ON TRACK FOR 4 YEARS OF MATH!

Discrete Mathematics and Modeling for High School Students and Teachers

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Goals of this session:

1. What professional development is needed for teachers to be “comfortable” teaching this course?
2. What are the Big Ideas in the PUHSD *Discrete Mathematics and Modeling* course?
3. What instruction and assessment should you see going on in this course?

How did we get here?

December 2008

- Graduating class of 2013 needs 4 years of Math credit
- One credit is equivalent to Algebra II

Fall Enrollment for upper level Math Courses

	Algebra 3	Algebra 3H	Pre-Calculus	Calculus I, Calculus II, AP, IB	Advanced Math	Upper Level Total for Year	Total Math Enrollment
2002	1690	382	670	45	257	3044	18856
	9.0%	2.0%	3.6%	0.2%	1.4%	**16.1%**	
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2008	2397	923	801	44	527	4692	25266
	9.5%	3.7%	3.2%	0.2%	2.1%	**18.6%**	

How did we get here?

- Only 18% of the student population were in Jr./Sr. level Math classes
- We needed more options for students not pursuing Calculus
- Aligned with the newly released AZ 2008 College and Career Readiness Standards

New Path

- MSP Grant:
  Mathematics Modeling Partnership: Preparing Urban Teachers for Implementing College and Work Readiness Standards

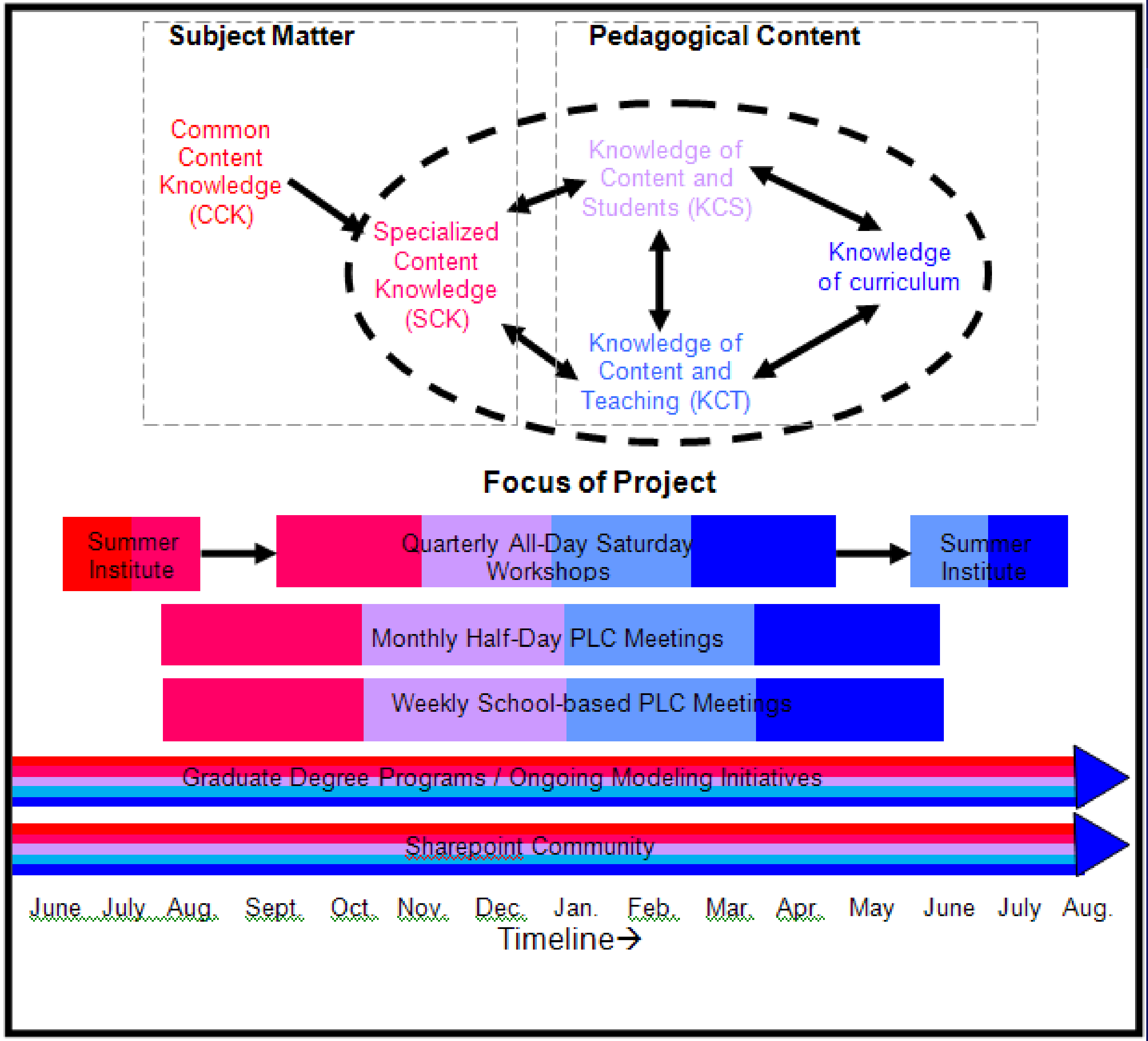
Partnership PUHSD & ASU

- Goals were to create curriculum for 4th year course
- Increase teacher content knowledge
- Increase student achievement

Where do we start?

135 hours of PD

- 1 week workshop first summer
  - Developing teacher content knowledge (TCK) and “comfort”
- 8 Saturdays throughout the school year
  - Build curriculum and TCK
  - Modeling instruction
- 1 week summer workshop at the end of the grant



Subject Matter
Pedagogical Content
Common Content Knowledge (CCK)
Specialized Content Knowledge (SCK)
Knowledge of Content and Students (KCS)
Knowledge of curriculum
Knowledge of Content and Teaching (KCT)
Focus of Project
Summer Institute
Quarterly All-Day Saturday Workshops
Summer Institute
Monthly Half-Day PLC Meetings
Weekly School-based PLC Meetings
Graduate Degree Programs / Ongoing Modeling Initiatives
Sharepoint Community
June July Aug. Sept. Oct. Nov. Dec. Jan. Feb. Mar. Apr. May June July Aug.
Timeline→

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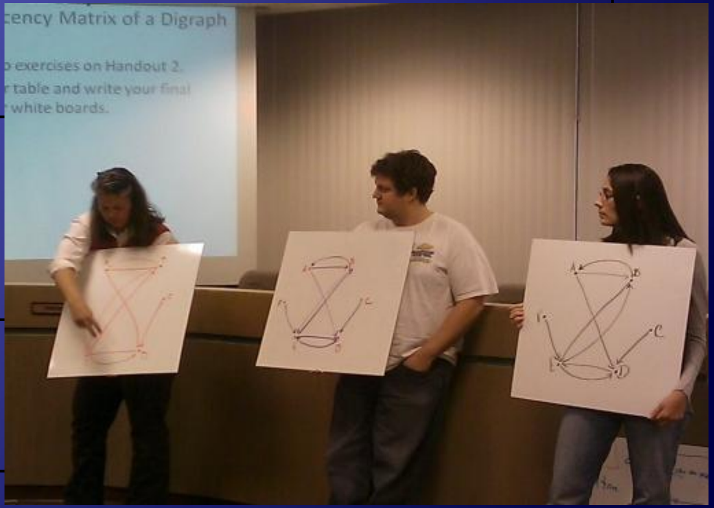
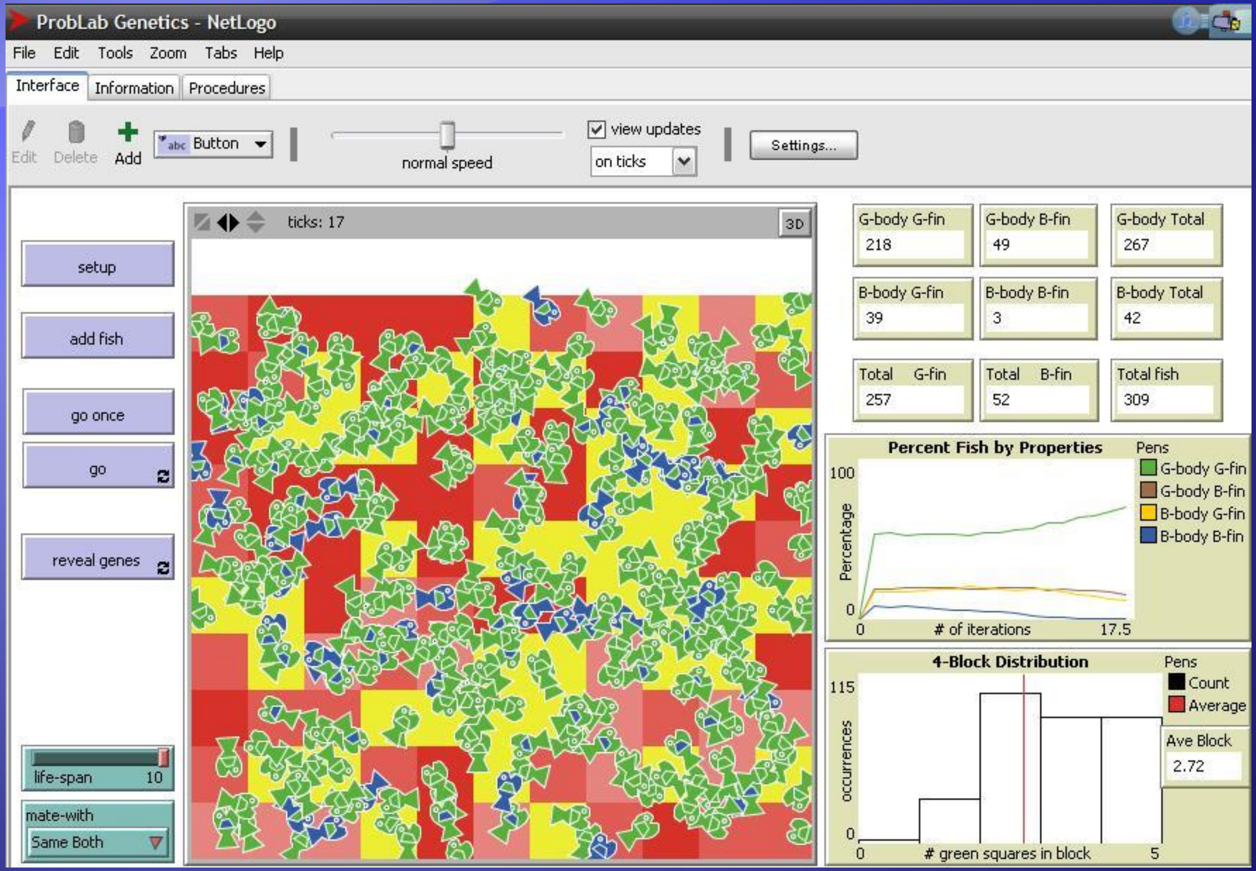
Curriculum Process

Concept unit	Big Ideas
Algebra Modeling	• Review linear functions • Modeling Physical/Phenomenon, Biology/financial etc. • Curve fitting families of curves • Linear, Polynomial, Exponential, Logistic
Data Modeling	• Distribution /Error • Measures of center and measures of dispersions • Graphical Analysis
Probability and Combinatorics	• Systematic Counting • Dependency/conditional • Rules of Probability • Permutations/ Combinations • Normal distribution
Vertex-Edge Graphs	• Euler Circuits/ Hamilton circuits • Traveling salesperson • Networks/Trees • Adjacency matrices
Complex Systems Modeling	• Analysis of change • Dynamical system • Iterative/recursive



Technology



http://ccl.northwestern.edu/netlogo/

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Explore a unit

Concept Unit Map

Key Concepts

- Determine an Euler path and circuit.
- Determine a Hamilton path and circuit.
- Create vertex edge graphs from an adjacency matrix.
- Create an adjacency matrix from a vertex edge graph.
- Explain the difference between a path, a circuit, and a walk.
- Devise, analyze and apply algorithms for solving vertex edge graphs.
- Interpret row sums and the nth power of adjacency matrices for vertex edge graphs.

Vertex Edge Modeling

Enduring Understandings:
There are many ways of representing information in mathematics, and carefully choosing the form you use can greatly simplify the problem-solving process. Vertex-edge graphs provide a useful way to represent and analyze real-world situations involving relationships among a finite number of elements, including scheduling, managing conflicts, and finding efficient routes.

While some vertex-edge graphical representations of information are complicated and difficult to use, adjacency matrices can provide the same information in a simpler format that can be mathematically manipulated to provide even more information than can be seen in the graph.

- What steps take a real-world pathway and abstract it to a circuit?
- What are the criteria to Eulerize a given path?
- What effect does Eulerizing have on a pathway event in a real-world situation?
- What is the importance of valences in determining an Euler Circuit?
- What are the criteria for creating a Hamiltonian circuit?
- What are the differences in criteria for Euler versus Hamiltonian circuits?
- What is the relationship between graphs and matrices?

Key Vocabulary
Trail
Loop
Walk
Circuit
Bridge
Euler Path
Euler Circuit
Hamiltonian Path
Hamiltonian Circuit
Tree
Spanning Tree
Vertex Coloring
Adjacency Matrix
Critical path
Directed graph
Connected graph
Degree
Traceable
Traveling Salesman Problem
Diagraph
Matrix

Resources
Angel, Abbott, & Runde. (2009). *Survey of Mathematics with Applications* (8th ed.)

Websites: Wolfram, Newfield

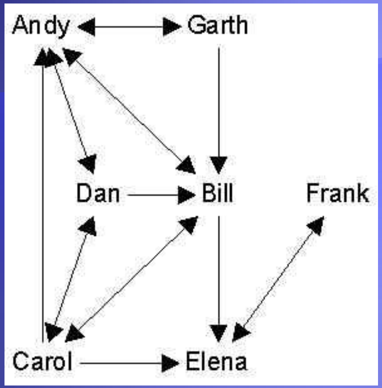
Textbook Resources

Examples

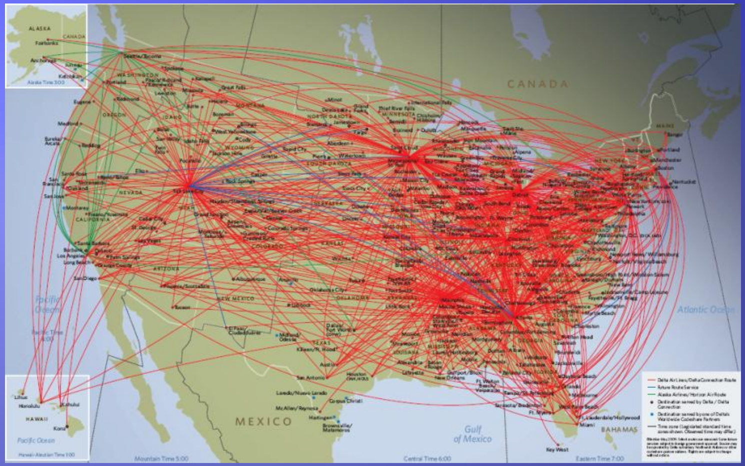
- Rank players in a round-robin tennis tournament using matrices and Hamiltonian paths.
- Determine how many different ways the American Idol judges can, traveling together, visit each of the cities for auditions and return to Hollywood.
- Create a schedule of sports for campers to ensure that they may participate in any or all of seven offered sports.

Graphs in Context

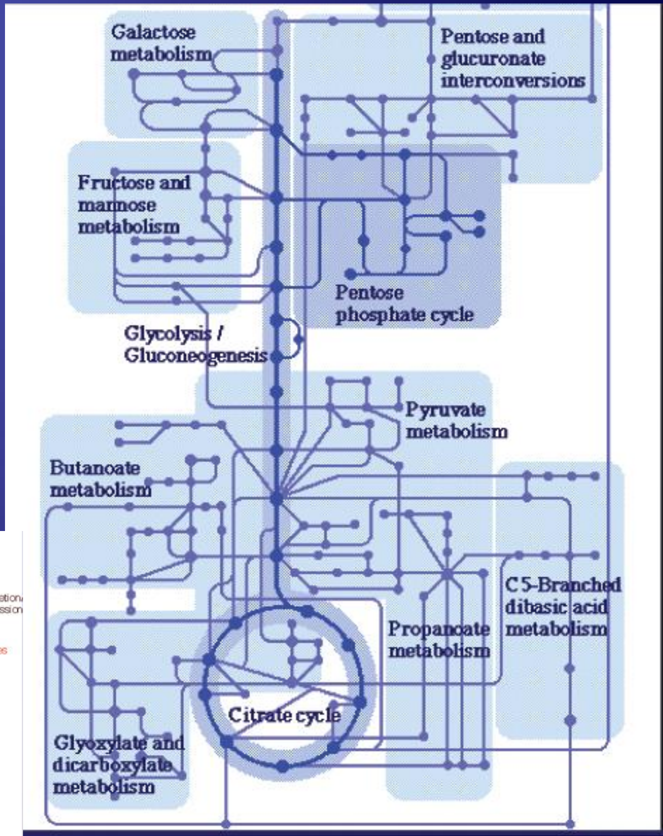
Social Networks



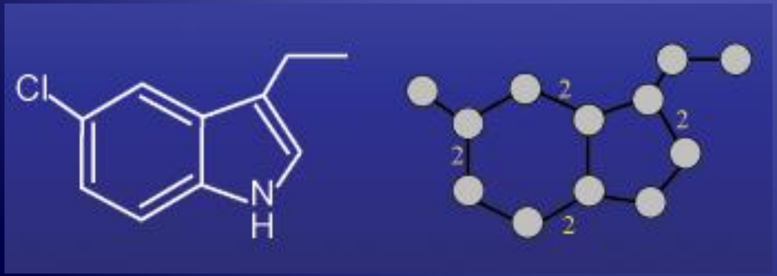
Transportation



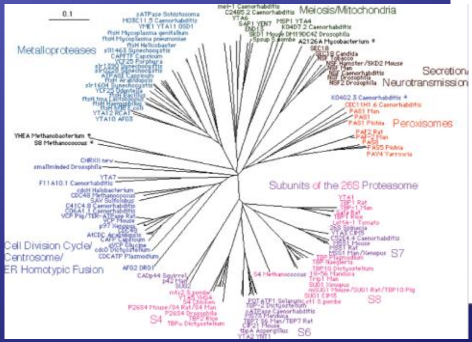
Metabolic Processes



Chemical compounds



Classification

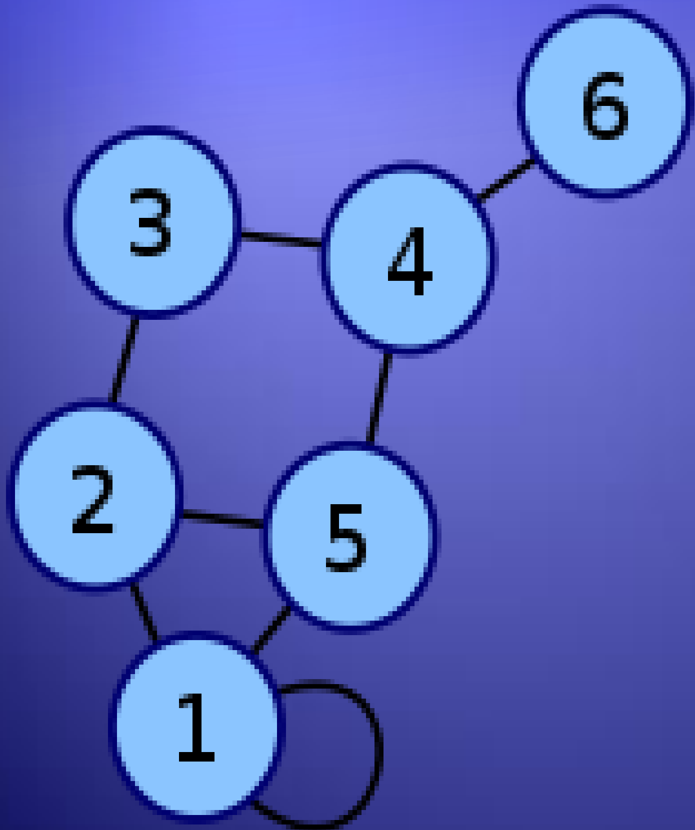


Networks

- Each group member will be given a clue to solve the problem
- One rule- you may not show your clue to anyone else. You may tell them about it, you may read it to them, but you cannot show it to them.
  - You have to tell others what you know
- Each clue contains connections between different barges that make up the city

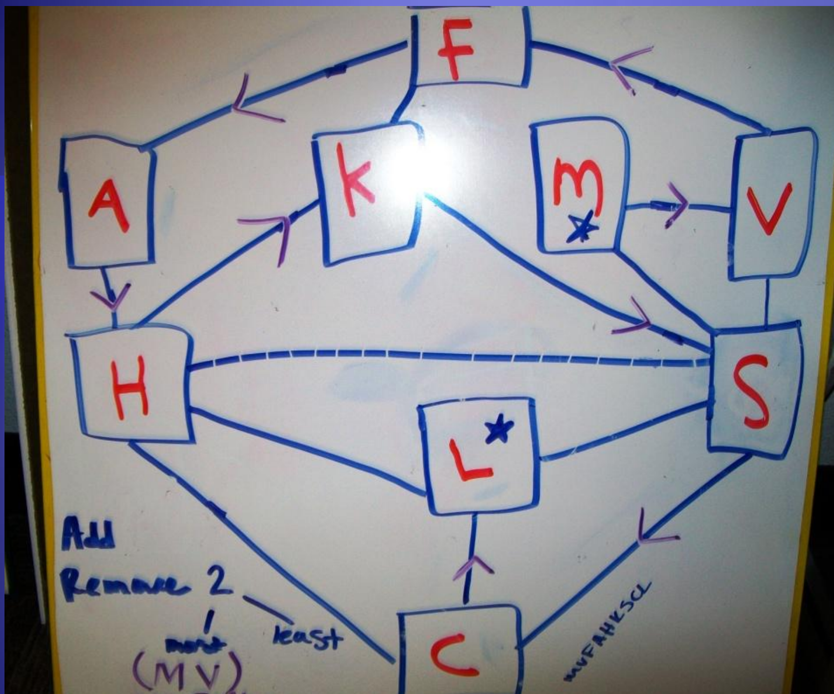
Adjacency Matrix

- **adjacency matrix:** a |*V*| x |*V*| array where each cell *i,j* contains the weight (or 1) of the edge between *vi* and *vj* (or 0 for no edge)



$$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

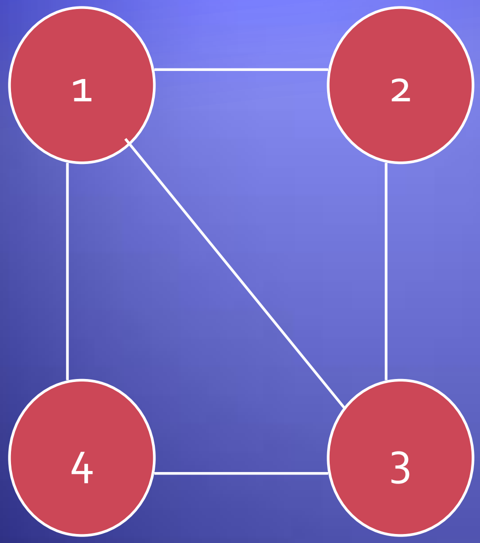
Adjacency Matrices



	Carp Condo	Library	Market	Museum	City Hall	Fat Albatross	Fish Company	Portside schools	Spindrift Villas
Carp Condo									
Library									
Market									
Museum									
City Hall									
Fat Albatross									
Fish Company									
Portside schools									
Spindrift Villas									

Coloring

- **chromatic number**: the smallest number of labels for a coloring of a graph
- **vertex coloring:** coloring the vertices such that no edge in E has two end-points with the same color
- What is the chromatic number of this graph?



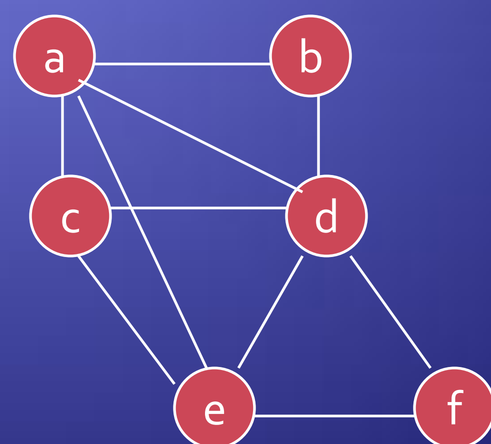
The Four Color Theorem

- The **four color theorem**, or the **four color map theorem**, states that given any separation of a plane into contiguous regions, called a *map*, the regions can be colored using at most four colors so that no two adjacent regions have the same color. Two regions are called *adjacent* only if they share a border segment, not just a point. This can be extended to all planar graphs.

Famous problem: Conflict graphs

- **Conflict graph:** a graph where each vertex represents a concept or resource and an edge between two vertices represents a conflict between these two concepts
- When the vertices represents intervals on the real line (such as time) the conflict graph is sometimes called an interval graph
- A coloring of an interval graph produces a schedule that shows how to best resolve the conflicts… a minimal coloring is the "best" schedule"
- This concept is used to solve problems in the physical mapping of DNA

	1	2	3	4
A	x	x	x	
B		x		
C			x	
D		x	x	x
E			x	x
F				x



Colors?

Assessment

- Must assess higher order thinking skills
- Rubrics to evaluate group progress

Student Assessment For Learning – Discrete Mathematics with Modeling: Unit 1

Name: ______________________________ Date: ____________________ Period: ______

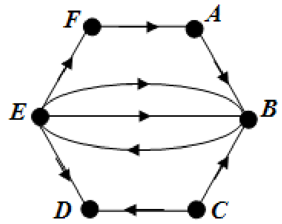
For each skill, mark whether or not you know this skill. For every review skill that ***you do not know***, write an action plan describing how you will acquire that skill prior to the test.

Skill	**Learning Target**	**Review**		**Test Assessment**	
		Know it?	Don't Know?	Correct or Incorrect?	Simple or Hard?
	General form for various models Linear $y = mx + b$ Quadratic $y = ax^2 + bx + c$ Polynomial $y = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$ Exponential $y = ab^{x-k} + k$ Logistic $y = \frac{c}{1 + ae^{-rt}}$				

Snapshot of an assessment

15. In a group of 8 friends, many have dated other members of the group. Phoebe has dated Eric, Scott, and Russell. Ruth has dated Scott; Claire has dated Eric and Russell; and Anne has dated Eric, Scott, Russell, and Matt. Draw a graph that models the dating relationships of the friends. (5 pts)

16. Find the adjacency matrix for the digraph below. (5 pts)



17. Draw a digraph with the adjacency matrix below. (5 pts)

$$\begin{array}{c} \boldsymbol{A} \\ \boldsymbol{B} \\ \boldsymbol{C} \\ \boldsymbol{D} \\ \boldsymbol{E} \end{array} \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

18. How many **two-step** paths are there altogether in the digraph in #17 above? (3 pts)

19. How many **three-step** paths from D to E are there in the digraph in #17 above? (3 pts)

How did we do?

Table 1.1 DMCM
Comparison of Participant and Comparison Groups
Group comparison on Post-test Scores

Discrete Mathematics Content Measure		
Group	**Post-score Mean (SD)**	**Significance p < .05**
Participant	**18.29** (3.29)	**.00**
Control	11.95 (3.17)	

Table 1.2 DMCM: Discrete Mathematics Content Measure
Participant Group Difference Pre-Post

Discrete Mathematics Content Measure: Participant Group			
Group	**Pre-score Mean (SD)**	**Post-score Mean (SD)**	**Significance p < .05**
Participant	**10.47** (3.41)	**18.29** (3.30)	**.00**

Table 1.3 DMCM: Discrete Mathematics Content Measure
Comparison Group Difference Pre-Post

Discrete Mathematics Content Measure			
Group	**Pre-score Mean (SD)**	**Post-score Mean (SD)**	**Significance p < .05**
Control	11.42 (2.72)	11.95 (3.17)	.24

How did we do?

Table 1.4 MSP Teacher Content Knowledge—Participants (Discrete Mathematics Content Measure)

PARTICIPANTS	
Project Name:	MSP
Test Name:	DMCM
My Name:	Haag
Year:	2010
P value <	0.00
Total number of records read:	27
Number of teachers with both pre-test and post-test scores:	27
Number of teachers with significant gains:	19

Table 1.5 MSP Teacher Content Knowledge—Comparison

COMPARISON	
Project Name:	MSP
Test Name:	DMCM
My Name:	Haag
Year:	2010
P value:	0.24
Total number of records read:	40
Number of teachers with both pre-test and post-test scores:	40
Number of teachers with significant gains:	0

How did we do?

Table 1.6 MSP Teacher RTOP Total—Participants

PARTICIPANTS	
	PUHSD ASU
Project Name:	MSP
Test Name:	RTOP
My Name:	Haag
Year:	2010
Z value:	-2.198
P value:	0.028
Total number of records read:	27
Number of teachers with both pre-test and post-test scores:	27
Number of teachers with significant gains:	20

- The Post RTOP score (M = 68.56, SD = 17.29) revealed a nine-point gain from the Pre RTOP score (M = 59.78, SD = 17.59), and the gain was significant (p = .028). PUHSD had a 15% increase in pre-post RTOP scores.

Observation Protocol

LESSON DESIGN AND IMPLEMENTATION

- 1. *The instructional strategies and activities respected students' prior knowledge and the preconceptions inherent therein.*
- 2. *The lesson was designed to engage students as members of a learning community.*
- 3. *In this lesson, student exploration preceded formal presentation*.
- 4. *This lesson encouraged students to seek and value alternative modes of investigation or of problem solving.*
- 5. *The focus and direction of the lesson was often determined by ideas originating with students.*

CONTENT

- Knowledge can be thought of as having two forms: knowledge of *what is* (Propositional Knowledge), and knowledge of *how to* (Procedural Knowledge). Both are types of content. The RTOP was designed to evaluate mathematics or science lessons in terms of both.
- **Propositional Knowledge**
- 6) *The lesson involved fundamental concepts of the subject.*
- 7) *The lesson promoted strongly coherent conceptual understanding.*

  8) *The teacher had a solid grasp of the subject matter content inherent in the lesson.*
- 9) *Elements of abstraction (i.e., symbolic representations, theory building) were encouraged when it was important to do so*.
- 10) *Connections with other content disciplines and/or real world phenomena were explored and valued.*
- **Procedural Knowledge**
- 11) *Students used a variety of means (models, drawings, graphs, symbols, concrete materials, manipulatives, etc.) to represent phenomena.*
- 12) *Students made predictions, estimations and/or hypotheses and devised means for testing them.*
- 13) *Students were actively engaged in thought-provoking activity that often involved the critical assessment of procedures.*
- 14) *Students were reflective about their learning.*
- 15) *Intellectual rigor, constructive criticism, and the challenging of ideas were valued.*

Observation Protocol

CLASSROOM CULTURE

- 16. *Students were involved in the communication of their ideas to others using a variety of means and media.*
- 17. *The teacher's questions triggered divergent modes of thinking.*
- 18. *There was a high proportion of student talk and a significant amount of it occurred between and among students.*
- 19. *Student questions and comments often determined the focus and direction of classroom discourse*.
- 20. *There was a climate of respect for what others had to say.*
- <u>Student/Teacher Relationships</u>
- 21. *Active participation of students was encouraged and valued.*
- 22. *Students were encouraged to generate conjectures, alternative solution strategies, and/or different ways of interpreting evidence.*
- 23. *In general the teacher was patient with students.*
- 24. *The teacher acted as a resource person, working to support and enhance student investigations.*
- 25. *The metaphor "teacher as listener" was very characteristic of this classroom.*

How did we do?

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2009	2786	1245	864	187	822		5904
	12.1%	5.4%	3.7%	0.8%	3.6%		25.6%
2010	3312	1252	885	197	1032	154	6832
	14.7%	5.6%	3.9%	0.9%	4.6%	0.7%	30.4%

Questions

- Jeanette Scott jscott@phxhs.k12.az.us
- Mona Toncheff toncheff@phxhs.k12.az.us

References

- https://sites.google.com/site/puhsddiscrete
- RTOP http://physicsed.buffalostate.edu/AZTEC/RTOP/RTOP_full/
- Netlogo http://ccl.northwestern.edu/netlogo/
- Dr. DeBellis http://discreteteaching.com/discrete_ii.html
- ***Navigating through Discrete Mathematics in Grades 6-12*** **NCTM 2008**
- COMAP *For all Practical Purposes :* ***mathematical literacy in today's world***
- ***United we Solve,*** Tim Erickson
- Jim Middleton jimbo@asu.edu
- Susan Haag- evaluator susan.haag@asu.edu