VOLUME 26 | ISSUE 1| FEBRUARY 2025

NCSM JOURNAL

OF MATHEMATICS EDUCATION LEADERSHIP

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EXAMINING DISTRICT MATHEMATICS LEADERS' SUPPORT FOR SCHOOL-BASED MATHEMATICS COACHES

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ABSTRACT

Mathematics coaching differs significantly from mathematics teaching, and many coaches transition to the role directly from teaching with limited opportunities to learn to work effectively with teachers. Although coach professional development can provide one source of support for coaches' learning, coaches might also benefit from close work with other accomplished facilitators of teachers' learning, such as district mathematics leaders. This study analyzed interviews with 15 district mathematics leaders to understand whether and how they supported school-based mathematics coaches. We found 13 of 15 leaders worked closely with coaches to support them, and we identified seven ways they did so (e.g., classroom visits with coaches). Our findings have significance for research on district leadership and district leaders' support for coaches.

Keywords: mathematics coaching, professional development, district leadership

Teaching that supports students' attainment of rigorous mathematics learning goals is highly challenging, complex work that involves, among other things, eliciting, attending to, and making use of students' reasoning (Lampert et al., 2010; National Council of Teachers of Mathematics, 2014). This kind of teaching differs significantly from the kind of instruction often seen in many U.S. mathematics classrooms (Schoenfeld, 2022; Stigler & Hiebert, 2009), meaning many mathematics teachers will require support for their learning if they are to teach in ways that can support students' mathematics learning. Schools and districts in the United States frequently support teachers by employing mathematics coaches, who work directly with teachers to help them improve their teaching and thus students' learning (Kraft & Hill, 2020; Russell et al., 2020).

Mathematics coaches are often hired because of their experience and prior successes as teachers (Chval et al., 2010). Yet, the work of mathematics coaching differs significantly from that of mathematics teaching (Kane & Saclarides, 2023; Saclarides & Kane, 2023), and mathematics coaches often transition to the role directly from the classroom with limited opportunities to learn the coaching-specific knowledge, perspectives, and practices necessary to support teachers' learning effectively (Stein et al., 2022). Many mathematics coaches therefore require support for their *own* learning if they are to provide teachers with the quality of coaching that can support improvements in mathematics teaching and learning (Kane & Saclarides, 2023; Saclarides & Kane, 2023).

One common type of support for coaches is pull-out professional development (PD), which we consider off-site coach PD that takes coaches from classrooms and schools to participate in self-contained training sessions or courses (Kane & Saclarides, 2022; Stein et al., 2022). However, mathematics coaches might also benefit from more individualized support through collaborative work with other accomplished facilitators of mathematics teachers' learning. District mathematics leaders (DMLs) are educators who bear significant responsibility for mathematics learning and teaching in a district (Bolyard & Baker, 2024) and who often design and facilitate PD for teachers. Because of their assumed expertise in math instruction and facilitating PD for mathematics teachers, DMLs may be uniquely positioned to work closely with school-based mathematics coaches to aid coaches in supporting teachers' learning. Yet, it is unclear how common it is for DMLs to work directly with school-based mathematics coaches and what that work might look like, especially considering the wide variation in district contexts (e.g., size, demographic makeup, distance to city center) across the United States.

In this paper, we report on a study examining DMLs' perceptions regarding the support they provide—or do not provide—to mathematics coaches. We share findings related to DMLs' perceptions of the percentage of time they work directly with school-based mathematics coaches. We also report on the ways in which DMLs reported to have interacted with school-based mathematics coaches. Because, as we discuss later, such interactions could constitute coach learning opportunities, their identification marks a key step toward a broader research agenda focused on DMLs' efforts to support school-based mathematics coaches' learning.

CONCEPTUAL GROUNDING

Mathematics Coaching

Mathematics coaching is a form of job-embedded support for mathematics teachers' learning that is becoming increasingly common in U.S. schools and districts (Kraft et al., 2018). Mathematics coaches, who are intended to be accomplished mathematics educators, work closely with teachers on activities central to the work of teaching (e.g., planning for lessons, implementing instructional activities, analyzing students' work). The primary goal of mathematics coaching is to support teachers in developing the effective and equitable instructional practices necessary to support all students' mathematics learning (Kraft & Hill, 2020; Russell et al., 2020). The conceptual rationale for mathematics coaching is based on the notion that working with an accomplished colleague on activities relevant to one's work can support professional learning (Lave & Wenger, 1991). Empirically, research has shown that coaching can support teachers' development when it is sustained and coaching interactions are of high quality (Gibbons & Cobb, 2017; Russell et al., 2020; Saclarides & Munson, 2021).

Prior research on coaching has identified coaching activities that can support teachers' learning (Gibbons & Cobb, 2017). These potentially productive coaching activities include, for example, modeling instruction, coteaching, and conducting one-on-one coaching cycles with teachers (Russell et al., 2020; Saclarides & Munson, 2021). Yet, the learning potential of these activities depends on when and how coaches enact the activities with teachers (Gibbons & Cobb, 2016). Further, the learning potential of coaching writ large depends, in part, on the extent to which coaches and teachers have opportunities to engage in sustained interactions together (Blazar & Kraft, 2015). We can conclude effective coaching requires coaches to develop, among other things, expertise in facilitating potentially productive coaching activities and in navigating schooling contexts to create sustained opportunities to engage teachers in such activities.

Yet, many mathematics coaches transition to the coach role directly from the classroom with limited opportunities to develop such coaching-specific expertise before starting their work with teachers (Stein et al., 2022). The transition from teacher to coach requires a significant adjustment in professional identity, skill set, and approach (Gallucci et al., 2010). Coaches must navigate complex interpersonal dynamics, build trust and credibility, and adapt to new challenges in working with adult learners. Chval et al. (2010) emphasized that addressing these struggles requires systemic support, clear role definitions, and ongoing PD for coaches. It is therefore important for mathematics coaches to have opportunities to develop the expertise necessary to maximize coaching's potential and support mathematics teachers' learning (Kane & Saclarides, 2023; Saclarides & Kane, 2023; Stein et al., 2022).

Supporting Mathematics Coaches' Learning

Although several recent studies of coaching have examined the learning opportunities that can arise when coaches engage in traditional, pull-out PD (Kane & Saclarides, 2022; Saclarides & Kane, 2023; Stein et al., 2022), few studies have examined other types of professional learning activities that can support mathematics coaches beyond traditional pull-out PD. Coaches are likely to benefit from job-embedded supports that situate coaches' learning in their own contexts (Kochmanski & Recore, 2024), just as teachers benefit from similar forms of support (e.g., coaching and professional learning communities [PLCs]; Cobb et al., 2018). For example, mathematics coaches might benefit significantly from close work with other accomplished facilitators of mathematics teachers' learning, just as teachers can benefit greatly from working with other educators. Put another way, accomplished facilitators of mathematics *teachers*' learning could support coaches' learning, just as coaches (who are assumed to be accomplished teachers of students) serve in a similar capacity with teachers.

As noted previously, DMLs are education professionals who frequently design and facilitate PD for teachers (Jackson et al., 2015). Because many DMLs routinely facilitate PD for teachers, they likely have developed into accomplished facilitators of teachers' learning. Consequently, it is worth exploring whether and to what extent DMLs serve in a mentorship capacity for school-based coaches. However, the coaching literature provides little guidance on whether DMLs see working with coaches as a core function of their role nor on the amount of time they might work with coaches—if they do at all. Further, it is an open question as to *how* DMLs might work to support coaches' learning, if at all. Understanding whether, the extent to which, and how such leaders can support mathematics coaches' learning can provide greater clarity regarding how coaches can be supported to develop the coaching-specific expertise necessary to support mathematics teachers' learning.

Research Questions

The following questions informed our investigation of DMLs' work with school-based mathematics coaches:

1. What percentage of their time do DMLs perceive to be spent in support of school-based mathematics coaches?
2. In what ways do DMLs interact with school-based mathematics coaches, if they do at all?

METHODS

Study Context

We investigated our research questions in the context of an ongoing, larger research project funded by the National Science Foundation. The project aims to understand whether and how the design and use of professional learning resources can support large-scale improvements in mathematics teaching and learning across a state in the southeastern United States. A significant conjecture of the project is that educators at all levels of systems (e.g., teachers, school-based coaches, school-based administrators, district leaders) will benefit from collaborative efforts to design solutions to common problems of practice related to district-wide instructional improvement initiatives. In line with this conjecture, the project encourages actors at different levels of school and district systems to collaborate, making it a best case context to investigate whether and in what ways DMLs interact with school-based mathematics coaches.

As part of the broader project, researchers conducted 28 semistructured interviews with mathematics instructional leaders across the focal state. The interviewed instructional leaders included mathematics educators working in various roles (e.g., school-based mathematics coaches, teacher leaders who split their time between classroom teaching and other leadership activities, and DMLs). Interviews included questions designed to clarify with whom the instructional leaders worked in their contexts, the nature of that work, and the goals of their work. Consequently, DMLs' interview responses had the potential to provide rich information regarding their work with school-based mathematics coaches, thereby enabling us to answer our research questions.

Participants

Fifteen of the 28 instructional leader interviews were conducted with DMLs. These 15 participants accounted for our study sample. Though formal DML titles varied, all participating leaders stated they worked across schools, were employed primarily by a district, did not report to a principal, and were responsible for mathematics learning and teaching in the district. As shown in Table 1, participating DMLs worked in districts of varying sizes and locations within the focal state, with some leaders working in smaller, rural districts and others working in large, urban school districts. For this paper and to maintain participant anonymity, we considered a small district to be any district serving fewer than 15,000 students, a midsized district to be any district serving more than 15,000 students but less than 100,000 students, and a large school district to be any district serving over 100,000 students. We also included the per-student expenditures in Table 1. The per-student expenditures ranged from a low of just over $11,000 per student to a high of over $18,000 per student, and we classified them into three categories: (a) low, which was anything less than $13,000; (b) medium, which was anything greater than $13,000 and less than $15,000; and (c) high, which was anything over $15,000. Together, the 15 DMLs represent a variety of contexts in which DMLs work and enabled us to explore our research questions. All DMLs were part of a statewide initiative to bring together mathematics educators at different system levels to support large-scale instructional improvement in mathematics. Consequently, it was highly likely that the 15 participants would interact directly with school-based coaches in some capacity.

Table 1
Participants' Demographics

District leader	District type	District size	Expenditure per student
Karen	Rural	Small	Low
Jessica	Suburban	Mid-sized	Medium
Celeste	Suburban	Small	Medium
Jasmine	Urban	Mid-sized	Medium
Grace	Urban	Mid-sized	Medium
Alice	Urban	Large	Low
Greg	Urban	Large	Low
Will	Urban	Large	Low
Sasha	Urban	Mid-sized	High
Mary	Urban	Large	Low
Mabel	Rural	Small	Low
Scarlett	Rural	Small	Low
Quinn	Rural	Small	High
Mia	Rural	Small	Low
Nancy	Rural	Small	Medium

Note. Names are pseudonyms.

Data Collection

The primary data for this study were semistructured interviews with DMLs. Semistructured interviews were appropriate for this study because this type of interview enabled us to be responsive to ideas we heard from DMLs and thus press them to elaborate on and further explain their responses to the interview questions in an organic way. This approach ensured we collected rich data on their thoughts, beliefs, and perceptions with regard to their roles and work with educators in their district, including mathematics coaches. Each interview lasted roughly 60 minutes and was conducted by a research team member. Interviewers went through a 1-hour training session in which the designers of the semistructured interview protocol (see appendix) outlined the purposes of each interview question, shared sample prompts to elicit interviewees' thinking further, conducted a mock interview as a model, and reserved time for questions. Interviews were conducted virtually and in person, and we recorded all interviews for later analysis. We conducted one interview with each of the 15 DMLs. Interviews occurred over two rounds of data collection during the late fall to early winter timeframe. This timing was ideal because it meant DMLs had already begun their instructional improvement efforts in earnest; thus, they had enough time to begin working directly with school-based coaches. If we had conducted the interviews at the start of the school year, then DMLs might have had far fewer opportunities to begin their instructional improvement efforts, with far fewer types of interactions with school-based coaches on which to report. Conducting the interviews at the close of the year also may have led to DMLs forgetting the type of interactions they had with coaches due to the intense push toward statewide testing.

Data Analysis

We answered our two research questions in turn. To answer our first research question, which focused on the percentage of time DMLs spent supporting school-based mathematics coaches, we analyzed DMLs' responses to two interview questions from the semistructured interviews. The first relevant prompt was: "With whom do you work closely?" The second relevant prompt was: "What percentage of your time do you spend working with teachers/other teachers/other leaders to improve instruction?" Notably, we did not provide DMLs with types of leaders to consider or percentage ranges to choose when answering the second relevant question. However, interviewers were encouraged to ask follow-up prompts that pressed DMLs for specificity in their responses. If, for example, a DML responded to the initial question by explaining that they typically spend 50% of their time working directly with educators in schools, the interviewer might press the DML to clarify the specific role groups the DML worked with and the percentage of time for each role group. In this example, the interviewer might ask follow-up questions such as: "Can you tell me a bit more about the specific groups of educators you work with in schools?" and "About what percentage of time do you work with those specific groups?" We recorded whether DMLs reported working directly with coaches and, if so, what percentage of their time they reported doing so.

Next, we answered our second research question, which focused on the ways in which DMLs interacted directly with school-based mathematics coaches. We analyzed those interviews in which DMLs reported working closely with school-based mathematics coaches. We included all DMLs who stated they worked with coaches, regardless of the percentage of time they reported.

To identify the types of interactions DMLs had with coaches, we analyzed DMLs' responses to three loosely related interview prompts intended to clarify what they do in support of large-scale instructional improvement. The first relevant prompt was: "Please walk us through a typical day in your role/position. What does it look like?" The second relevant prompt was: "What do you spend the rest of your time doing when not working directly with teachers? What does this work look like? Are there particular activities you do?" The third prompt was: "Who would you consider to be your community, if you feel you have one? Who participates in this community? How do you work in this community? What does this work look like?" Although the three questions did not ask DMLs to report directly on their work with school-based mathematics coaches, they provided extensive opportunities for the DMLs to explain and elaborate upon their daily work. Because this second step in our analysis focused only on the DMLs who self-reported working closely with school-based mathematics coaches, we surmised that, in describing their daily work, they would articulate how they interacted with those coaches.

We were unaware of a coding scheme for district leaders' work with coaches, so we used grounded methods (Corbin & Strauss, 2014) to analyze the DMLs' responses to interview questions. We first listened to each district leader's interview and marked relevant episodes in which the DMLs reported on or described their work with school-based mathematics coaches. We considered a relevant episode to begin when the DML started describing a specific aspect of their work with school-based coaches and to end when the DML shifted the topic of conversation to something else. For example, in one interview, a DML described several activities she did regularly across the district in response to the question, "What do you spend the rest of your time doing when not working directly with teachers?" We marked a relevant episode as the beginning when this DML began to describe the classroom observations she conducted frequently with school-based coaches in her district. We considered the relevant episode to have ended when the DML changed the topic of conversation by discussing a different activity—in this case, coordinating additional support for currently struggling students.

The first and fourth authors then listened to relevant episodes in each interview and used inductive coding to characterize the types of interactions DMLs had with school-based mathematics coaches. For example, one DML said she was "in charge of the summer coaching academy," which she described as PD "for [school-based] coaches" in the district. She described setting the agenda for the academy, designing sessions, and facilitating sessions for coaches. We used the code "design and facilitate coach PD" for this interaction. This same DML reported that she "collaborates with coaches"

to "design and lead PD for teachers." For this interaction, we used the code "prepare and cofacilitate teacher PD with coaches." The first and fourth authors coded all relevant episodes separately and met to reach a consensus on the codes. This process ensured consistency in our coding scheme application, which was appropriate given the limited number of cases we analyzed in the study.

Having conducted an initial round of inductive coding, we looked across codes to identify broader themes in the types of interactions DMLs had with coaches. For example, we noticed many DMLs referenced the activity of visiting classrooms to observe instruction with school-based coaches. However, they used a variety of terms to describe this type of interaction, including "learning walks," "visiting teachers' classrooms together," and "observing teachers." We classified each of these codes under the theme "classroom visits with coaches." We then applied these themes to each episode we coded, thereby accounting for the *types* of interactions each DML in our study reported with school-based mathematics coaches.

Finally, we wrote an analytic memo (Lempert, 2007) documenting the types of interactions each DML reported with their school-based coaches. In this memo, we focused on whether DMLs mentioned the type of interaction in the interview, not on the number of episodes in which each DML mentioned the type of interaction. This focus on type of interaction was because we were interested in the range of interactions DMLs might have with coaches, not how often they referred to those interactions in their interviews. This approach enabled us to see how many DMLs in our sample reported working with school-based coaches in the ways we identified over the course of our analysis. In other words, this memo enabled us to count the number of DMLs who had specific types of interactions with coaches.

Findings

We report our findings in two sections. First, we share findings regarding the percentage of time DMLs reported working with school-based mathematics coaches—if they reported to do so. In doing so, we answer our first research question. We then turn to our second research question and report on the ways in which DMLs reported interacting with school-based mathematics coaches.

Percentage of Time in Support of School-Based Mathematics Coaches

Table 2 shows that 13 of 15 interviewed DMLs self-reported working closely with school-based mathematics coaches. The two DMLs who did not work closely with school-based mathematics coaches were in small, rural districts that did not employ school-based mathematics coaches. All DMLs in districts employing school-based mathematics coaches spent some portion of their time interacting directly with those coaches.

Table 2
Percentage of Time Working With School-Based Mathematics Coaches

District leader	District type	District size	Expenditure per student	Works with mathematics coaches	Percentage of time with mathematics coaches
Karen	Rural	Small	Low	No	0
Jessica	Suburban	Mid-sized	Medium	Yes	50
Celeste	Suburban	Small	Medium	Yes	50–60
Jasmine	Urban	Mid-sized	Medium	Yes	50
Grace	Urban	Mid-sized	Medium	Yes	Not shared
Alice	Urban	Large	Low	Yes	40
Greg	Urban	Large	Low	Yes	50
Will	Urban	Large	Low	Yes	20–30
Sasha	Urban	Mid-sized	High	Yes	Not shared
Mary	Urban	Large	Low	Yes	Not shared
Mabel	Rural	Small	Low	No	0
Scarlett	Rural	Small	Low	Yes	75
Quinn	Rural	Small	High	Yes	25
Mia	Rural	Small	Low	Yes	35–40
Nancy	Rural	Small	Medium	Yes	33

DMLs reported a wide range in the percentage of their time devoted to working with coaches. For example, Will reported spending 20%–30% of his time working closely with school-based mathematics coaches. On the other hand, Scarlett reported spending nearly 75% of her time working to support school-based mathematics coaches. Interestingly, those DMLs who spent a third or more of their time working with school-based mathematics coaches stated they did so because they thought their work with school-based coaches would have a greater impact on teachers and students than working directly with individual teachers in individual schools. In other words, they viewed working closely with the coaches, who would then work closely with teachers, as important. As Scarlett put it, she cannot be in every school at once, so a major part of her job is "to build the capacity of [her] coaches," so they can "work effectively with teachers," which suggests that, for some DMLs, a significant portion of their work in schools may be devoted to supporting school-based mathematics coaches or instructional leaders in learning to support teachers better.

Types of Interactions Between DMLs and School-Based Mathematics Coaches

We identified seven ways in which DMLs in this study interacted directly with school-based mathematics coaches: (a) facilitating PD for coaches, (b) engaging in strategic planning with school-based coaches, (c) providing individualized support for coaches in conducting one-on-one coaching with teachers, (d) visiting classrooms with coaches, (e) training coaches to deliver district PD at their schools, (f) preparing and cofacilitating PD for teachers, and (g) cofacilitating PLCs with coaches. Table 3 summarizes our findings and shows the types of interactions each DML self-reported having with school-based mathematics coaches. Interestingly, no DMLs reported having all seven types of interactions with school-based mathematics coaches. The most common interactions were visiting classrooms with coaches (n = 10), facilitating pull-out PD for coaches (n = 7), and cofacilitating PLCs with coaches (n = 7). Next, we describe each type of interaction the DMLs in this study reported having with school-based mathematics coaches, following the order in which they are listed in Table 3. In describing each type of interaction, we also provide examples from our interviews with DMLs.

Facilitating PD for Coaches

Seven of the 15 DMLs reported facilitating traditional pull-out PD for school-based mathematics coaches in their district. As the DMLs described, this PD involved coaches leaving their school sites to attend working sessions at a centralized location, such as the district offices. DMLs cited several different goals for the coach PD. Many DMLs noted the coach PD focused on effective mathematics teaching practices. For example, Greg explained that he offered training for school-based mathematics coaches focusing on "things they should consider doing [from] a math lens as an instructional facilitator in their building." He also has offered "progression training" to coaches, where he has helped coaches understand the progression of standards and the

Table 3
Interactions With School-Based Mathematics Coaches by District Mathematics Leader

	Types of interactions with mathematics coaches						
District leader	**Facilitating PD for coaches**	**Engaging in strategic planning**	**Providing individualized support for coaches**	**Visiting classrooms with coaches**	**Training coaches to deliver district PD**	**Preparing and cofacilitating PD for teachers**	**Cofacilitating PLCs with coaches**
Karen							
Jessica	✓	✓				✓	
Celeste	✓	✓	✓	✓			
Jasmine	✓			✓			✓
Grace	✓			✓	✓		
Alice				✓	✓	✓	✓
Greg	✓						✓
Will			✓				
Sasha			✓	✓			✓
Mary			✓	✓			✓
Mabel							
Scarlett		✓		✓		✓	✓
Quinn	✓			✓	✓		✓
Mia	✓			✓		✓	
Nancy				✓		✓	
Total	7	3	4	10	3	5	7

progression of mathematical ideas students are expected to learn in certain grade bands.

Other DMLs explained that the PD they provided to coaches was intended to give them opportunities to connect and discuss common problems of practice they could then work to address collectively. Mia, for example, described the primary goal of the monthly PD she facilitated for mathematics coaches as getting the coaches to talk to one another, so they feel less alone in doing their coaching work. In her interview, she explained:

> *I meet with [the coaches] once a month…We try to get them to talk to each other, that's the big thing. Because they're almost always the only [coach] at their school. When they can reach out and talk to the other [coaches] that makes them, they can flourish…So we do problems of practice with them. So that's huge for [the coaches] because, like, meetings are hard. They don't like being pulled out of their schools. So [we ask them], bring a problem that you're having in your school, and we can all solve it together. So, sort of like brainstorming…and it doesn't have to be a fire, but what can we do to sort of like address this [problem] in a structured way, in a way that makes sense.*

Finally, two DMLs noted they led PD focused on supporting coaches in developing effective coaching practices. The relatively limited number of DMLs who focused coach PD on *coaching* practices aligns with prior research indicating PD for mathematics coaches often focuses on mathematics and the teaching of mathematics as opposed to the knowledge and practices specific to coaching (Saclarides & Kane, 2023). In both cases, DMLs reported the PD focused on how coaches can lead coaching cycles or PLCs effectively with teachers. Celeste, for example, explained that she led PD for coaches that focused on how to "go into a PLC and help a PLC plan through a launch, explore, discuss [lesson]." She explained that she has worked with the coaches to "think about what that [kind of work] looks like on a daily basis with PLCs."

Engaging in Strategic Planning With Coaches

Three of the 15 DMLs reported engaging in ongoing strategic planning with school-based mathematics coaches. For all three DMLs, this involved meeting with school-based coaches to determine which teachers in the building the coach would support directly to maximize their impact. Scarlett, for example, described how she has worked with coaches to bring together student achievement data and data on instruction to determine which teachers need direct support from coaches. Scarlett shared:

> *One visit could be that we look at student data together, and then formulate next steps based on data. One visit might be that I do learning walks with them, we try to do nonjudgmental data collection. And then we come back, we triangulate our data between the lesson plans, and what they have talked about in PLCs, with the teachers, and then we come forward with if that teacher needs more support.*

As another example, Jessica described working with coaches to identify teachers in their buildings who were likely to stay at the school longer term, so coaches could prioritize working with those teachers. Jessica argued this step was worth doing because "coaches can be more impactful" when they establish "longer standing relationships" with teachers.

Beyond thinking strategically with school-based coaches about which teachers they should support, Alice reported working with coaches to analyze student achievement data. She explained that data are a big deal in her district and are "only getting bigger" as they "head through to the end of the year." Because of this impact, she described meeting with the school-based coaches to discuss "formative assessments" and "how they can use [data]," including how the coaches "can identify what needs to happen" to get teachers where they want them. These conversations were intended to help coaches think strategically about the different teacher supports they implemented in their buildings.

Providing Individualized Support for Coaches

Four of the 15 DMLs reported providing individualized support to school-based coaches beyond strategic planning. All four DMLs explained that they provided side-by-side support to coaches as they interacted with teachers and met with coaches afterward to discuss their decisions. Nancy, for example, explained that she sometimes has worked "side-by-side" with coaches to plan for and lead PLCs. Afterward, she met with them to debrief the PLC and discuss how it went. She explained that her goal was to figure out what she could "do to further support what [the coach] is trying to do to move [the teachers] forward." Like Nancy reported providing side-by-side support during PLCs, Alice reported going to schools with coaches to conduct coaching cycles with them. Her goal in doing so was to help the coach identify what "the next steps are" and then figure out the appropriate "bite sized pieces that can move instruction forward."

Two DMLs who provided individualized support to school-based coaches explained that this support was sustained in nature. For example, Celeste noted:

> *I try to meet with [new coaches] on a weekly basis in the beginning of the year and then eventually move to just every 2 weeks. A lot of that [early work] is problem solving. Like, I sent this email [to a teacher], and it did not go very well. I'm like, "Okay, well, let's talk about why we should have reworded that." . . . So I do a lot of [early work] to help them navigate the coaching world.*

In contrast, the other two DMLs reported providing individualized support only in response to one-off requests from coaches or principals. For example, Will explained that he provides individualized support to coaches, but it usually "ends up being more one off" and a response to a "question about something" he can answer.

Visiting Classrooms With Coaches

Ten of the 15 DMLs we interviewed conducted classroom visits with coaches, making this type of interaction the most common DMLs in this study had with school-based coaches. DMLs reported that these classroom visits typically involved the DML and school-based coach doing observations of

several mathematics classrooms in the school, with the goal of understanding the current state of instruction in the building. As a brief clarifying note, we distinguish classroom visits from strategic planning because of the depth of conversation described. DMLs reported that classroom visits typically resulted in brief conversations in which they supported coaches regarding potential improvement goals for the teachers they had seen. In contrast, DMLs who engaged in strategic planning with their coaches described this work as involving in-depth conversations in which the coach and DML discussed school-wide and teacher-specific improvement goals.

Jasmine's interview illustrates this type of interaction. In her interview, she reported that she would often "go to a school in the morning and a school in the afternoon" to see instruction. During these visits, she would "sit and observe some classrooms with the math coach." For Jasmine, this was a beneficial interaction because it meant she could support the mathematics coach in learning what to look for in the classroom to see whether teachers are implementing district curriculum effectively. DMLs also noted meeting with coaches frequently after conducting classrooms visits to determine next steps for teachers whose instruction they had observed.

Training Coaches to Deliver District PD

Three of the 15 DMLs also support school-based coaches by training them to deliver district-provided PD sessions at their schools. All three coaches described this work as following a train-the-trainer model, wherein they support the coaches in learning to deliver teacher PD sessions originally developed by the district. For example, Alice described leading PD that has "been a kind of train-the-trainer model" where school-based coaches learn something new about mathematics teaching, "and they take that learning back to the teachers." As another example, Quinn reported designing PD for teachers and then "training [coaches] to actually implement" PD sessions with teachers over the summer. This focus on learning to deliver specific PD sessions contrasts with designing and facilitating coach PD; the latter focuses primarily on supporting coaches to improve their capacity to work effectively with teachers, not on learning to lead a specific PD session designed for teachers.

Preparing and Cofacilitating PD for Teachers

Four DMLs reported working with coaches to support them in designing and facilitating school-based PD for teachers. Unlike the prior activity, which focuses on training school-based coaches to lead district-designed PD, this type of interaction focuses on supporting coaches in developing and leading their own PD sessions that are responsive to the teachers in coaches' current school contexts. DMLs who interacted with coaches this way noted interactions usually involved meeting with the coach to develop PD activities and then joining the PD session to provide the coach with added support. For example, Mia explained she has often worked with school-based coaches to "tailor" school-level PD experiences to teachers' current practices. As another example, Scarlett reported meeting with coaches "if they're doing any PD" so she can "support them as much as possible." She explained that this support often involved planning the PD together, and then she would attend the PD to see how it went.

Cofacilitating PLCs

Seven of the 15 DMLs joined their school-based coaches in their buildings to cofacilitate PLCs between teachers, making this one of the most reported activities. DMLs cited several reasons for cofacilitating the PLCs. Some DMLs noted that their presence in PLCs was intended to provide the coaches with a visible show of support. Others noted joining PLCs to support coaches and teachers in conducting in-depth analyses of student-level and instructional data. For example, Scarlett explained that, just a few weeks before her interview, she visited a school struggling in fifth-grade math. On this visit, she "worked with the fifth-grade math team" and led the PLC in "looking at benchmark data" by "grade level and standard." For this visit, the "coach was there 100% of the time and was engaged" in the activities. Scarlett explained that she led the PLC through an "item analysis" where they "looked at the type of problem that was most frequently missed." Others noted joining PLCs to ensure consistency in messaging and feedback from the district to the school administration to the school-based coach. Overall, all seven DMLs who mentioned this activity appeared to see the cofacilitation of PLCs as both supporting coaches and directly supporting teachers' learning, meaning it often served two parallel purposes.

Discussion

This study had two primary goals. First, we aimed to better understand whether DMLs see the support of school-based mathematics coaches as a primary component of their jobs. Of the 15 DMLs we interviewed, 13 noted that they saw the support of school-based mathematics coaches as a key component of their job function. Second, we aimed to clarify the ways in which DMLs interacted directly with school-based mathematics coaches, if they did so. Our rationale for pursuing this latter goal was that clarifying these types of interactions serves as an initial step toward a greater understanding of how DMLs might work to support school-based mathematics coaches' learning. We identified seven types of interactions that DMLs in this study had with school-based mathematics coaches.

Our findings surface several key issues of significance for research on mathematics coaching and supporting mathematics coaches. First, we found most DMLs saw the support of school-based mathematics coaches as a component of their work. This finding suggests DMLs often interact directly with school-based mathematics coaches, meaning these educators have the potential to support school-based mathematics coaches' learning. This finding is significant for research on coaches' learning (Kane & Saclarides, 2022; Saclarides & Kane, 2023; Stein et al., 2022) because it highlights an additional source of support for coaches beyond traditional, pull-out PD that could aid in coaches' development. It is also significant for research on DMLs—it clarifies an often-nebulous role by detailing a key component of DMLs' daily work. Our study indicates DMLs devote considerable time and energy to working closely with school-based coaches, in addition to other common activities (e.g., designing and facilitating PD for teachers; Jackson et al., 2015).

Second, we found considerable variation in the percentage of time DMLs reported working with school-based mathematics coaches. When we began our study, we suspected DMLs working in larger, relatively well-resourced districts might have more opportunities to work closely with school-based coaches because there might have been more available coaches with whom to work and thus more opportunities to support their learning. We also suspected DMLs working in smaller, less resourced districts might spend less time supporting coaches because there might be fewer coaches and more responsibilities for the district leader. The evidence in support of our conjecture is mixed. On one hand, the two DMLs who reported that they did not work closely with school-based mathematics coaches came from smaller, more rural school districts. Further, 4 of the 5 DMLs who reported spending over 50% of their time supporting school-based coaches worked in urban or suburban school districts. These findings lend support to our early suspicions. On the other hand, the DML who reported spending the greatest percentage of time supporting school-based mathematics coaches worked in a small, rural school district. This finding suggests other factors may influence district leaders' support of school-based coaches.

Third, by identifying the ways in which the DMLs we interviewed interacted with school-based mathematics coaches, we took steps to better understand how DMLs might support school-based mathematics coaches' learning. Because of the nature of this study, we were unable to determine the extent to which the seven types of interactions we identified supported coaches' learning. However, all seven interactions we identified appear to have the potential to support school-based mathematics coaches' learning. That said, we also recognize their potential is largely contingent on the nature of the interaction. For example, working with coaches to prepare and cofacilitate PD might constitute a significant learning opportunity for a school-based coach *if the DML supports the coach in seeing the codesign and cofacilitation experience as a case from which to learn.* This co-facilitation might involve the DML holding framing conversations before and after the collaborative experience in which the DML presses and supports the coach to identify principles of effective PD design and facilitation that the coach might then take up when developing other PDs for teachers.

In contrast, this type of interaction might have limited potential for supporting coaches' learning if the DML approaches it from a "helping hands" perspective and focuses exclusively on designing and facilitating the PD without discussing what the coach learned from the experience. Because the types of interactions we identified describe what DMLs might do when working with teachers, we suggest they can serve as the initial basis for the delineation of a topology of how DMLs can support school-based mathematics coaches' learning. Developing and validating such a topology would be a highly beneficial step forward for research examining coaches' learning (Stein et al., 2022) and the design of *systems* of support for coaches' learning (Kochmanski & Recore, 2024).

Finally, regarding implications for practice, we see the identification of the seven types of interactions as beneficial for DMLs in other districts who spend a significant percentage of their time working closely with school-based mathematics coaches. We suggest such DMLs might find the types of interactions informative for their work, as we described possible interactions DMLs might aim to have with coaches. If any types of interactions prove new or novel, DMLs might try them out with school-based coaches with whom they work.

Limitations and Directions for Future Research

Though this study provided valuable insights into how DMLs support school-based mathematics coaches, there were several limitations. First, the findings were based on interviews with 15 DMLs, which may not capture the full diversity of roles, experiences, and responsibilities of DMLs across varied districts and contexts. Further, the interviews were part of a larger study, and the interview had other foci (e.g., what DMLs perceived to be high-quality mathematics instruction). Second, although the study identified seven types of interactions between DMLs and coaches, it did not assess the impact of these interactions on the learning or professional growth of the coaches, focusing instead on potential rather than verified outcomes. Third, the study did not investigate whether the seven interaction types are exhaustive or if other significant interactions are not in this data set, which would require additional interviews in other contexts, including DMLs working in different states. Fourth, although we acknowledge the effectiveness of these interactions likely depends on their quality, this study did not attend directly to the quality of interactions due to the nature of the data we analyzed. Fifth, while we acknowledge systemic factors (e.g., district size and resources), the study did not explore how broader organizational structures or leadership practices influence DML–coach interactions, leaving the role of district-level policies and priorities underexamined. These limitations highlight the need for further research to understand comprehensively how DMLs effectively support school-based mathematics coaches and the systemic factors that shape their interactions.

Turning now to directions for future research, we suggest researchers might investigate the coach learning potential of the types of activities identified in this study. Our current work is descriptive, and due to the available data, we could only analyze interviews with DMLs in which they described how they worked with coaches. Future research might collect data on district leaders' and coaches' interactions as they engage in activities described previously to look closely at whether the activities can give rise to coach learning opportunities. Researchers also might look closely at the kinds of expertise necessary for DMLs to facilitate these activities effectively with coaches, such that they support coaches in learning to support teachers better. For example, just as it is useful to understand when and why coaches choose to enact coaching activities with teachers (Gibbons & Cobb, 2016; Kochmanski & Cobb, 2023; Witherspoon et al., 2021), it may be similarly useful to look closely at when

and why DMLs choose to engage school-based coaches in particular types of support. Finally, as noted, we initially thought school districts' size and available resources might have influenced the amount of time district leaders worked with school-based coaches; however, the size of the district did not appear to explain discrepancies in the percentage of time spent working with coaches for this subset of 15 district leaders. Future research might build on this analysis by investigating explanations for the differences we observed in how much time district leaders devoted to supporting school-based coaches' learning and in how they went about working with coaches.

Conclusion

Mathematics coaching is an increasingly common strategy for supporting improvements in teaching and learning. The transition from teacher to coach is significant and requires new and novice coaches to develop new forms of knowledge and practice (Stein et al., 2022). As education professionals who often design and implement professional learning experiences for teachers, DMLs can serve as facilitators of coaches' learning. In this study, we found most DMLs we interviewed devoted at least a portion of their workday to supporting the learning of school-based mathematics coaches. We also found seven types of activities that DMLs reported enacting with coaches to support their learning. These results suggest DMLs have an essential role in the support and success of school-based mathematics coaches.

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Acknowledgments and Declarations
The authors have no conflicts of interest on which to report.

The authors wish to thank Olu Adefope, Jenna Fernandez, Christine Fisher, Cathy Holl-Cross, Katherine Mawhinney, Lanre Oriowo, Catherine Schwartz, and Michelle Stephan for their collaboration on the larger research study and Allana Durst for assisting with data collection.

This report is based upon work supported by the National Science Foundation under grant numbers DRL – 2100895, DRL – 2100903, DRL – 2100833, and DRL – 2100947. Any opinions, findings, and conclusions or recommendations expressed herein are those of the authors and do not necessarily reflect the views of the National Science Foundation.

Appendix

Relevant Section of Semistructured DML Interview Protocol

PART C - DESCRIBE YOUR WORK

In this next part of the interview, we would like to better understand what it looks like for you to do your work as a a math instructional leader

Please walk us through a typical day in your role/position? What does it look like?

IF they say there is no typical day: Please talk us through the kinds of things you often do in your role/position.

IF they say they visit schools or classrooms: How do you decide which schools/classrooms to visit? Do you consult with anyone about this decision?

What percentage of your time do you spend working with teachers/other teachers/other leaders to improve instruction?

PROBE on working with teachers: What does this work look like? Are there particular activities you do when working with teachers/other teachers/other leaders (i.e., modeling, planning with teachers, etc.)?

What do you spend the rest of your time doing?

PROBE on this by asking: What does this work look like? Are there particular activities you do? Particular expectations for this other work?

PROBE on curriculum development/planning: What role do you have in curriculum planning? What does this look like? Do you have a say in decisions around school- or district-wide curriculum? Who else do you work with to make these decisions?

PROBE on teaching students in the classroom: Do you teach students in the classroom? If so, what percentage of your time focuses on this? How does this impact or influence your work with other teachers? Your work with curriculum?

PROBE on other things: What else haven't we asked about that you actually do?

PROBE on comparison to expectations: Imagine you are one of those memes, where it shows a list of what everyone thinks you do during your day, and then there is your box saying, "This is what I actually do." What do other people *think* you do during the day? Who are your "others"? How does this compare to what you actually do?

With whom do you work closely? Note that this can include people you support, people who support you, your boss, people who you see as peers, etc.

Who would you consider to be your community, if you feel you have one?

PROBE on specific communities: Who participates in this community? How do you work in this community? What does the work look like?