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INTRODUCING ROUGH DRAFT MATH TO SUPPORT TEACHER'S EFFORTS TO FOSTER STUDENT ENGAGEMENT AND LEARNING

EXAMINING DISTRICT MATHEMATICS LEADERS' SUPPORT FOR SCHOOL-BASED MATHEMATICS COACHES

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ROUGH DRAFT MATH FOR ENGAGED LEARNING

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JMEL 2025 INTRODUCING ROUGH DRAFT MATH TO PRESERVICE AND NOVICE MATHEMATICS TEACHERS TO SUPPORT THEIR EFFORTS TO FOSTER STUDENT ENGAGEMENT AND LEARNING

ABSTRACT

Mathematics educators often face the challenge of students disliking mathematics or experiencing a negative relationship with the subject. Intentional teaching practices can be a mechanism to mitigate this challenge; thus, preservice and novice teachers would benefit from opportunities to develop such teaching practices. This research explored the potential of rough draft math (RDM) to support teachers in addressing these issues. RDM is a pedagogical approach where students discuss and share their preliminary mathematical ideas without the fear of being wrong. Teachers welcome rough draft thinking, which gives students explicit opportunities to revise their work or thinking. This study examined the impact of RDM on preservice and novice secondary mathematics teachers through their written reflections on opportunities to learn about the approach through readings. Findings suggested that the readings can promote preservice and novice teachers' awareness of how RDM can foster a more comfortable and engaging learning environment, highlighting the importance of teachers holding a nonevaluative stance toward students' thinking and teachers' roles in facilitating mathematical discourse.

Keywords: mathematics teacher education, teaching practices, mathematical discourse, rough draft math

Background
The challenge of mathematics instruction in modern K–12 classrooms in the United States is multifaceted, encompassing the need to foster student engagement, instill confidence in students, and improve their mathematics performance. Despite the critical importance of mathematical literacy in a technology-driven world, many students report disliking mathematics, feeling anxious about it, and often failing to see its relevance to their lives (Ashcraft & Krause, 2007; Boaler, 2016, 2024). These challenges have been amplified since the COVID-19 global pandemic began in March 2020 (Hornstra et al., 2022). Such negative perceptions and experiences can lead to a vicious cycle of students avoiding challenges and performing poorly in mathematics, ultimately affecting their academic and professional futures.

The Problem: Pedagogical Practices Contributing to Students' Mathematics Disengagement
Numerous studies have documented widespread disengagement with mathematics among students. Rather than situating the issue solely with students, mathematics disengagement can be understood as a rational response to traditional pedagogical practices dominating mathematics classrooms. Hembree (1990) found mathematics anxiety affects a significant proportion of students, often leading to avoidance behaviors and diminished achievement. However, disengagement is not restricted to struggling students; even high-performing students frequently report a lack of enjoyment and intrinsic motivation (Eccles & Wigfield, 2002). This issue stems from students' limited opportunities to engage in meaningful and powerful learning experiences. Research also has shown that mathematics student engagement is malleable and influenced by teaching practices (Irvine, 2020; Zavala & Aguirre, 2023). Traditional approaches to mathematics education, which prioritize quick, accurate, and procedurally conforming solutions, can alienate students by focusing on rote learning rather than fostering deep conceptual understanding (Boaler, 2016, 2024). The roots of disengagement are in these historical and pedagogical choices, highlighting the need for instructional shifts that prioritize learning as an evolving process.

A Possible Solution: Rough Draft Math
Rough draft math (RDM; Jansen, 2020; Jansen et al., 2016) offers a potential solution to disrupting trends of student disengagement. Inspired by practices in writing instruction, where students are encouraged to produce and revise multiple drafts, RDM applies a similar approach to mathematical thinking. RDM teaching practices include (a) fostering a learning community that welcomes mistakes, unfinished thinking, and ideas in progress; (b) enacting tasks that invite students to share reasoning or multiple strategies; (c) highlighting strengths in students' drafts; (d) inviting students to revise; and (e) asking students to reflect on how their thinking changed (Jansen, 2020). In an RDM learning space, students can share their rough draft thinking verbally or through written work. By framing students' initial ideas as

rough drafts, teachers can create a classroom culture where exploring and making mistakes is normalized and valued. This process aligns with creating a safe classroom where learning from mistakes is safe (Lampert, 2001).

The RDM approach to teaching creates a learning environment that emphasizes growth in understanding over time. RDM is an example of an instructional approach that aligns with the growth mindset principles (Dweck, 2006), emphasizing that abilities can be developed through effort and practice. Educating students about a growth mindset and encouraging them to have one is not enough; students also need to experience a classroom where teaching practices communicate that growth and changes in their thinking are valued.

Purpose of the Study

In this study, we examined the impact of two relatively minimal interventions to support preservice and novice teachers learning about RDM. The first intervention involved engaging preservice secondary mathematics teachers (PSTs) in reading a journal article written for practitioners about RDM (Jansen et al., 2016) to investigate their perceptions of the RDM approach. The second minimal intervention explored novice secondary mathematics teachers' perceptions of RDM after reading an entire book about RDM (Jansen, 2020) and implementing the approach in their classrooms. We considered these interventions "minimal" because they were relatively low lifts for teacher–leaders to support teachers by engaging them in reading and subsequent reflections, in contrast with extensive coaching, for example. We wondered about the degree to which this minimal intervention could have impacts on PSTs' thinking, novice teachers' thinking, and novice teachers' practice. The study sought to provide valuable information about PSTs' and novice secondary mathematics teachers' perceptions of RDM under different conditions, such as exposure to RDM through reading a short article, a more in-depth study through reading an entire book, and attempts to enact RDM in their classroom practices after completing the readings. Understanding the impact of different opportunities to learn from PSTs and novice secondary mathematics teachers' perceptions of RDM can inform mathematics teacher–leaders' practices.

LITERATURE REVIEW

Mathematics Anxiety and Its Impacts

Mathematics anxiety presents a significant barrier to student engagement and performance. It is characterized by feelings of tension, apprehension, and fear, interfering with students' abilities to manipulate numbers and solve mathematical problems in a wide variety of ordinary life and academic situations (Ashcraft & Krause, 2007). High levels of anxiety can lead to avoidance behaviors, where students resist taking advanced mathematics courses, participating in class discussions, or even attempting to solve problems (Hembree, 1990). Such avoidance also can result in a lack of foundational skills and a negative feedback loop, further entrenching students' fears and dislike of mathematics.

In a meta-analysis of 747 effect sizes from 1992–2018, Barroso et al. (2021) found a small-to-moderate, negative correlation between mathematics anxiety and mathematics achievement, moderated by factors such as grade level and types of mathematics assessments—with the effect starting in childhood and remaining significant through adulthood. In a meta-analysis of 177 studies involving 906,311 participants, Caviola et al. (2022) also found mathematics anxiety and test anxiety impacted mathematics performance significantly. Mathematics anxiety is often linked to a fear of making mistakes, which is tied closely to performance-avoidance goals (Skaalvik, 2018). Students with these goals strive to avoid situations where their peers might notice their struggles or mistakes in solving problems. Mathematics anxiety and test anxiety are highly correlated with one another (Kazelskis et al., 2000), so students may be anxious about being wrong while anxious about performing well when doing mathematics. Reducing the pressure of these experiences when students feel judged for not being immediately correct might decrease students' avoidance of challenging learning opportunities. In a study of 2,551 secondary students, Fiorella et al. (2021) found test anxiety was correlated negatively with mathematics achievement. Additionally, in a meta-analysis of 57 studies, Finell et al. (2022) found a negative correlation between mathematics anxiety and mathematics performance, confirming working memory significantly mediated this relationship. We posit that RDM could reduce pressure for students to be correct during initial attempts at mathematics problem solving (a) if students' rough drafts are treated as valuable resources for everyone's learning and (b) if multiple attempts are welcomed via opportunities for revision in mathematics classes.

Engagement and Discourse in Mathematics Classrooms

Engaging in mathematical discourse is essential for fostering deeper conceptual understanding and enhancing learning outcomes. Kazemi and Stipek (2001) emphasized the role of sociomathematical norms (i.e., requiring students to explain their reasoning and explore connections among strategies) in promoting meaningful mathematical discussions. These practices encourage students to articulate their thinking, justify their solutions, and engage with their peers' perspectives. Boaler (2016, 2024) further highlighted that traditional approaches often prioritize procedural fluency, neglecting the critical value of dialogue in understanding mathematical concepts. By integrating discourse into instruction, teachers create a collaborative learning environment where students feel supported in navigating challenges and developing a more profound mastery of mathematics.

Revising, as part of mathematical discourse, plays a crucial role in refining and deepening understanding. Errors and misunderstandings, when addressed openly, become valuable opportunities for the reconceptualization and exploration of alternative strategies (Kazemi & Stipek, 2001). Boaler (2016) noted encouraging students to view revision as a natural and essential part of the learning process helps shift their focus from correctness to growth and discovery. In this context, discourse allows students to reexamine their solutions, compare them with peers, and refine their ideas

collaboratively. As Boaler (2024) later suggested, such iterative processes nurture mathematical mindsets by normalizing mistakes and emphasizing perseverance and creativity in problem solving. Through this cycle of discussion and revision, students enhance their understanding and develop a resilient and confident approach to learning mathematics.

The Concept of Rough Drafts in Education

The idea of using rough drafts is well established in writing instruction, where teachers encourage students to produce multiple drafts of their work, receive feedback, and make revisions. This process helps students develop their ideas and improve their writing skills over time (Murray, 1972). Applying a similar approach to mathematics can help students view their initial ideas as a starting point for further exploration and refinement rather than as final products to be judged and graded (Jansen, 2020; Jansen et al., 2016).

The process of rough drafts and revising in mathematics was inspired by the concept of exploratory talk (Barnes, 2008). As Barnes (2008) described, students often experience classroom discussions as being asked to perform what they know, which can feel like a final draft; however, if students engage in open discussion and the community learns together, this process feels more exploratory. When the second author read about exploratory talk with secondary teachers, they decided to rename Barnes's idea of exploratory talk as "rough draft talk" (Jansen et al., 2016), because they conjectured that the label of rough draft talk might carry more meaning for students than the label of exploratory talk. If discussions are referred to as rough draft talk, students can view classroom discussions as sites for continued learning rather than performing what they know for others. Rough draft talk conversations can help students revise their thinking while learning in community with and from their peers and teachers.

RDM in Practice

RDM involves several key practices that support mathematical discourse and student engagement. These practices include (a) fostering a learning community where mistakes and unfinished thinking are accepted, (b) enacting tasks that invite students to share their reasoning and strategies, (c) highlighting strengths in students' drafts, (d) inviting students to revise their work, and (e) asking students to reflect on how their thinking has changed (Jansen, 2020). Rathouz et al. (2023) found framing online discussion boards as RDM learning spaces encouraged every student to share their diverse mathematical approaches, perspectives, and ideas. Thanheiser and Jansen (2016) also showed how providing learners the opportunity to consider their perceptions of the completeness and correctness of their work before sharing it publicly helped learners feel more comfortable sharing. As a result, learners recognized the value of sharing, and their metacognitive skills improved. These prior studies conducted in mathematics educators' courses at the university level also have focused on engaging future teachers in RDM within the context of learning mathematics for teaching; however, there is still much for mathematics education leaders to understand about how to support future teachers' learning about RDM in pedagogical methods courses for teaching mathematics.

Teachers' Perceptions of Teaching Approaches and Their Impact on Implementation

Understanding mathematics teachers' perceptions of an instructional approach (e.g., RDM) is essential for facilitating its successful implementation. Such understanding enables educational communities to provide targeted support, address barriers, and customize approaches to fit various educational contexts, ultimately leading to improved teaching and learning outcomes. Several strands of educational research have substantiated the need to understand mathematics teachers' perceptions of a teaching approach to support their effective implementation of that approach.

First, teachers' perceptions influence their teaching practices and willingness to adopt new methodologies significantly. According to Pajares (1992), teachers' beliefs about education are linked closely to their instructional decisions and classroom behaviors. When math teachers view a teaching approach favorably, they are more likely to implement it with fidelity, subsequently enhancing its potential benefits for student learning (Richardson, 1996).

Moreover, understanding teachers' perceptions helps identify potential barriers to implementation. Beswick's (2007) research indicated teachers often face external and internal barriers when integrating new teaching approaches. External barriers include a lack of resources and support, whereas internal barriers involve beliefs and attitudes toward the approach. By understanding these perceptions, educational leaders can address specific concerns, tailor professional development (PD) programs, and provide the necessary resources to overcome these barriers.

Additionally, teachers' perceptions are crucial for adapting teaching approaches successfully to different educational contexts. As Spillane et al. (2002) pointed out, teachers interpret and adapt new approaches based on existing knowledge, experiences, and specific needs of their students. Understanding these perceptions allows for the customization of support strategies, ensuring the teaching approach is integrated effectively into diverse classroom environments (Darling-Hammond & Bransford, 2007).

Furthermore, research by Fullan (2001) emphasized that change in educational practice is a complex process that requires understanding and addressing teachers' subjective experiences. Teachers' perceptions provide valuable insights into the practical challenges and successes they encounter, thereby informing more effective and sustainable support mechanisms. To understand teachers' perceptions of RDM, we investigated how PSTs and novice teachers made sense of RDM after reading about the approach, including how novice teachers reported enacting RDM after the readings.

Research Questions

We addressed the following research questions for this study:

1. What are PSTs' initial perceptions and understandings of RDM before their internship?
2. How do PSTs' perspectives on RDM evolve after gaining classroom experience?

3. Which RDM teaching practices do PSTs and novice teachers find most salient, feasible, and challenging, and what are the reasons behind their choices?
4. What factors influence PSTs' and novice teachers' decisions to implement RDM practices, and how do these factors shape their teaching approaches?

METHODS

Opportunities for Teachers to Learn About RDM

We conjectured that exposing PSTs to the RDM approach through the minimal intervention of reading and reflecting on an article (Jansen et al., 2016) would plant a seed that could potentially grow during their field experiences. We anticipated PSTs might be skeptical, especially if they had not experienced RDM approaches as mathematics students. We thought experiencing RDM as students would help preservice teachers better understand the approach. Therefore, the first author modeled the RDM approach during participants' university coursework throughout the study. We were also concerned that participants may become more skeptical after attempting to implement RDM approaches in their classrooms. Given our concerns, the first author asked the second group of novice teachers to read and discuss the full *Rough Draft Math: Revising to Learn* book (Jansen, 2020). We conjectured that reading and discussing the book would have a positive impact on novice teachers' confidence, knowledge, and skills. Although we anticipated all teachers would desire some support in implementing the approach, we were uncertain which practices teachers would consider most challenging to implement.

Participants

The study involved 19 secondary mathematics teachers from a rural region in the southern United States. All participants were enrolled in mathematics education programs at a large research institution in the southern United States. Thirteen participants were undergraduate seniors enrolled in a secondary mathematics teacher preparation program. These 13 PSTs were invited to participate in the study by reading an article about RDM (Jansen et al., 2016) and providing reflections in an individual online assignment immediately after the reading. The reading and reflection assignments were required course assignments in a teaching secondary mathematics course, which was taught by the first author; however, PSTs could elect whether to participate in the study. All 13 PSTs gave their consent to participate. Four of the 13 PSTs identified as men, and nine identified as women. Two of the 13 PSTs were Black, and 11 were White. The first author modeled the RDM approach throughout the course so PSTs could experience the approach from a student's perspective.

Six months later, 13 PSTs and six additional novice teachers who read the full RDM book (Jansen, 2020) were individually asked additional questions in an online assignment. By this point, PSTs had opportunities to implement the RDM approach during their full-time internships. The six novice teachers were enrolled in a graduate-level teaching secondary mathematics course, also taught by the first author. One of the six novice teachers obtained her teaching license through an undergraduate secondary mathematics education teacher preparation program. This novice teacher was in her 4th year of teaching secondary math. The five other novice teachers had earned bachelor's degrees in kinesiology, information technology, physics, business administration, and meteorology; were each in their 1st years of teaching; and had obtained their secondary mathematics teaching licenses through an alternate route graduate degree program. All six novice teachers were White. Four novice teachers identified as men, and two identified as women. All worked at rural public schools. The six novice teachers were also invited to participate in the study by reading a book about RDM (Jansen, 2020) and sharing their reflections. All six novice teachers gave their consent to participate. As with the undergraduate course, the first author was the instructor and modeled the RDM approach throughout the graduate course so teachers could experience the approach from a student's perspective.

Positionality of Authors

The authors of this article are two mathematics teacher educators. The first author was the instructor of the undergraduate and graduate-level secondary mathematics education courses and designed follow-up prompts for the study in consultation with the second author. The second author was a recognized expert on the RDM approach, having authored an influential article and a book on the subject that teachers in this study read and discussed. We are strong advocates for the RDM approach, believing in its potential to influence students' beliefs and actions positively regarding mathematics. We both model RDM when we teach courses in mathematics education.

Still, we acknowledged the importance of ensuring the reliability of findings, particularly given our personal investment in the success of the RDM approach. To mitigate potential bias, we implemented several measures during the analysis. First, we engaged in reflexive practices, regularly discussing assumptions and ensuring we approached data with openness to positive and negative outcomes. We also employed member checking by seeking participants' feedback to validate the accuracy of our interpretations. Finally, we conducted peer debriefings with colleagues outside the project to scrutinize the findings further (Saldaña, 2013). These strategies helped to ensure results were grounded in the data and not overly influenced by personal RDM advocacy.

Data Collection

Data were collected through written reflections from the 13 participating PSTs and six novice teachers. To address Research Question 1 regarding PSTs' initial perceptions and understanding of RDM before their internship, the 13 PSTs who read an article about RDM (Jansen et al., 2016) responded to the following prompts:

1. What did you learn?
2. What did you find interesting?
3. A question you have.

To address Research Question 2 regarding how PSTs' perspectives on RDM evolved, 6 months after trying RDM with their mathematics students during their internships,

four PSTs completing their full-time teaching internships responded to Prompt 4:

4. How has your thinking about RDM changed?

To address Research Questions 3 and 4 regarding which practices PSTs and novice teachers found most salient, feasible, or challenging—and what factors influenced their decisions to implement these practices in their classrooms—the four PSTs who were completing their full-time teaching internships and the six additional novice teachers who read the full RDM book (Jansen, 2020) responded to Prompts 5–8:

5. Which of the following RDM teaching practices do you consider salient (i.e., most important to you)? Why/how?
    a. fostering a learning community where mistakes, unfinished thinking, ideas in progress, and ideas that you are not sure about are okay;
    b. enacting tasks that invite students to share reasoning and/or multiple strategies;
    c. highlighting strengths in students' drafts;
    d. inviting students to revise; and
    e. asking students to reflect on how their thinking changed.

6. Which of the following RDM teaching practices do you consider feasible (i.e., most possible to put into practice in your classroom)? Why/how?
    a. fostering a learning community where mistakes, unfinished thinking, ideas in progress, and ideas that you are not sure about are okay;
    b. enacting tasks that invite students to share reasoning and/or multiple strategies;
    c. highlighting strengths in students' drafts;
    d. inviting students to revise; and
    e. asking students to reflect on how their thinking changed.

7. Which of the following RDM teaching practices would you like help with? Why/how?
    a. fostering a learning community where mistakes, unfinished thinking, ideas in progress, and ideas that you are not sure about are okay;
    b. enacting tasks that invite students to share reasoning and/or multiple strategies;
    c. highlighting strengths in students' drafts;
    d. inviting students to revise; and
    e. asking students to reflect on how their thinking changed.

8. What factors influence your decisions to use RDM or not? How and why do they influence your decisions?

The data corpus comprised 13 PST responses to Prompts 1–3, four PST responses to Prompt 4, and 10 responses (i.e., four PSTs and six novice teachers) to Prompts 5–8, resulting in 83 responses. We typed and organized the responses into a spreadsheet file to facilitate the coding process.

Data Analysis

We conducted a thorough examination of data using thematic analysis, as Braun and Clarke (2006) outlined. This method enabled us to uncover recurring patterns, overarching themes, and categories in the data set, providing a deeper understanding of PSTs' and novice teachers' responses (Braun & Clarke, 2006). Our analysis involved several key steps—(a) familiarizing ourselves with data, (b) coding, (c) developing themes, and (d) interpreting the findings—ensuring the process was rigorous and reliable (Nowell et al., 2017).

Once we identified codes, we shifted to deductive coding (Bingham & Witkowsky, 2022) to categorize responses. This step allowed us to organize data based on key themes identified in previous analyses. Subsequently, we analyzed data for representative and exceptional quotes to enrich our understanding of the PSTs' and novice teachers' perspectives. To ensure the consistency and accuracy of our analysis, two mathematics teacher–educators coded each response independently, working in an anonymous manner. Any discrepancies were resolved through collaborative discussions.

To uncover PSTs' initial perceptions and understanding of RDM before their internship (i.e., Research Question 1), we asked PSTs to read an article about RDM (Jansen et al., 2016) and share what they learned, found interesting, and questions they had. Overall, PSTs reported a perception that RDM can promote student confidence by creating a low-pressure environment conducive to risk taking and learning. Initially skeptical, they ultimately saw value in RDM talk as a strategy for improving student engagement and comfort with ambiguity; however, their concerns about implementation reflected the practical realities teachers can face in translating theory into practice, indicating successful adoption of RDM will likely require ongoing support and adaptation to various instructional contexts. Next, we elaborate upon this finding by sharing participants' voices.

RDM Fosters a Safe Environment for Risk Taking and Learning

PSTs consistently highlighted that RDM, particularly through RDM talk, provided students with a safe space to share their ideas without fear of being wrong. The participants perceived this sort of supportive atmosphere could help students feel comfortable taking risks, which would be critical for deeper learning and participation in mathematical discourse. For example, one PST stated, "It [RDM] allows the students to more comfortably share their thoughts and ideas about a given topic in math without having to worry about being wrong or right, which leads to higher confidence." Another PST shared, "RDM creates a more positive, safe classroom for thinking." PSTs recognized that RDM shifted classroom dynamics by reducing pressure to always be correct, fostering greater student engagement. This shift in classroom dynamics was seen as crucial for enhancing confidence, especially among students who typically felt hesitant to participate in class discussions.

Skepticism Evolving Into Appreciation of RDM's Impact
Many PSTs initially expressed skepticism about the efficacy of RDM talk in promoting meaningful classroom participation; however, after reflecting on the positive student responses presented in the article, they came to appreciate how RDM normalized mistakes as part of the learning process, thereby validating students' contributions regardless of correctness. For example, one PST shared, "I was a little skeptical at first. . . . But the quotes from the students [in the 2016 article] about how this strategy made them feel okay with being wrong made me believe that this would work." This shift from doubt to acceptance underscored how evidence of student experiences can reshape teacher perceptions. PSTs moved from questioning RDM's feasibility to recognizing its potential for cultivating a more inclusive and reflective classroom environment.

Concerns About Practical Implementation
Although PSTs appreciated the pedagogical value of RDM, they also raised concerns about its practical application. Key considerations included the time required to implement RDM talk, how it might fit into different subject areas, and its scalability across grade levels. For example, one PST stated, "How much time does this take out of the classroom, and is the amount it takes harmful against lecture time, or other time to be practicing problems?" Although PSTs were intrigued by RDM's potential, their concerns about logistical challenges suggested the need for further PD to integrate such practices into diverse classroom contexts effectively. Their reservations highlighted the balance between instructional innovation and practical feasibility.

Research Question 2
To uncover how PSTs' perspectives on RDM evolved 6 months later, after gaining classroom experience (i.e., Research Question 2) and trying the RDM approach in their classrooms during their teaching internship, we asked the PSTs how their thinking about RDM had changed. Overall, PSTs' perspectives evolved to reflect a more realistic understanding of RDM's application. Although they continued to believe in its potential to create a supportive learning environment, they also recognized the need for more active teacher involvement to overcome student resistance and facilitate deeper engagement. This finding highlighted the importance of scaffolding in RDM practices to help students become more confident and independent thinkers. We elaborate further on this finding by sharing participants' voices next.

Even after gaining classroom experience, PSTs maintained their belief that RDM can foster a positive, low-stress atmosphere conducive to student participation. These participants continued to view RDM as a valuable tool for encouraging students to engage in mathematical thinking without fear of making mistakes. Their initial understanding of RDM as a strategy for reducing student anxiety and promoting open discussion appeared to persist throughout their internship experiences; for example, one PST stated, "I still think it fosters a less stressful environment for students. I would still want to foster this mindset in my classroom." PSTs remained committed to using RDM to cultivate a safe space for student expression and risk taking, even as they recognized challenges of implementing it consistently.

Realizing the Challenges of Student Engagement and the Need for Scaffolding
Although PSTs upheld the benefits of RDM, their classroom experience highlighted practical challenges in student engagement. The participants observed many students were hesitant to take risks, struggled to start solving problems independently, and often required teacher intervention to begin their thought processes. Classroom experiences deepened PSTs' understanding of RDM's limitations when applied in practice. The PSTs realized although RDM can establish a supportive environment, the approach does not lead to active student engagement automatically. For example, one PST shared, "I have noticed that students do not know where to start when answering problems. Oftentimes, I have to ask students questions about the problems to start their thought process." These participants also recognized teachers need to provide additional scaffolding (e.g., asking guiding questions) to help students overcome hesitation and initiate their problem-solving efforts.

To address Research Question 3, we asked interning PSTs and novice teachers which RDM practices they found most salient, feasible, and challenging. PSTs and novice teachers clearly valued the practice of fostering a learning community where mistakes are embraced, viewing it as essential for creating a supportive classroom atmosphere. This practice increased student participation and laid groundwork for other RDM strategies to be effective. PSTs and novice teachers found inviting students to revise their work as the most feasible RDM practice, believing it fit well with their existing classroom routines and assessments. In contrast, encouraging students to reflect on their thinking posed a significant challenge, as it required deeper engagement and metacognitive skills with which students often struggled. This tension between fostering a supportive environment and promoting more complex cognitive tasks highlighted the need for further strategies and support to encourage reflection in students. Further elaboration and participant voices that reflected these findings are presented next.

Salient Practice: Fostering a Learning Community That Embraces Mistakes
PSTs and novice teachers overwhelmingly identified the importance of creating a classroom environment where mistakes, unfinished ideas, and ongoing thinking are accepted and encouraged. These participants viewed this practice as crucial for promoting student engagement, confidence, and intellectual risk taking. Many participants believed fostering this supportive community was the foundation for other RDM practices to succeed. PSTs and novice teachers saw fostering a mistake-friendly classroom culture as the most important RDM strategy. One representative quote was, "I think fostering a learning community where mistakes are okay is the most salient to me. This allows your students to feel safe in your classroom just as they should at home." Another participant said, "If the teacher can provide or create this [community],

then the others [RDM practices] will happen. Without it, I do not think [the other RDM practices] will happen naturally." The participants believed when students felt safe to make mistakes, they were more likely to engage in learning and share their reasoning, thereby deepening their understanding. This practice was seen as a prerequisite for other RDM activities, such as revision and reflection.

Feasible Practice: Inviting Students to Revise Their Work

When considering feasibility, PSTs and novice teachers found the practice of inviting students to revise their work to be the most actionable in their classrooms. They perceived this practice as easy to implement, often integrating revisions into assessments like quizzes and tests. PSTs and novice teachers appreciated the opportunity revising provided for students to learn from their mistakes and improve their understanding. They also viewed revision as a natural extension of fostering a supportive environment. PSTs and novice teachers found inviting students to revise their work highly feasible because they could incorporate it seamlessly into existing classroom structures, such as assessments. One participant reported, "Inviting students to revise would be one of the most feasible ones to put into practice. It would be easy to give students a quiz or test and ask them to revise any incorrect responses." Another participant wrote, "This [inviting students to revise] allows higher scores than just taking [assessments] at face value." Participants appeared to perceive that inviting revisions encouraged deeper learning by giving students additional opportunities to reflect on and improve their work, making it a practical and beneficial practice in the RDM framework.

Challenging Practice: Encouraging Student Reflection

One of the most challenging RDM practices for PSTs and novice teachers was engaging students in reflecting on how their thinking had changed. PSTs and novice teachers struggled with students' reluctance or difficulty articulating their thought processes, particularly in mathematics, where metacognition could be less natural for many students due to a lack of opportunities to reflect on their learning. Participants also discussed facing challenges in helping students see the value of reflecting on and learning from their mistakes. One participant wrote, "Asking the students to reflect on how their thinking changed is hard . . . students have a difficult time articulating their thoughts." Another wrote, "I think actually having students pinpoint how their actual thinking changed is hard for multiple reasons, but mostly due to students having a difficult time articulating their thoughts." PSTs and novice teachers found encouraging students to reflect on their thinking particularly difficult, as it required students to engage in metacognitive processes that many found uncomfortable or unnatural. The challenge was not only in getting students to reflect meaningfully but also in helping them recognize the importance of this reflection in their learning. Teachers expressed a need for strategies to facilitate and support student reflection.

Research Question 4

To address Research Question 4, we asked interning PSTs and novice teachers what factors influenced their decisions to implement RDM in their classrooms. Findings indicated PSTs and novice teachers were influenced deeply by their students' engagement levels when deciding to implement RDM practices. Their desire to create a supportive and mistake-friendly environment reflected their commitment to fostering student learning. However, time constraints and standardized testing pressures posed significant challenges that at times limited their effective RDM implementation. PSTs and novice teachers needed to navigate these challenges while balancing their pedagogical ideals with the practical realities of classroom constraints, suggesting a need for systemic changes that support more flexible teaching approaches.

Student Engagement and Responses as Driving Factors

Students' engagement and responsiveness are significant factors influencing PSTs' and novice teachers' decisions to implement RDM practices. PSTs and novice teachers expressed that their strategies were heavily guided by how students reacted to different teaching methods. For instance, when students showed interest and participation, teachers were more likely to continue using RDM practices. Conversely, apathy or disengagement from students prompted participants to reconsider or modify their approaches. One participant shared, "As a teacher, you have to know your students and what they respond to positively. I think ultimately how my students respond influences whether I use RDM throughout the class or not." Another participant wrote, "The responses I receive from my students influence my decisions to use RDM. If my students respond well to a strategy, I will continue using it." PSTs and novice teachers emphasized that understanding their students' needs and preferences is crucial for effective RDM implementation. Participants highlighted the importance of creating a classroom culture where mistakes were viewed positively because it allowed students to feel comfortable engaging in the learning process. Such awareness drove PSTs and novice teachers to adapt their methods based on student feedback, ultimately fostering a more interactive and supportive learning environment; however, novice teachers may not have recognized the significant influence they had on their students' responses to the RDM practices. If their students resisted RDM initially, they could engineer experiences where students experience the benefits of RDM, leading them to recognize RDM's values.

Time Constraints and Curriculum Alignment as Challenges

Although PSTs and novice teachers recognized the potential benefits of RDM, they also cited significant challenges that influenced their decisions. Time constraints, driven primarily by testing schedules and curriculum demands, were mentioned frequently as barriers to implementing RDM practices fully. PSTs and novice teachers expressed concern that the structured nature of state testing often limited their ability to allow for open discussions, revisions, and collaborative learning experiences, which are central to RDM. A participant reported, "The biggest factor in using RDM is time constraints and simplicity of information... sometimes information and problems presented do not align themselves well with RDM." Another participant wrote, "Several of the ideas [from RDM] seemed achievable, but there are many time constraints due to state testing schedules." The perceived pressure of curriculum

requirements and standardized testing appeared to shape how these early career teachers approached RDM. Many participants felt the rigidity of their schedules did not align with the more flexible, discussion-oriented nature of RDM practices. Consequently, this mismatch could deter them from incorporating RDM strategies, as they prioritized covering essential content over facilitating a more open learning environment. However, if teachers do not invest time in having students draft and revise, students may not develop a full understanding of the material, and teachers may need to spend time reteaching it. Accordingly, investing time to draft and revise thinking in mathematics may be worth it.

Comparing PSTs and Novice Teachers

Both novices and PSTs indicated they valued RDM and used RDM practices; however, novices reported a more comprehensive integration of RDM into their teaching practices than PSTs. Novices also reflected on the transformative impact RDM practices had on their students' learning. Novices demonstrated a nuanced understanding of RDM practices and the purpose of enacting RDM practices. Novices connected RDM practices to fostering student confidence and deeper engagement. One novice stated, "I want students to see the progress in their abilities as well as know that my focus is on their progress and not just their initial thoughts." This quote highlighted the novice's commitment to prioritizing growth over correctness. Another novice reflected, "Students need to feel comfortable enough in class to make mistakes but still share their reasoning with other students to form deeper understanding." This quote demonstrated the novice's understanding of how RDM fosters collaboration and exploration. By explicitly recognizing the role of mistakes and revisions in conceptual learning, the novices illustrated their integration of RDM practices into their broader instructional goals.

In comparison, PSTs often focused on the logistics of implementing specific RDM practices but lacked deeper connections to the impact on student learning. One PST noted, "I think fostering a learning community where mistakes are ok is the most salient to me. I think this allows your students to feel safe in your classroom just as they should at home." Although this response indicated the PST valued their students' emotional safety, the participant failed to elaborate on the implications for mathematical thinking. Another PST described challenges with promoting student reflection, stating, "It is kind of hard to get students to reflect on their thinking or see how it has changed." This response indicated the PST struggled to implement a key RDM practice effectively. Although PSTs recognized the value of RDM practices, their reflections often highlighted difficulties in execution, suggesting their understanding of how to leverage these practices to support deeper learning remains in development.

Discussion

Findings from this study illuminated critical insights into the implementation of RDM practices among PSTs and novice teachers, revealing the perceived effectiveness of varied approaches to introducing RDM to teachers and challenges they reported encountering during classroom implementation. One significant discovery was the marked disparity in effectiveness between the different approaches to introducing RDM, particularly the advantages of reading a full-length book on RDM compared to a single article on RDM. The depth and comprehensive nature of the book appeared to foster a more profound understanding of RDM practices and philosophies among novice teachers compared to reports from preservice teachers. Although the article provided valuable introductory information, the book allowed novice teachers to engage with the material in more meaningful ways, encouraging them to delve into specific case studies, reflective exercises, and practical strategies for implementation. Such an in-depth exploration equipped the teachers with a broader range of tools and insights to adapt to their unique classroom contexts. The greater depth of understanding demonstrated by novice teachers compared to PSTs may also be attributed to their extended time in the field and their status as more experienced, older students. This insight reinforced the importance of mathematics education leaders selecting professional learning materials that align with the developmental needs of educators. Mathematics teacher leaders should consider integrating full-length texts as foundational elements of PD programs while designing supplementary guided discussions and reflections to deepen teacher learning.

Moreover, this study underscored that PSTs' and novice teachers considered it critically important to cultivate a supportive learning environment that emphasizes the acceptance of mistakes and unfinished thinking as part of the learning process. PSTs and novice teachers consistently expressed a desire to create classrooms where students feel safe taking risks and learning from their errors, aligning with RDM's foundational principles. Data reflected that many PSTs and novice teachers perceived fostering such an environment can boost student engagement and enhance the overall learning experience. This insight emphasizes the responsibility of mathematics education leaders to model these practices in PD sessions. Teacher–leaders should demonstrate how fostering a community of learners—where risk taking and revision are valued—can transform classroom cultures to align with the principles of RDM. By doing so, PD can serve as a mirror for classroom practices leaders hope to see implemented by teachers.

However, PSTs and novice teachers experienced challenges when implementing RDM, particularly concerning time constraints and curriculum alignment. This study revealed many novice teachers perceived and experienced systemic pressures, particularly from standardized testing schedules, which often limited their opportunities to engage their students in the reflective and collaborative processes essential to RDM. Several novice teachers noted the rigid nature of their curricular requirements sometimes clashed with RDM's ideals, making it difficult to incorporate practices that encouraged discussion and revision. This finding echoed previous studies (Horn, 2012; Lampert et al., 2010), highlighting the tension between ambitious instructional practices and institutional constraints. Mathematics

education leaders must advocate for policies that allow flexible teaching methodologies and promote practices such as RDM, emphasizing the value of reflection and collaboration. By collaborating with policymakers and district leaders, teacher–leaders can work to reduce systemic barriers and create conditions for sustained implementation of RDM.

Another critical theme emerging from the data was the influence of student responses and behaviors on teachers' decisions to adopt RDM practices. Many PSTs and novice teachers articulated the level of student engagement and willingness to embrace mistakes impacted their commitment to implementing RDM in their classrooms significantly. This relationship is vital because it emphasizes the need for teacher–leaders to prepare PSTs and novice teachers to cultivate a classroom environment that encourages risk taking and open communication. Building on prior literature (Boaler, 2016; Jansen et al., 2024), this study highlighted how teacher–leaders can leverage PD opportunities to equip educators with strategies to implement RDM practices. For example, teacher–leaders might include explicit training on inviting revisions, purposeful task selection, and reframing mistakes as learning opportunities in professional learning communities (PLCs), which can help teachers see RDM practices as salient and feasible (Jansen et al., 2024).

The small sample size of this study presented limitations that also warrant discussion. Although findings provided significant insights into participants' perceptions and practices, their generalizability across broader contexts remains uncertain. Future researchers should aim to include a larger and more diverse cohort of participants to strengthen the validity of the findings and provide a richer data set for analysis. Incorporating a pre- and post-survey also would have enhanced our ability to measure changes in PSTs' and novice teachers' thinking before and after engaging with the article or book and applying RDM in their classrooms. Such a survey would have facilitated a clearer understanding of their shifts in perspectives and practices resulting from exposure to RDM. This recommendation aligns with frameworks for measuring teacher growth, such as those Guskey (2002) proposed, which emphasize the need for longitudinal data collection to capture the sustained impact of PD. Furthermore, longitudinal studies could provide insights into how participants implement RDM practices over time and how their perceptions evolve as they gain experience. A comprehensive approach to research on RDM could reveal consistent patterns and challenges in early career teachers, which can provide insight to inform the design of more targeted interventions to support teachers' learning.

In light of these findings, future iterations of the intervention should be redesigned to incorporate more interactive components, such as collaborative workshops or peer mentoring opportunities, alongside reading assignments. These elements would encourage PSTs and novice teachers to engage more actively with the material and learn from one another's experiences. Providing opportunities for real-time practice and feedback on RDM implementation also can better prepare these educators for the complexities of teaching mathematics. Technology-based tools, such as virtual coaching platforms or asynchronous forums, could facilitate sustained educator collaboration, further enhancing professional growth. Mathematics education leaders can play a pivotal role in facilitating these initiatives, ensuring teacher preparation programs and PD offerings are grounded in research-based principles and adapted to meet the contextual needs of teachers.

For example, since conducting this analysis, the second author has been experimenting with supporting PSTs by having more explicit engagement with RDM. In a recent pedagogical methods course for middle school PSTs, the second author modeled a discussion with PSTs about the value of rough drafting and revising in mathematics class. Then, the second author engaged PSTs in a mathematics learning experience that involved drafting and revising, which has happened regularly throughout the semester. At the end of the mathematics experience, PSTs reflected on how their thinking changed and the value of revising. Then, in their field placements or internships, PSTs (a) engaged their own middle school students in a conversation about rough drafts and revising; (b) enacted a three-act math task that involved establishing a problem to investigate through noticing and wondering, estimating possible answers, and then revising their thinking; and (c) had their students reflect on how their thinking changed. Their reflections from their practice were initially promising, but future analyses on this approach's effectiveness in teacher education are needed. Experiencing explicit modeling of what they can do with their students and immediately applying this approach in a classroom could impact PSTs' and novice teachers' learning to enact RDM.

This study highlighted the effectiveness of varied approaches to introducing RDM among PSTs and novice teachers. Implementation challenges, particularly regarding time constraints and student engagement, necessitate systemic support and comprehensive training in RDM philosophies. Mathematics teacher leaders can better equip future educators to embrace RDM practices by (a) redesigning the intervention to emphasize depth of understanding through reading the longer book rather than only the shorter article; (b) fostering collaborative learning environments, so PSTs and novice teachers experience RDM as learners and providing opportunities for immediate enactment of RDM in a classroom after reading about it; and (c) addressing implementation barriers. This approach to teacher preparation has promise for cultivating classrooms that prioritize growth, learning, and resilience in hopes of leading to improved mathematical understanding and confidence among students. The goal of mathematics teacher–leaders moving forward should be to create an educational ecosystem that nurtures both teachers and students, fostering an environment where every learner feels empowered to engage deeply with mathematics, learn from their mistakes, and develop a love for learning that lasts a lifetime.

Conclusion

This study explored PSTs' and novice teachers' perceptions of RDM and its potential impact on student beliefs and actions. Findings suggest the minimal intervention of exposure to

the RDM approach through reading an article (Jansen et al., 2016) can catalyze changes in PSTs' perceptions. After implementing RDM practices, PSTs continued to view RDM as a promising approach to address longstanding challenges in mathematics education, such as student disengagement, anxiety, and underperformance. By creating a learning environment where mistakes, unfinished thinking, and ideas in progress are accepted, PSTs perceived that RDM fostered a comfortable and engaging atmosphere for students to participate in mathematical discourse. The RDM approach aligns with principles of a growth mindset, encouraging students to view their abilities as malleable and capable of development through effort and practice. Results also revealed the teacher plays a critical role in facilitating RDM practices effectively. PSTs and novice teachers believed adopting a nonevaluative stance, highlighting strengths in students' drafts, and inviting them to revise their thinking were key strategies for supporting meaningful mathematical discourse and learning.

However, the PSTs' and novice teachers' responses also revealed potential challenges in implementing RDM, such as engaging all students in sharing their reasoning, providing constructive feedback on incomplete work, and eliciting meaningful self-reflection from students. Practical constraints, such as time limitations and curriculum transitions, may also influence the feasibility of adopting RDM practices. Despite these challenges, the potential benefits of RDM in improving student engagement, confidence, and learning in mathematics warrant further exploration and support. Results revealed reading and discussing the full book (Jansen, 2020) enhanced novice teachers' knowledge, skills, and confidence. PSTs and novice teachers also expressed a desire for support in implementing the approach. Ongoing PD and collaboration among teachers could help address the identified challenges and facilitate the effective integration of RDM into classroom practices. Future research could examine the long-term impacts of RDM on student outcomes, such as academic achievement, mathematical confidence, and attitudes toward the subject. Investigating the strategies and classroom practices that support the successful implementation of RDM also could provide valuable insights into teacher education and PD. This study builds on prior literature by highlighting the effectiveness of tailored approaches to introducing RDM among PSTs and novice teachers while addressing challenges hindering implementation. By expanding PD to emphasize depth of understanding through texts, fostering collaborative learning experiences, and providing immediate opportunities for classroom enactment, mathematics education leaders can better support educators in adopting RDM practices.

This work contributes to an emerging vision for mathematics education—one that prioritizes teacher agency, student engagement, and equitable practices to create vibrant learning environments for all. In conclusion, RDM offers a promising approach to address the multifaceted challenges that mathematics educators face. By creating a safe and supportive learning environment, fostering mathematical discourse, and promoting a growth mindset, RDM has the potential to engage students, build their confidence, and enhance their learning experiences in mathematics classrooms. We hope the analysis of how minimal interventions supported PSTs' and novice teachers' learning provides insight for other mathematics teacher leaders who want to support teachers with enacting RDM.

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