and video work, this would be contrary to the design

and values of the materials because it changes the opportunity to examine the mathematics prior to focusing in on the video episode (an explicit design decision and underlying value).

- If there is a need to concentrate on a few key concepts deeply, provide additional mathematical experiences that may include combining consecutive sessions that focus on a few particular mathematical purposes. This has no impact because attending to participants needs and furthering key mathematical conceptual learning are both aligned with the design and values of these materials.

Fatal: Adaptations that would be considered fatal in relationship to fidelity if they are contrary to the design and values of the materials. These fatal adaptations can include such facilitator decisions to:

- Accept unquestioningly whatever participants say or produce as good and accurate, in the spirit of trying to affirm their efforts but not identifying their limited knowledge. This adaptation is fatal because it is not consistent with the mathematical learning goals of the designed materials.
- Rely solely on the idea that participants will figure this out on their own eventually, without facilitator assistance via synthesis, reflection, or explication of participants' ideas or even by direct instruction (though by design, not right away). This adaptation is fatal because it is contrary to the underlying value that facilitators are teachers who can and should play an active role in supporting teacher learning.
- Use a lecture mode in the first session to bolster limited content knowledge, aiming to teach participants certain concepts ("here's what recursive means") and thereby violating the spiraling design of the curriculum during a first encounter with a concept.[5]

Productive: Adaptations that would be considered productive in relationship to fidelity of LTLF materials' use, include such facilitator decisions such as:

- Strategic choice of using participant work or explanations (lifting up and highlighting) to direct participants' attention to particular content knowledge (e.g., facilitator's choice about when to introduce term "recursive" and what work/comments from the participants work to link the term to). This is a adaptation that is consistent with the underlying design and values of the materials in that it takes seriously both the mathematical learning goals of the curricula and the participants ideas.
- Aim for mathematical ideas that are more familiar and more accessible (e.g., rate of change versus formalizing ideas related to recursion) and relate the more accessible concepts to more abstract concepts. This is productive in that the adaptation attempts to better address the learners' needs by creating mathematical accessibility without compromising the key mathematical learning goals.
- Make more extensive use of visual representations and meanings ascribed to them (e.g., labeling). This adaptation is productive in that it is consistent with the design and underlying values around content and learners. This adaptation recognizes that often participants with limited content knowledge are more likely to produce or understand visual representations that keep their emerging understanding of the concept intact.
- Use the video to revisit the mathematics. This adaptation is productive in that it is using student ideas within the video to raise additional work on the mathematics, thereby creating opportunities to deepen and expand the mathematical learning of the participants.

Conclusion

This paper has put forth the notion that fidelity and adaptation can not only co-exist, but also work together productively. In using the Learning and Teaching Linear Functions materials to illustrate the argument about the continuum of adaptation practices, from fatal to productive, it was argued that productive adaptations are ones that are consonant with underlying values and are therefore expressions of fidelity. By playing out various scenarios where adaptation is called for, the intention of this paper was to shed light on how well specified professional development materials can provide facilitators access and opportunity for adaptations to make adaptations with fidelity.

These categories of adaptations can not only apply to issues of use with fidelity within these particular materials, but potentially apply to other well-specified materials as well. While the specifics of what counts as fatal or productive may vary, the fact that one could use a well specified set of materials consistent with or contrary to its underlying

[5] This is not to suggest that a facilitator never tells, rather timing and purpose make a difference in how ready learners might be to attach the meaning of what is being told. [Lobato, et.al, 2005)

design principles and values generalizes beyond the LTLF materials. It seems logical that adaptation of other well-specified professional development materials[6] could also be categorized from fatal to productive, depending upon how those adaptations line up with the authors' underlying values and design principles.

In general, developers of PD materials could support use with fidelity by ensuring that they specify their underlying principles and intentions explicitly enough that users can make informed decisions about productive adaptations. Likewise, users of PD materials could look for (and demand) PD materials that are well specified, thereby allowing them a window into the authors' intentions and creating the opportunity for them to make informed decisions about their adaptations.

In addition to PD developers and users responsibilities, research can play a role in furthering the knowledge about the relationship between fidelity and adaptation. Borko encourages us as a field to investigate the relationship between fidelity and adaptation across multiple settings, in order to consider which elements of a program must be preserved to ensure the adaptations aligned to underlying goals and principles. Even though examining one set of materials is useful, it is clearly not enough. We need studies across multiple programs to examine questions such as: *Do well-specified materials yield more likelihood of adaptation with fidelity? Or, what resources support productive adaptation?* We as a field could also gain in knowledge if we studied whether these proposed scales of adaptation cut across multiple and varied programs and/or disciplines, e.g. *Can fatal to productive adaptations can be understood in professional development materials in science?* Clearly much investigation is ahead of us. Hopefully this paper has opened the door to possible conversations, new thinking and future research.

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[6] e.g. as Developing Mathematical Ideas (Schifter, et al 1999), Fostering Algebraic Thinking Toolkit (Driscoll, et. al 2001) , and Using Cases to Transform Mathematics Teaching and Learning (Smith et al, 2005) materials

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